CBSE Class–12 Mathematics

NCERT solution

Chapter -12

Linear Programming - Exercise 12.2

1. Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs Rs. 60/kg and Food Q costs Rs. 80/kg. Food P contains 3 units/kg of vitamin A and 5 units/kg of vitamin B while Food Q contains 4 units/kg of vitamin A and 2 units/kg of vitamin B. Determine the minimum cost of the mixture.

Ans. Let food P consists of x kg and food Q consists of y kg.

Minimum Z = $60x+80y$

According to question $3x+4y\geq 8$ and $5x+2y\geq 11$

$x\geq 0$ and $y\geq 0$

	A	B	C
x	0	4	-4
y	2	-1	5

Consider $3x+4y\geq 8$

Let $3x+4y=8 \Rightarrow y=\dfrac{8-3x}{4}$

But (0, 0) does not satisfy this inequation, therefore the required half-plane does not contain (0, 0).

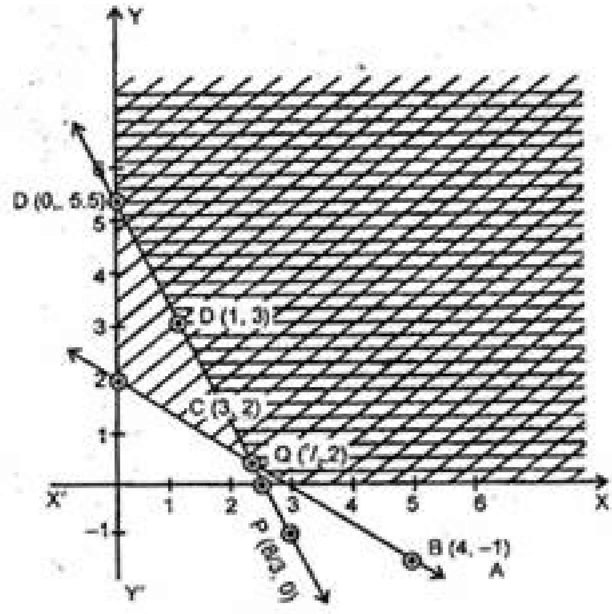
Again consider $5x+2y\geq 11$

Let $5x+2y=11 \Rightarrow y=\dfrac{11-5x}{2}$

	D	E	F

x	0	1	3
y	5.5	3	-2

Again (0, 0) does not satisfy this inequation, therefore the required half-plane does not contain (0, 0).



The double shaded region is our solution set. The corners of this region are D (0, 5.5), Q (2, 1.5) and P $\left(\frac{8}{3}, 0\right)$.

Now Z = $60x+80y$

At D(0,5.5)

Z = 60 x 0 + 80 x 5.5 = 440

At Q (2, 1.5)

Z = 60 x 2 + 80 x 1.5 = 160

AtP $\left(\frac{8}{3}, 0\right)$

Z = 60 x $\frac{8}{3}$ 80 x 0 = 160

Hence, minimum Z = 160 at $x=2, y=1.5$ means minimum cost Z = Rs. 160 when Reshma mixes food P = 2 kg and food Q = 1.5 kg.

2. One kind of cake requires 200g of flour and 25 g of fat and another kind of cake requires 100 g of flour and 50 g of fat. Find the maximum number of cake which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes.

Ans. Let number of cakes made of first kind are x and that of second kind is y.

$\therefore$ Let to maximize Z = $x+y$

According to question $200x+100y \le 5000$ and $25x+50y \le 1000$

$x \ge 0, y \ge 0$

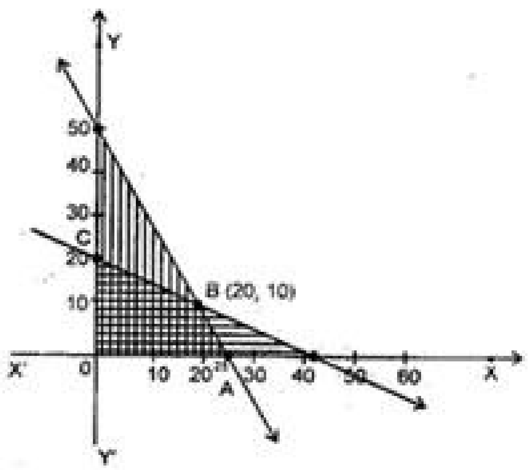
Consider $200x+100y \le 5000$

Let $200x+100y = 5000$

$\Rightarrow 2x+y=50$

$$\Rightarrow \frac{x}{25}+\frac{y}{50}=1$$

Here, (0, 0) satisfies this inequation, therefore the required half plane contains (0, 0).



Again consider $25x+50y \le 1000$

Let $25x+50y=1000$

$\Rightarrow x+2y=40$

$\Rightarrow \frac{x}{40}+\frac{y}{20}=1$

Here, again (0, 0) satisfies this inequation, therefore the required half plane contains (0, 0).

The double shaded region is the feasible region which is solution set.

The corner points of this region are O (0, 0), A (25, 0), B (20, 10) and C (0, 20).

$\therefore$ Z = $x+y$

At O (0, 0) Z = 0 + 0 = 0

At A (25, 0) Z = 25 + 0 = 25

At B (20, 10) Z = 20 + 10 = 30

AtC (0, 20) Z = 0 + 20 = 20

Hence, maximum number of cakes Z = 30 when $x=20, y=10.$

3. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftman's time.

(i) What number of rackets and bats must be made if the factory is to work at full capacity?

(ii) If the profit on a racket and on a bat is Rs. 20 and Rs. 10 respectively, find maximum profit of the factory when it works at full capacity.

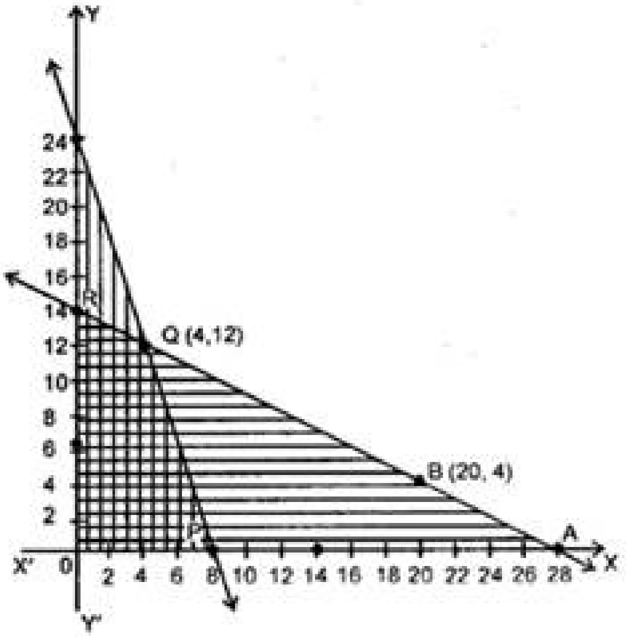
Ans. Let number of rackets = x and number of bats = y

	A	B	C

x	28	20	0
y	0	4	14

(i) To maximize Z = $x+y$

According to question $1.5x+3y\le 42$ and $3x+y\le 24$



$x\ge 0,\ y\ge 0$

Consider $1.5x+3y\le 42$

Let $1.5x+3y=42 \Rightarrow x+2y=28$

$\Rightarrow\ x=28-2y$

Here (0, 0) satisfies this inequaiton, therefore the required half-plane contains (0, 0).

Again consider

$3x+y\le 24$

Let $3x+y=24$

$\Rightarrow\ \frac{x}{8}+\frac{y}{24}=1$

Here (0, 0) satisfies this inequaiton, therefore the required half-plane contains (0, 0).

Now the feasible region is the double-shaded region i.e., the solution set. The corner points of this region are O (0, 0), P (8, 0), Q (4, 12) and R (0, 14).

Now Z = $x+y$

At O (0, 0) Z = 0 + 0 = 0

At P (8, 0) Z = 8 + 0 = 0

At Q (4, 12) Z = 4 + 12 = 16

At R (0, 14) Z = 0 + 14 = 14

Now maximum Z = 16 at $x=4, y=12.$

(ii) The profit on a racket = Rs. 20 and the profit on a bat = Rs. 10

Let maximum profit P = $20x+10y$

Hence maximum profit P = 20 x 4 + 10 x 12 = Rs.200

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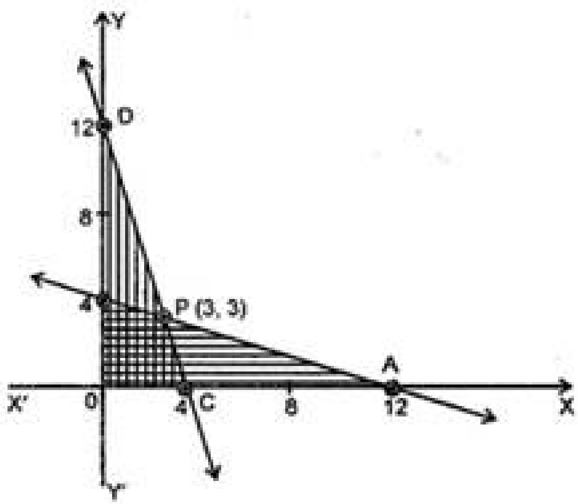
4. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs. 17.50 per package on nuts and Rs. 7.00 per package on bolts. How many packages of each should be produced each day so as to maximize his profit, if he operates his machines for at the most 12 hours a day?

Ans. Let number of package of nuts = x and number of packages of bolts = y

To maximum profit Z = $17.50x+7y$

According to question $x+3y\le12$ and $3x+y\le12$ $x\ge0, y\ge0$



Consider $x+3y \le 12$

Let $x+3y=12 \Rightarrow \frac{x}{12}+\frac{y}{4}=1$

Therefore, points are A (12, 0), B (0, 4). Also, (0, 0) satisfies this inequation, therefore the required half-plane contains (0, 0).

Again consider $3x+y \le 12$

Let $3x+y=12$

$$\Rightarrow \frac{x}{4}+\frac{y}{12}=1$$

Therefore, points are C (4, 0), D (0, 12).

Again (0, 0) satisfies this inequation, therefore the required half-plane contains (0, 0).

The double shaded region is OBPCO and its corners are O (0, 0), B (0, 4), D (3, 3), C (4, 0).

Now Z = $17.50x+7y$

At O (0, 0) Z = 17.5 x 0 + 7 x 0 = 0

At B (0, 4) Z = 17.5 x 0 + 7 x 4 = 28

At P (3, 3) Z = 17.5 x 3 + 7 x 3 = 73.50

At C (4, 0) Z = 17.5 x 4 + 7 x 0 = 70

$\therefore$ Maximum profit = Rs. 73.50 at $x = 3, y = 3$

Hence maximum profit Z = Rs. 73.50 when he produces number of packets of nuts = 3 and number of packets of bolts = 3.

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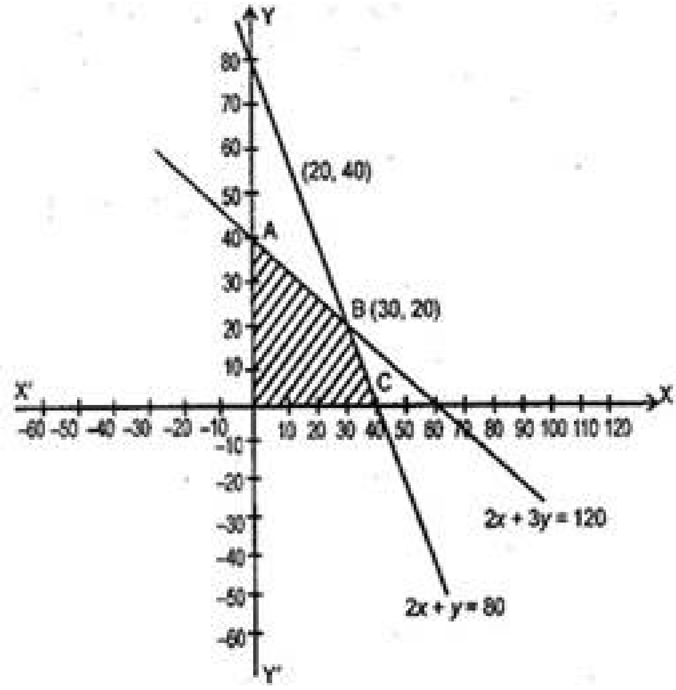
5. A factory manufacturers two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screw A, while it takes 6 minutes on automatic and 3 minutes on hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of Rs. 7 and screws B at a profit of Rs. 10. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce a day in order to maximize his profit? Determine the maximum profit.

Ans. Let the manufacturers produces x packages of screws A and y packages of screws B, then time taken by x packages of screws A and y packages of screws B on automatic machine = $(4x+6y)$ minutes. And time taken by x packages of screws A and y packages of screws B on hand operated machine = $(6x+3y)$ minutes.

Since, each machine is available for at the most 4 hours, i.e., 4 x 60 = 240 minutes.

Therefore, we have $4x+6y \le 240 \Rightarrow 2x+3y \le 120$ and $6x+3y \le 240 \Rightarrow$ $2x+y \le 80$.

Profit on selling x packages of screws A and y packages of screws B = Z = $7x+10y$



$\therefore$ To find: x and y such that Z = $7x+10y$ is maximum subject to $2x+3y \le 120$, $2x+y \le 80$, $x \ge 0$ and $y \ge 0$.

Consider $2x+3y \le 120$

Let $2x+3y=120$

$$\Rightarrow \frac{x}{60}+\frac{y}{40}=1$$

$\therefore$ A (60, 0) and B (0, 40)

Here, (0, 0) satisfies this inequation, therefore the required hal-plane contains (0, 0).

Again $2x+y \le 80$

Let $2x+y=80$

$$\Rightarrow \frac{x}{40}+\frac{y}{80}=1$$

$\therefore$ C (40, 0) and D (0, 80)

Here also (0, 0) satisfies this inequation, therefore the required hal-plane contains (0, 0).

The feasible portion of the graph satisfying the inequalities $2x+3y \le 120$ and $2x+y \le 80$ is OABC which is shown shaded in the figure.

Co-ordinates of O, A, B and C are (0, 0), (0, 40), (30, 20) and (40, 0) respectively.

Now Z = $7x+10y$

At O (0, 0) Z = 7 x 0 + 10 x 0 = 0

At A (0, 40) Z = 7 x 0 + 10 x 40 = 400

At B (30, 20) Z = 7 x 30 + 10 x 20 = 410

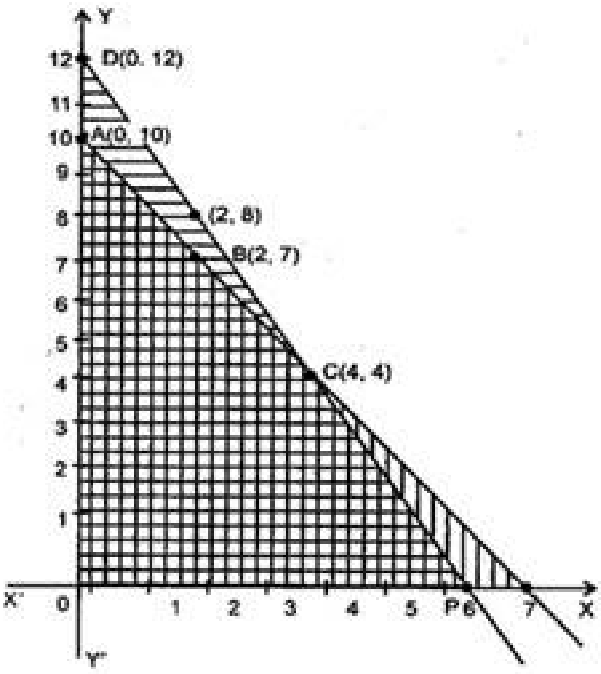
At C (40, 0) Z = 7 x 40 + 10 x 0 = 280

$\therefore$ Maximum profit Z = Rs. 410 at $x=30, y=20$

Hence, if the manufacturer produces 30 screws of type A and 20 screws of type B, he earn a maximum profit of Rs. 410.

6. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is Rs.5 and that from a shade is Rs.3. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximize his profit?

Ans. Let the manufacturer produces x pedestal lamps and y wooden shades, then the time taken by x pedestal lamps and y wooden shades on grinding/cutting machines = $(2x+y)$ hours and the time taken by x pedestal lamps and y wooden shades on the sprayer = $(3x+2y)$ hours.



Since grinding/cutting machine is available for at the most 12 hours, i.e., $2x+y\leq 12$ and sprayer is available for at the most 20 hours, i.e., $3x+2y\leq 20$

Profit from the sale of x lamps and y shades Z = $5x+3y$

$\therefore$ To find: x and y such that Z = $5x+3y$ is maximum subject to constraints $2x+y\leq 12$, $3x+2y\leq 20$, $x\geq 0$, $y\geq 0$.

Consider $3x+2y\leq 20$

Let $3x+2y=20$

$$\Rightarrow y=\frac{20-3x}{2}$$

	A	B	C
x	0	2	4
y	10	7	4

Now the area represented by $3x+2y\leq 20$ is the half-plane containing (0, 0) as (0, 0) satisfies the inequaiton.

Consider $2x+y\leq 12$

Let $2x+y=12$

$\Rightarrow \ y=12-2x$

	D	E	C
x	0	2	4
y	12	8	4

The inequation consists of the half-plane containing (0,) as (0, 0) satisfies this inequation.

The double shaded region OPCAO is our solution where O (0, 0), P (6, 0), C (4, 4), A (0, 10).

Now Z = $5x+3y$

At O (0, 0) Z = 5 x 0 + 3 x 0 = 0

At P (6, 0) Z = 5 x 6 + 3 x 0 = 30

At C (4, 4) Z = 5 x 4 + 3 x 4 = 32

At A (0, 10) Z = 5 x 0 + 3 x 10 = 30

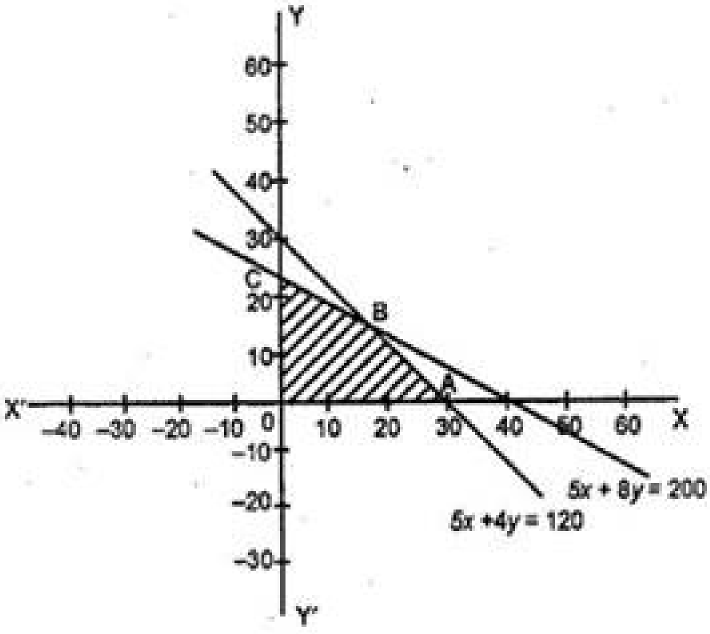
Now maximum Z = 32 at $x=4, y=4$

Hence, maximum profit Z = Rs. 32 when he manufactures 4 pedestal lamps and 4 wooden shades.

7. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A requires 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit is Rs.5 each for type A and Rs.6 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximize the profit?

Ans. Let the company manufactures x souvenirs of type A and y souvenirs of type B, then time taken for cutting x souvenirs of type A and y souvenirs of type B = $(5x+8y)$ minutes and time taken for assembling x souvenirs of type A and y souvenirs of type B =

$(10x+8y)$ minutes.



Since 3 hours 29 minutes i.e., 200 minutes are available for cutting, therefore we have $5x+8y \le 200.$

Also since 4 hours i.e., 240 minutes are available for assembling, therefore we have $10x+8y \le 240 \Rightarrow 5x+4y \le 120$

Thus, our L.P.P. is to maximize profit Z = $5x+6y$ subject to constraints $5x+8y \le 200$, $5x+4y \le 120$, $x \ge 0, y \ge 0.$

Now $5x+8y \le 200$

Let $5x+8y = 200 \Rightarrow \frac{x}{40}+\frac{y}{25}=1$

x	40	0
y	0	25

Again $5x+4y \le 120$

Let $5x+4y = 120$

$\Rightarrow \frac{x}{24}+\frac{y}{30}=1$

x	24	20	0
y	0	5	30

In both the equations origin (0, 0) satisfies them and therefore the required half planes are there which contains (0, 0).

The portion of graph satisfying the inequalities $5x+8y \le 200$ and $5x+4y \le 120$ is OABC and is shown in shaded in the figure. Coordinates of the points O, A, B and C are (0, 0), (24, 0), (8, 20) and (0, 25) respectively.

Now Z = $5x+6y$

At O (0, 0) Z = 5 x 0 + 6 x 0 = 0

At A (24, 0) Z = 5 x 24 + 6 x 0 = 120

At B (8, 20) Z = 5 x 8 + 6 x 20 = 160

At C (0, 25) Z = 5 x 0 + 6 x 25 = 150

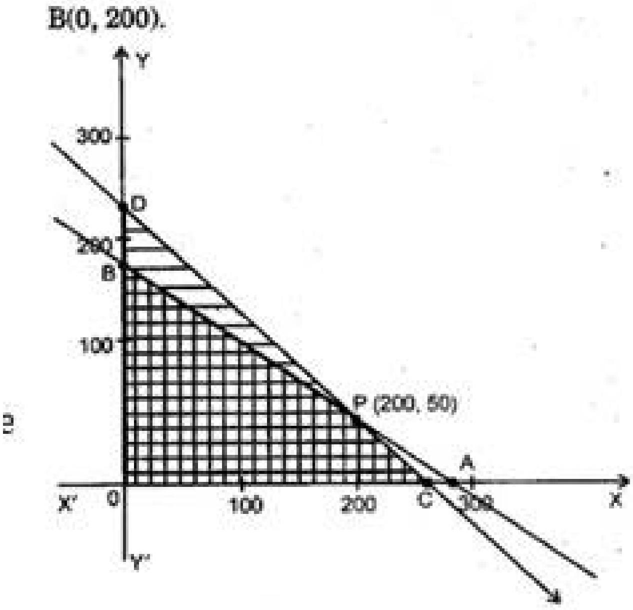
Now maximum Z = 160 at $x=8, y=20$

Hence, maximum profit = Rs. 160 when he manufactures 8 souvenirs of type A and 20 souvenirs of type B.

8. A merchant plans to sell two types of personal computers – a desktop model and a portable model that will cost Rs. 25000 and Rs. 40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs, 70 lakhs and if his profit on the desktop model is Rs. 4500 and on portable model is Rs. 5000.

Ans. Let number of desktop model computer = x and number of a portable model computer = y

To maximum profit Z = $4500x + 5000y$ subject to $25000x + 40000y \le 70,00,000$, $x + y \le 250,\ x \ge 0, y \ge 0$



Consider $25000x + 40000y \le 70,00,000 \Rightarrow 5x + 8y \le 1400$

Let $5x + 8y = 1400$

$$\Rightarrow \frac{x}{280} + \frac{y}{175} = 1$$

$\therefore$ Points are A (280, 0), B (0, 175)

Now (0, 0) satisfies the inequation, therefore the required half-plane contains (0, 0).

Again consider $x + y \le 250$

Let $x + y = 250$

$$\Rightarrow \frac{x}{250} + \frac{y}{250} = 1$$

$\therefore$ Points are C (250, 0), D (0, 250).

Again (0, 0) satisfies the inequation, therefore the required half-plane contains (0, 0).

The double shaded region OCPBO is the solution set. Its corners are O (0, 0), C (280, 0), P (200, 50), B (0, 200).

Now Z = $4500x + 5000y$

AtO (0, 0) Z = 4500 x 0 + 5000 x 0 = 0

At C (280, 0) Z = 4500 x 280 + 5000 x 0 = 12,60,000

(Rejected as $x \geq 250$)

At P (200, 50) Z = 4500 x 200 + 5000 x 50 = 11,50,000

At B (0, 200) Z = 4500 x 0 + 5000 x 200 = 10,00,000

Now maximum Z = Rs. 11,50,000 at $x = 200, y = 50$

Hence, maximum profit = Rs. 11,50,000 when he sells 200 desktop models and 50 portable models.

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9. A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs Rs. 4 per unit food and F_2 costs Rs. 6 per unit. One unit of food F_1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.

Ans. Let the number of units of food F_1 = x and number of units of food F_2 = y

To minimize Z = $4x + 6y$ subject to $3x + 6y \geq 80$, $4x + 3y \geq 100$, $x \geq 0, y \geq 0$

Consider $3x + 6y \geq 80$

Let $3x + 6y = 80 \Rightarrow y = \dfrac{80 - 3x}{6}$

	A	B	C
x	0	5	10

y	$\frac{40}{3}$	$\frac{65}{6}$	$\frac{50}{6}$

Clearly, (0, 0) is not included in the half-plane.

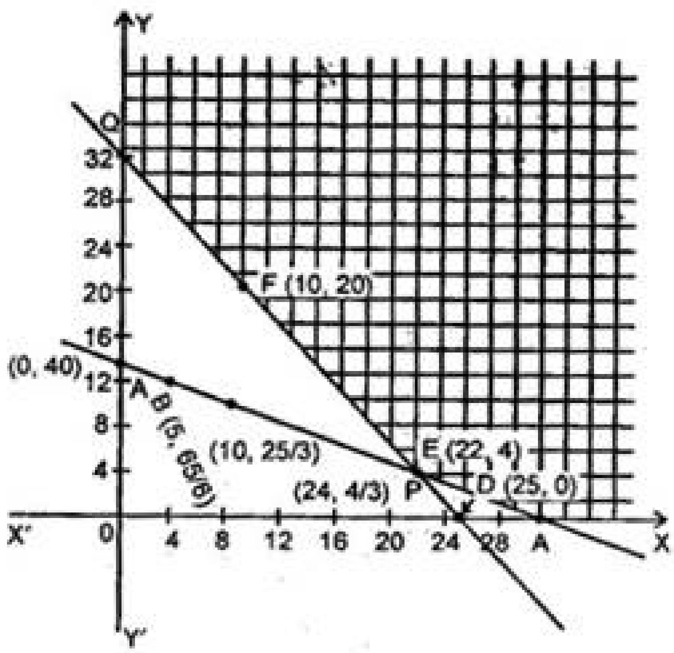
Now again consider $4x+3y\geq 100$

Let $4x+3y=100 \Rightarrow x=\dfrac{100-3y}{4}$

	D	E	F
x	25	22	10
y	0	4	20

Here, also (0, 0) is not included in the required half-plane.

The double shaded region is our feasible region and its corners are



A$\left(\dfrac{80}{3},0\right)$, P$\left(24,\dfrac{4}{3}\right)$ and Q$\left(0,\dfrac{100}{3}\right)$

Now Z = $4x+6y$

At A$\left(\frac{80}{3}, 0\right)$ Z = 4 x $\frac{80}{3}$ + 6 x 0 = 106.67

At P$\left(24, \frac{4}{3}\right)$ Z = 4 x 24 + 6 x $\frac{4}{3}$ = 104

At Q$\left(0, \frac{100}{3}\right)$ Z = 4 x 0 + 6 x $\frac{100}{3}$ = 200

Hence Z = Rs. 104 is minimum at $x = 24, y = \frac{4}{3}$.

Therefore, minimum cost is Rs.104 when 24 units of food F_1 and mixed with 4/3 units of food F_2.

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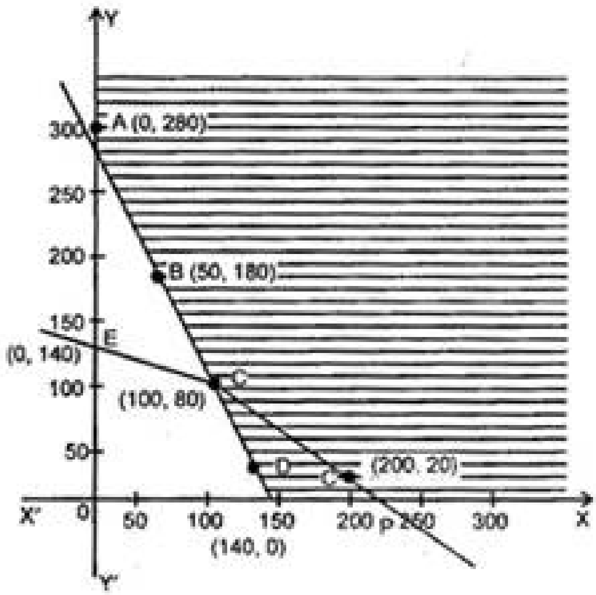
10. There are two types of fertilizers F_1 and F_2. F_1 consists of 10% nitrogen and 6% phosphoric acid and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If F_1 costs Rs. 6/kg and F_2 costs Rs. 5/kg, determine how much of each type of fertilizer should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?

Ans. Let the quantity of F_1 = x kg and the quantity of F_2 = y kg

We have to minimize Z = $6x+5y$ subject to $\frac{10x}{100}+\frac{5y}{100} \geq 14$, $\frac{6x}{100}+\frac{10y}{100} \geq 14$

$x \geq 0, y \geq 0$

Consider $\frac{10x}{100}+\frac{5y}{100} \geq 14$



Let $\frac{10x}{100}+\frac{5y}{100}=14$

$\Rightarrow 10x+5y=1400$

$\Rightarrow\ 2x+y=280$

$\Rightarrow\ y=280-2x$

	A	B	C	D
x	0	50	100	140
y	280	180	80	0

Here, (0, 0) is not included in the required half plane and (0, 0) does not satisfy this inequation.

Again consider $\frac{6x}{100}+\frac{10y}{100}\geq 14$

Let $\frac{6x}{100}+\frac{10y}{100}=14\ \Rightarrow\ 6x+10y=1400$

$\Rightarrow\ 3x+5y=700\ \Rightarrow\ \ y=\frac{700-3x}{5}$

	E	C	G
x	0	100	200
y	140	80	20

Again Here, (0, 0) is not included in the required half plane and (0, 0) does not satisfy this inequation.

The shaded region XPCAY is our feasible region. Its corners are P $\left(\frac{700}{3}, 0\right)$, C (100, 80) and A (0, 280).

Now Z = $6x+5y$

At P $\left(\frac{700}{3}, 0\right)$ Z = 6 x $\frac{700}{3}$ + 5 x 0 = 1400

At C (100, 80) Z = 6 x 100 + 5 x 80 = 1000

At A (0, 280) Z = 6 x 0 + 5 x 280 = 1400

Now minimum cost Z = Rs. 1000 at $x=100, y=80.$

Therefore, Minimum cost is Rs.1000 when the farmer used 100 kg of fertilizer F_1 and 80 kg of fertilizer F_2.

11. The corner points of the feasible region determined by the following system of linear inequalities:

$2x+y\le 10,\ x+3y\le 15,\ \ x, y\ge 0$ **are (0, 0), (5, 0), (3, 4) and (0, 5). Let** $Z=px+qy,$ **where** $p, q>0.$ **Condition on** p **and** q **so that the maximum of Z occurs at both (3, 4) and (0, 5) is:**

(A) $p=q$

(B) $p=2q$

(C) $p=3q$

(D) $q = 3p$

Ans. To maximize Z = $px + qy, \quad p, q > 0$ subject to $2x + y \le 10$, $x + 3y \le 15, x \ge 0, y \ge 0$

The corner points are (0, 0), (5, 0), (3, 4) and (0, 5).

Maximum of Z occurs at both (3, 4) and (0, 5)

Z = $px + qy$

At (3, 4) Z = $3p + 4q$

At (0, 5) Z = $5q$

Now maxima occurs at both points,

$3p + 4q = 5q$

$\Rightarrow 3p = q$

$\Rightarrow q = 3p$

Hence option (D) is correct.