CBSE Class–12 Mathematics

NCERT solution

Chapter - 7

Integrals -Exercise 7.7

Integrate the functions in Exercises 1 to 9.

1. $\sqrt{4-x^2}$

Ans. $\int \sqrt{4-x^2}\ dx$

$= \int \sqrt{2^2-x^2}\ dx$

$= \frac{x}{2}\sqrt{2^2-x^2} + \frac{2^2}{2}\sin^{-1}\frac{x}{2} + c$

$\left[\because \int \sqrt{a^2-x^2}\ dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$

$= \frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2} + c$

2. $\sqrt{1-4x^2}$

Ans. $\int \sqrt{1-4x^2}\ dx$

$= \int \sqrt{1^2-(2x)^2}\ dx$

$= \dfrac{\left(\frac{2x}{2}\right)\sqrt{1^2-(2x)^2} + \frac{1^2}{2}\sin^{-1}\left(\frac{2x}{1}\right)}{2 \rightarrow \text{Coeff. of } x \text{ in } 2x} + c$

$\left[\because \int \sqrt{a^2-x^2}\ dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$

$= \frac{1}{2}\left[x\sqrt{1-4x^2}+\frac{1}{2}\sin^{-1}2x\right]+c$

$= \frac{x}{2}\sqrt{1-4x^2}+\frac{1}{4}\sin^{-1}2x+c$

$= \frac{1}{4}sin^{-1}2x + \frac{x}{2}\sqrt{1-4x^2}+c$

3. $\sqrt{x^2+4x+6}$

Ans. $\int \sqrt{x^2+4x+6}\ dx = \int \sqrt{x^2+4x+4+6-4}\ dx$

$= \int \sqrt{(x+2)^2+\left(\sqrt{2}\right)^2}\ dx$

$= \left(\frac{x+2}{2}\right)\sqrt{(x+2)^2+\left(\sqrt{2}\right)^2}+\frac{\left(\sqrt{2}\right)^2}{2}\log\left|x+2+\sqrt{(x+2)^2+\left(\sqrt{2}\right)^2}\right|+c$

$\left[\because \int \sqrt{x^2+a^2}\ dx = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|\right]$

$= \left(\frac{x+2}{2}\right)\sqrt{x^2+4x+6}+\frac{2}{2}\log\left|x+2+\sqrt{x^2+4x+6}\right|+c$

$= \left(\frac{x+2}{2}\right)\sqrt{x^2+4x+6}+\log\left|x+2+\sqrt{x^2+4x+6}\right|+c$

4. $\sqrt{x^2+4x+1}$

Ans. $\int \sqrt{x^2+4x+1}\ dx$

= $\int \sqrt{x^2+4x+4+1-4}\ dx$

$= \int \sqrt{(x+2)^2-(\sqrt{3})^2}dx$

$= \left(\frac{x+2}{2}\right)\sqrt{(x+2)^2-(\sqrt{3})^2} - \frac{(\sqrt{3})^2}{2} log|x+2+\sqrt{(x+2)^2-(\sqrt{3})^2}| + c$

$\left[\because \int \sqrt{x^2-a^2}\ dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|\right]$

= $\left(\frac{x+2}{2}\right)\sqrt{x^2+4x+1} - \frac{3}{2}\log\left|x+2+\sqrt{x^2+4x+1}\right| + c$

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5. $\sqrt{1-4x-x^2}$

Ans. $\int \sqrt{1-4x-x^2}\ dx$

= $\int \sqrt{-x^2-4x+1}\, dx$

= $\int \sqrt{-\left(x^2+4x-1\right)}\ dx$

= $\int \sqrt{-\left(x^2+4x+4-4-1\right)}\ dx$

= $\int \sqrt{-\left[\left(x+2\right)^2+\left(\sqrt{5}\right)^2\right]}\ dx$

= $\int \sqrt{\left(\sqrt{5}\right)^2-\left(x+2\right)^2}\ dx$

= $\frac{x+2}{2}\sqrt{\left(\sqrt{5}\right)^2-\left(x+2\right)^2} + \frac{\left(\sqrt{5}\right)^2}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right) + c$

$\left[\because \int \sqrt{a^2-x^2}\ dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$

= $\frac{x+2}{2}\sqrt{1-4x-x^2} + \frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right) + c$

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6. $\sqrt{x^2+4x-5}$

Ans. $\int \sqrt{x^2+4x-5}\ dx$

= $\int \sqrt{x^2+4x+4-4-5}\ dx$

$= \int \sqrt{(x+2)^2-(3)^2}dx$

$= \left(\frac{x+2}{2}\right)\sqrt{(x+2)^2-(3)^2} - \frac{(3)^2}{2}log|x+2+\sqrt{(x+2)^2-(3)^2}| + c$

$\left[\because \int \sqrt{x^2-a^2}\ dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|\right]$

= $\left(\frac{x+2}{2}\right)\sqrt{x^2+4x-5} - \frac{9}{2}\log\left|x+2+\sqrt{x^2+4x-5}\right| + c$

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7. $\sqrt{1+3x-x^2}$

Ans. $\int \sqrt{1+3x-x^2}\ dx$

= $\int \sqrt{-x^2+3x+1}\ dx$

= $\int \sqrt{-\left(x^2-3x-1\right)}\ dx$

= $\int \sqrt{-\left(x^2-3x+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2-1\right)}\ dx$

$= \int \sqrt{-\left[\left(x-\frac{3}{2}\right)^2+\left(\frac{\sqrt{13}}{2}\right)^2\right]}\,dx$

$= \int \sqrt{\left(\frac{\sqrt{13}}{1}\right)^2-\left(x-\frac{3}{2}\right)^2}\,dx$

$= \dfrac{x-\frac{3}{2}}{2}\sqrt{\left(\frac{\sqrt{13}}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}+\dfrac{\left(\frac{\sqrt{13}}{2}\right)^2}{2}\sin^{-1}\left(\dfrac{x-\frac{3}{2}}{\frac{\sqrt{13}}{2}}\right)+c$

$\left[\because \int \sqrt{a^2-x^2}\,dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$

$= \left(\frac{2x-3}{4}\right)\sqrt{1+3x-x^2}+\frac{13}{8}\sin^{-1}\left(\frac{2x-3}{\sqrt{13}}\right)+c$

8. $\sqrt{x^2+3x}$

Ans. $\int \sqrt{x^2+3x}\,dx$

$= \int \sqrt{x^2+3x+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2}\,dx$

$= \int \sqrt{\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2}\,dx$

$= \dfrac{x+\frac{3}{2}}{2}\sqrt{\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2}-\dfrac{\left(\frac{3}{2}\right)^2}{2}\log\left|x+\frac{3}{2}+\sqrt{\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2}\right|+c$

$$\left[\because \int \sqrt{x^2-a^2}\ dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|\right]$$

$$= \frac{2x+3}{4}\sqrt{x^2+3x} - \frac{9}{8}\log\left|x+\frac{3}{2}+\sqrt{x^2+3x}\right| + c$$

9. $\sqrt{1+\frac{x^2}{9}}$

Ans. $\int \sqrt{1+\frac{x^2}{9}}\ dx$

$$= \int \sqrt{\frac{9+x^2}{9}}\ dx$$

$$= \int \frac{\sqrt{x^2+3^2}}{3}\ dx$$

$$= \frac{1}{3}\int \sqrt{x^2+3^2}\ dx$$

$$= \frac{1}{3}\left[\frac{x}{2}\sqrt{x^2+3^2} + \frac{3^2}{2}\log\left|x+\sqrt{x^2+3^2}\right|\right] + c$$

$$\left[\because \int \sqrt{x^2+a^2}\ dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|\right] + c$$

$$= \frac{x}{6}\sqrt{x^2+9} + \frac{3}{2}\log\left|x+\sqrt{x^2+9}\right| + c$$

Choose the correct answer in Exercise 10 to 11.

10. $\int \sqrt{1+x^2}\ dx$ **is equal to:**

(A) $\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|\left(x+\sqrt{1+x^2}\right)\right|+C$

(B) $\frac{2}{3}\left(1+x^2\right)^{\frac{3}{2}}+C$

(C) $\frac{2}{3}x\left(1+x^2\right)^{\frac{3}{2}}+C$

(D) $\frac{x^2}{2}\sqrt{1+x^2}+\frac{1}{2}x^2\log\left|\left(x+\sqrt{1+x^2}\right)\right|+C$

Ans. $\int\sqrt{1+x^2}\;dx$

= $\int\sqrt{x^2+1^2}\;dx$

= $\frac{x}{2}\sqrt{x^2+1^2}+\frac{1^2}{2}\log\left|x+\sqrt{x^2+1^2}\right|+C$

= $\frac{x}{2}\sqrt{x^2+1}+\frac{1}{2}\log\left|x+\sqrt{x^2+1}\right|+C$

$\left[\because\int\sqrt{x^2+a^2}\;dx=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|\right]+c$

Therefore, option (A) is correct.

11.

$\int\sqrt{x^2-8x+7}\,dx$ is equal to

(A) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}+9\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

(B) $\frac{1}{2}(x+4)\sqrt{x^2-8x+7}+9\log\left|x+4+\sqrt{x^2-8x+7}\right|+C$

(C) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-3\sqrt{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

(D) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

Ans. $\int\sqrt{x^2-8x+7}\,dx$

$= \int\sqrt{x^2-8x+16-16+7}\,dx$

$= \int\sqrt{(x-4)^2-3^2}\,dx$

$= \left(\frac{x-4}{2}\right)\sqrt{(x-4)^2-3^2}-\frac{3^2}{2}\log\left|x-4+\sqrt{(x-4)^2-3^2}\right|+C$

$\left[\because \int\sqrt{x^2-a^2}\,dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|\right]$

$= \left(\frac{x-4}{2}\right)\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

Therefore, option (D) is correct.