CBSE Class–12 Mathematics

NCERT solution

Chapter - 13

Probability - Exercise 13.1

1. Given that E and F are events such that P (E) = 0.6, P (F) = 0.3 and P (E$\cap$ F) = 0.02, find P $(E|F)$ and P$\left(F|E\right)$.

Ans. Given: P (E) = 0.6, P (F) = 0.3 and P (E $\cap$ F) = 0.2

$$\therefore \ P\left(E|F\right) = \frac{P(E \cap F)}{P(F)} = \frac{0.2}{0.3} = \frac{2}{3}$$

And $$P\left(F|E\right) = \frac{P(E \cap F)}{P(E)} = \frac{0.2}{0.6} = \frac{1}{3}$$

2. Compute P$\left(A|B\right)$ if P (B) = 0.5 and P (A$\cap$ B) = 0.32

Ans. Given: P (B) = 0.5, P (A $\cap$ B) = 0.32

$$\therefore \ P\left(A|B\right) = \frac{P(A \cap B)}{P(B)} = \frac{0.32}{0.50} = \frac{32}{50} = 0.64$$

3. If P (A) = 0.8, P (B) = 0.5 and P$\left(B|A\right)$ = 0.4, find:

(i) P (A$\cap$B)

(ii) P$\left(A|B\right)$

(iii) P (A$\cup$ B)

Ans. (i) P (A $\cap$ B) = P$\left(B|A\right)$. P (A) = 0.4 x 0.8 = 0.32

(ii) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.32}{0.50} = \frac{32}{50} = 0.64$

(iii) P (A $\cup$ B) = P(A) + P (B) – P (A $\cap$ B) = 0.8 + 0.5 – 0.32 = 0.98

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4. Evaluate P (A $\cap$ B) if 2P (A) = P (B) = $\frac{5}{13}$ and $P(A|B) = \frac{2}{5}$.

Ans. Given: 2 P (A) = P (B) = $\frac{5}{13}$, $P(A|B) = \frac{2}{5}$

$\therefore$ P (A) = $\frac{5}{26}$

Now P (A $\cap$ B) = $P(B|A)$. P (B) = $\frac{2}{5} \times \frac{5}{13} = \frac{2}{13}$

And P (A $\cup$ B) = P(A) + P (B) – P (A $\cap$ B) = $\frac{5}{26} + \frac{5}{13} - \frac{2}{13} = \frac{11}{26}$

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5. If P (A) = $\frac{6}{11}$, P (B) = $\frac{5}{11}$ and P (A $\cup$ B) = $\frac{7}{11}$. Find (i) $P(A \cap B)$, (ii) $P(A|B)$, (iii) $P(B|A)$

Ans. (i) P (A $\cup$ B) = P(A) + P (B) – P (A $\cap$ B)

$\Rightarrow \frac{7}{11} = \frac{6}{11} + \frac{5}{11} -$ P (A $\cap$ B)

$\Rightarrow$ P (A $\cap$ B) = $\frac{6}{11} + \frac{5}{11} - \frac{7}{11} = \frac{4}{11}$

(ii) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{4/11}{5/11} = \frac{4}{5}$

(iii) P $\left(B|A\right)$ = $\dfrac{P(A \cap B)}{P(A)}$ = $\dfrac{4/11}{6/11} = \dfrac{4}{6} = \dfrac{2}{3}$

6. Determine P $\left(E|F\right)$: A coin is tossed three times.

(i) E : heads on third toss, F : heads on first two tosses.

(ii) E : at least two heads, F : at most two heads.

(iii) E : at most two tails, F : at least one tail.

Ans. A coin tossed three times, i.e.,

S = (TTT, HTT, THT, TTH, HHT, HTH, THH, HHH)

$\Rightarrow$ $n\left(S\right)$ = 8

(i) E : heads on third toss

E = (TTH, HTH, THH, HHH)

$\Rightarrow n\left(E\right) = 4$

P (E) = $\dfrac{n(E)}{n(S)} = \dfrac{4}{8} = \dfrac{1}{2}$

F : heads on first two tosses

F = (HHT, HHH)

$\Rightarrow n\left(F\right)$ =2

P (F) = $\dfrac{n(F)}{n(S)} = \dfrac{2}{8} = \dfrac{1}{4}$

$\therefore$ E $\cap$ F = (HHH)

$\Rightarrow n(E \cap F) = 1$

$\therefore P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{1}{8}$

And $P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1/8}{2/8} = \frac{1}{2}$

(ii) E : at least two heads

E = (HHT, HTH, THH, HHH)

$\Rightarrow n(E) = 4$

$P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

F : at most two heads

F = (TTT, HTT, THT, TTH, HHT, HTH, THH)

$\Rightarrow n(F) = 7$

$P(F) = \frac{n(F)}{n(S)} = \frac{7}{8}$

$\therefore E \cap F$ = (HHT, HTH, THH)

$\Rightarrow n(E \cap F) = 3$

$\therefore P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{3}{8}$

And $P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{3/8}{7/8} = \frac{3}{7}$

(iii) E : at most two tails

E = (HTT, THT, TTH, HHT, HTH, THH, HHH)

$\Rightarrow n(\mathrm{E}) = 7$

P (E) = $\frac{n(\mathrm{E})}{n(\mathrm{S})} = \frac{7}{8}$

F : at least one tail

F = (TTT, HTT, THT, TTH, HHT, HTH, THH)

$\Rightarrow n(\mathrm{F})$ = 7

P (F) = $\frac{n(\mathrm{F})}{n(\mathrm{S})} = \frac{7}{8}$

$\therefore$ E $\cap$ F = (HTT, THT, TTH, HHT, HTH, THH)

$\Rightarrow n$ (E $\cap$ F) = 6

$\therefore$ P (E $\cap$ F) = $\frac{n(\mathrm{E} \cap \mathrm{F})}{n(\mathrm{S})} = \frac{6}{8}$

And P$(\mathrm{E}|\mathrm{F})$ = $\frac{\mathrm{P}(\mathrm{E} \cap \mathrm{F})}{\mathrm{P}(\mathrm{F})} = \frac{6/8}{7/8} = \frac{6}{7}$

7. Determine P $(\mathrm{E}|\mathrm{F})$: Two coins are tossed once.

(i) E : tail appears on one coin, F : one coin shows head.

(ii) E : no tail appears, F : no head appears.

Ans. S = (HH, TH, HT, TT) $\Rightarrow$ $n(\mathrm{S})$ = 4

(i) E : tail appears on one coin

E = (TH, HT) $\Rightarrow n(E) = 2$

P (E) = $\frac{n(E)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

F : one coin shows head

F = (TH, HT) $\Rightarrow n(F)$ = 2

P (F) = $\frac{n(F)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

$\therefore$ E $\cap$ F = (TH, HT) $\Rightarrow n$ (E $\cap$ F) = 2

$\therefore$ P (E $\cap$ F) = $\frac{n(E \cap F)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

And $P(E|F)$ = $\frac{P(E \cap F)}{P(F)} = \frac{1/2}{1/2} = 1$

(ii) E : no tail appears

E = (HH) $\Rightarrow n(E) = 1$

P (E) = $\frac{n(E)}{n(S)} = \frac{1}{4}$

F : no head appears

F = (TT) $\Rightarrow n(F)$ = 1

P (F) = $\frac{n(F)}{n(S)} = \frac{1}{4}$

$\therefore\ E \cap F = \phi \Rightarrow n\ (E \cap F) = 0$

$\therefore$ P (E $\cap$ F) = $\dfrac{n(E \cap F)}{n(S)} = \dfrac{0}{4} = 0$

And P$\left(E|F\right)$ = $\dfrac{P(E \cap F)}{P(F)}$ = $\dfrac{0}{1/4} = 0$

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8. Determine P $\left(E|F\right)$: A dice is thrown three times.

E : 4 appears on the third toss, F : 6 and 5 appears respectively on first two tosses.

Ans. Since a dice has six faces. Therefore $n\left(S\right)$ = 6 x 6 x 6 = 216

E = (1, 2, 3, 4, 5, 6) x (1, 2, 3, 4, 5, 6) x (4)

F = (6) x (5) x (1, 2, 3, 4, 5, 6)

$\Rightarrow n\left(F\right)$ = 1 x 1 x 6 = 6

P (F) = $\dfrac{n(F)}{n(S)} = \dfrac{6}{216}$

$\therefore\ E \cap F$ = (6, 5, 4)

$\Rightarrow n\ (E \cap F) = 1$

$\therefore$ P (E $\cap$ F) = $\dfrac{n(E \cap F)}{n(S)} = \dfrac{1}{216}$

And P$\left(E|F\right)$ = $\dfrac{P(E \cap F)}{P(F)}$ = $\dfrac{1/216}{6/216} = \dfrac{1}{6}$

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9. Determine $\left(E|F\right)$: Mother, father and son line up at random for a family picture.

E : Son on one end, F : Father in middle.

Ans. S = (MFS, MSF, SFM, SMF, FMS, FSM) $\Rightarrow n(\text{S}) = 6$

E : Son on one end

E = (MFS, SFM, SMF, FMS) $\Rightarrow n(\text{E}) = 4$

F : Father in middle

F = (MFS, SFM) $\Rightarrow n(\text{F})$ = 2

P (F) = $\frac{n(\text{F})}{n(\text{S})} = \frac{2}{6} = \frac{1}{3}$

$\therefore$ E $\cap$ F = (MFS, SFM) $\Rightarrow n$ (E $\cap$ F) = 2

$\therefore$ P (E $\cap$ F) = $\frac{n(\text{E} \cap \text{F})}{n(\text{S})} = \frac{2}{6} = \frac{1}{3}$

And P$\left(\text{E}|\text{F}\right)$ = $\frac{\text{P}(\text{E} \cap \text{F})}{\text{P}(\text{F})} = \frac{1/3}{1/3} = 1$

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10. A black and a red die are rolled.

(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Ans. (a) $n(\text{S})$ = 6 x 6 = 36

Let A represents obtaining a sum greater than 9 and B represents black die resulted in a 5.

A = (46, 64, 55, 36, 63, 45, 54, 65, 56, 66) $\Rightarrow n(\text{A})$ = 10

P (A) = $\frac{n(A)}{n(S)} = \frac{10}{36}$

B = (51, 52, 53, 54, 55, 56) $\Rightarrow n(B)$ = 6

P (B) = $\frac{n(B)}{n(S)} = \frac{6}{36}$

A $\cap$ B = (55, 56) $\Rightarrow n(A \cap B)$ = 2

P (A $\cap$ B) = $\frac{2}{36}$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2/36}{6/36} = \frac{2}{6} = \frac{1}{3}$

(b) Let A denoted the sum is 8

$\therefore$ A = {(2, 6). (3, 5), (4, 4), (5, 3), (6, 2)}

B = Red die results in a number less than 4, either first or second die is red

$\therefore$ B = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)}

P (B) = $\frac{n(B)}{n(S)} = \frac{18}{36} = \frac{1}{2}$

A $\cap$ B = {(2, 6), (3, 5) $\Rightarrow n(A \cap B)$ = 2

P (A $\cap$ B) = $\frac{2}{36}$ = $\frac{1}{18}$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/18}{1/2} = \frac{2}{18} = \frac{1}{9}$

11. A fair die is rolled. Consider events E = {1, 3, 5}, F = {2, 3} and G = {2, 3, 4, 5}. Find:

(i) P $\left(\mathrm{E}|\mathrm{F}\right)$ **and P** $\left(\mathrm{F}|\mathrm{E}\right)$

(ii) P $\left(\mathrm{E}|\mathrm{G}\right)$ **and P** $\left(\mathrm{G}|\mathrm{E}\right)$

(iii) P $\left[\left(\mathrm{E}\cup\mathrm{F}\right)|\mathrm{G}\right]$ **and P** $\left[\left(\mathrm{E}\cap\mathrm{F}\right)|\mathrm{G}\right]$

Ans. S = (1, 2, 3, 4, 5, 6) $\Rightarrow$ $n\left(\mathrm{S}\right)$ = 6

E = (1, 3, 5) F = (2, 3) G = (2, 3, 4, 5)

$\Rightarrow$ $n\left(\mathrm{E}\right)$ = 3 $n\left(\mathrm{F}\right)$ = 2 $n\left(\mathrm{G}\right)$ = 4

(i) P (E) = $\dfrac{n\left(\mathrm{E}\right)}{n\left(\mathrm{S}\right)}=\dfrac{3}{6}$ P (F) = $\dfrac{n\left(\mathrm{F}\right)}{n\left(\mathrm{S}\right)}=\dfrac{2}{6}$

E $\cap$ F = (3) $\Rightarrow$ $n\left(\mathrm{E}\cap\mathrm{F}\right)$ = 1

P (E $\cap$ F) = $\dfrac{n\left(\mathrm{E}\cap\mathrm{F}\right)}{n\left(\mathrm{S}\right)}=\dfrac{1}{6}$

$\mathrm{P}\left(\mathrm{E}|\mathrm{F}\right)$ = $\dfrac{\mathrm{P}\left(\mathrm{E}\cap\mathrm{F}\right)}{\mathrm{P}\left(\mathrm{F}\right)}=\dfrac{1/6}{2/6}=\dfrac{1}{2}$ and $\mathrm{P}\left(\mathrm{F}|\mathrm{E}\right)$ = $\dfrac{\mathrm{P}\left(\mathrm{E}\cap\mathrm{F}\right)}{\mathrm{P}\left(\mathrm{E}\right)}=\dfrac{1/6}{3/6}=\dfrac{1}{3}$

(ii) P (E) = $\dfrac{n\left(\mathrm{E}\right)}{n\left(\mathrm{S}\right)}=\dfrac{3}{6}$ P (G) = $\dfrac{n\left(\mathrm{G}\right)}{n\left(\mathrm{S}\right)}=\dfrac{4}{6}$

E $\cap$ G = (3, 5) $\Rightarrow$ $n\left(\mathrm{E}\cap\mathrm{G}\right)$ = 2

P (E $\cap$ G) = $\dfrac{n\left(\mathrm{E}\cap\mathrm{G}\right)}{n\left(\mathrm{S}\right)}=\dfrac{2}{6}$

$P(E|G) = \frac{P(E \cap G)}{P(G)} = \frac{2/6}{4/6} = \frac{2}{4} = \frac{1}{2}$ and $P(G|E) = \frac{P(E \cap G)}{P(E)} = \frac{2/6}{3/6} = \frac{2}{3}$

(iii) P (G) = $\frac{n(G)}{n(S)} = \frac{4}{6}$

$E \cup F$ = (1, 2, 3, 5) and G (2, 3, 4, 5)

$(E \cup F) \cap G$ = (2, 3, 5) $\Rightarrow n[(E \cup F) \cap G]$ = 3

$$P[(E \cup F) \cap G] = \frac{3}{6}$$

$$P(E \cup F|G) = \frac{P[(E \cup F) \cap G]}{P(G)} = \frac{3/6}{4/6} = \frac{3}{4}$$

Again $E \cap F$ = (3)

$(E \cap F) \cap G$ = (3) $\Rightarrow n[(E \cap F) \cap G]$ = 1

$$P[(E \cap F) \cap G] = \frac{1}{6}$$

$$P(E \cap F|G) = \frac{P[(E \cap F) \cap G]}{P(G)} = \frac{1/6}{4/6} = \frac{1}{4}$$

12. Assume that each child born is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that (i) the youngest is a girl (ii) at least one is a girl?

Ans. Let first and second girl are denoted by G_1 and G_2 and boys B_1 and B_2.

$\therefore$ S = {(G_1G_2), (G_1B_2), (G_2B_1), (B_1B_2)}

Let A = Both the children are girls = (G_1G_2)

B = Youngest child is girl = $\{(G_1G_2), (B_1G_2)\}$

C = at least one is a girl = $\{(G_1B_2), (G_1G_2), (B_1G_2)\}$

$A \cap B = (G_1G_2) \Rightarrow P(A \cap B) = \frac{1}{4}$

$A \cap C = (G_1G_2) \Rightarrow P(A \cap C) = \frac{1}{4}$

$P(B) = \frac{2}{4}$ and $P(C) = \frac{3}{4}$

(i) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{2/4} = \frac{1}{2}$

(ii) $P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{1/4}{3/4} = \frac{1}{3}$

13. An instructor has a test bank consisting of 300 easy True/False questions, 200 difficult True/False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the test bank, what is the probability that it will be an easy question given that it is a multiple choice question?

Ans. Number of easy True/False questions = 300

Number of difficult True/False questions = 200

Number of easy multiple choice questions = 500

Number of difficult multiple choice questions = 400

Total number of all such questions = $n(S)$ = 1400

Let E represents an easy question and F represents a multiple choice question.

$\therefore$ $n(\mathrm{E})$ = 300 + 500 = 800 and $n(\mathrm{F})$ = 500 + 400 = 900

P (F) = $\frac{n(\mathrm{F})}{n(\mathrm{S})} = \frac{900}{1400}$

$n(\mathrm{E} \cap \mathrm{F})$ = 500 $\Rightarrow$ P (E $\cap$ F) = $\frac{n(\mathrm{E} \cap \mathrm{F})}{n(\mathrm{S})} = \frac{500}{1400}$

$\mathrm{P}(\mathrm{E}|\mathrm{F})$ = $\frac{\mathrm{P}(\mathrm{E} \cap \mathrm{F})}{\mathrm{P}(\mathrm{F})} = \frac{500/1400}{900/1400} = \frac{500}{900} = \frac{5}{9}$

14. Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.

Ans. S = (1, 1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1)

(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2)

(1, 3), (2, 3), (3, 3), (4, 3), (5, 3), (6, 3)

(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4)

(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5)

(1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 6)

$\therefore$ $n(\mathrm{S})$ = 36

Let A represents the event "the sum of numbers on the dice is 4" and B represents the event "the two numbers appearing on throwing two dice are different".

Therefore, A = {(1, 3), (2, 2), (3, 1)} $\Rightarrow$ $n(\mathrm{A})$ = 3

P (A) = $\frac{n(A)}{n(S)} = \frac{3}{36}$

Also B = {(2, 1), (3, 1), (4, 1), (5, 1), (6, 1), (1, 2), (3, 2), (4, 2), (5, 2), (6, 2),(1, 3), (2, 3), (4, 3), (5, 3), (6, 3), (1, 4), (2, 4), (3, 4), (5, 4), (6, 4),(1, 5), (2, 5), (3, 5), (4, 5), (6, 5), (1, 6), (2, 6), (3, 6), (4, 6), (5, 6)}

$n(B)$ = 30

P (B) = $\frac{n(B)}{n(S)} = \frac{30}{36}$

Now A $\cap$ B = {(1, 3), (3, 1)}

$\Rightarrow n(A \cap B)$ = 2

P (A $\cap$ B) = $\frac{n(A \cap B)}{n(S)} = \frac{2}{36}$

Hence, P$(A|B)$ = $\frac{P(A \cap B)}{P(B)}$

$$= \frac{2/36}{30/36} = \frac{2}{30} = \frac{1}{15}$$

15. Consider the experiment of throwing a die, if a multiple of 3 comes up throw the die again and if any other number comes toss a coin. Find the conditional probability of the event “the coin shows a tail”, given that “at least one die shows a 3”.

Ans. S = {(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

(1, H), (2, H), (3, H), (4, H), (5, H), (1, T), (2, T), (3, T), (4, T), (5, T)}

$\therefore n(S)$ = 20

P (first die shows a multiple of 3) = $\frac{12}{36} = \frac{1}{3}$

P (first die shows a number which is not a multiple of 3) = $\frac{4}{6} \times \frac{1}{2} + \frac{4}{6} \times \frac{1}{2} = \frac{8}{12} = \frac{2}{3}$

Let A = the coin shows a tail = {(1, T), (2, T), (4, T), (5, T)}

B = at least one die shows a 3 = {(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)}

A $\cap$ B = ϕ

$n(\text{A})$ = 4, $n(\text{B})$ = 6, $n(\text{A} \cap \text{B})$ = 0

P (B) = $\frac{6}{36} = \frac{1}{6}$ and P (A $\cap$ B) = 0

$\text{P}(\text{A}|\text{B})$ = $\frac{\text{P}(\text{A} \cap \text{B})}{\text{P}(\text{B})}$ = $\frac{0}{6/36} = 0$

In each of the following choose the correct answer:

16. If P (A) = $\frac{1}{2}$, P (B) = 0, then $\text{P}(\text{A}|\text{B})$ is:

(A) 0

(B) $\frac{1}{2}$

(C) not defined

(D) 1

Ans. P (A) = $\frac{1}{2}$, P(B) = 0

$\therefore$ $\text{P}(\text{A} \cap \text{B})$ = 0

$\therefore\ P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{0}$ = not defined

Therefore, option (C) is correct.

17. If A and B are events such that $P(A|B) = P(B|A)$, then:

(A) $A \subset B$

(B) A = B

(C) $A \cap B = \phi$

(D) P (A) = P (B)

Ans. $P(A|B) = P(B|A)$

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{P(B \cap A)}{P(A)}$$

$\Rightarrow$ P (A) = P (B)

Therefore, option (D) is correct.