CBSE Class–12 Mathematics

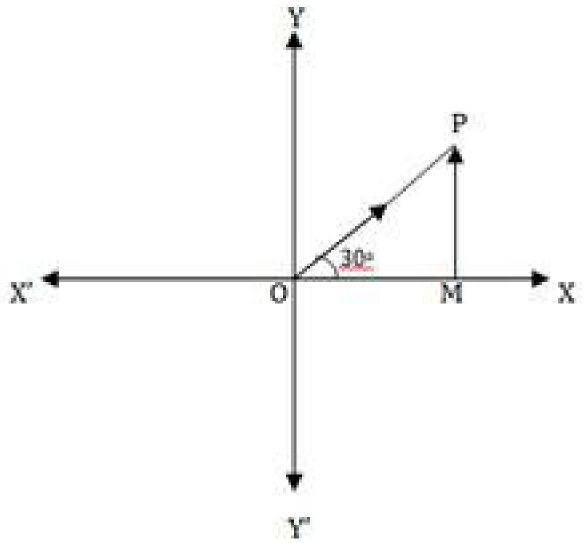
NCERT solution

Chapter - 10

Vector Algebra - Miscellaneous Exercise

1. Write down a unit vector in XY-plane making an angle of 30° in anti-clockwise direction with the positive direction of $x-$ axis.

Ans. Let $\overrightarrow{OP}$ be the unit vector in XY-plane such that $\angle$ XOP = 30°.



Therefore, $\left|\overrightarrow{OP}\right| = 1 \Rightarrow$ OP = 1 ……….(i)

By Triangle Law of Addition of vectors,

In ΔOMP, $\overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{MP}$ = $(OM)\hat{i} + (MP)\hat{j}$

[Unit vector along OX is $\hat{i}$ and that is along OY is $\hat{j}$]

$\Rightarrow \overrightarrow{OP} = OP\dfrac{OM}{OP}\hat{i} + OP\dfrac{MP}{OP}\hat{j}$ [Dividing and multiplying by OP in R.H.S.]

$\Rightarrow \overrightarrow{OP} = (1)\cos 30^\circ \hat{i} + (1)\sin 30^\circ \hat{j}$ [Using eq. (i)]

$\Rightarrow \overrightarrow{OP} = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$

2. Find the scalar components and magnitude of the vector joining the points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2).

Ans. Given points are P (x_1, y_1, z_1) and Q (x_2, y_2, z_2)

$\Rightarrow$ Position vector of point P = (x_1, y_1, z_1) = $x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

And Position vector of point Q = (x_2, y_2, z_2) = $x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

Now $\overrightarrow{PQ}$ = Position vector of Q – Position vector of P

= $x_2\hat{i} + y_2\hat{j} + z_2\hat{k} - \left(x_1\hat{i} + y_1\hat{j} + z_1\hat{k}\right)$

= $x_2\hat{i} + y_2\hat{j} + z_2\hat{k} - x_1\hat{i} - y_1\hat{j} - z_1\hat{k}$ = $(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

$\therefore$ Scalar components of the vector $\overrightarrow{PQ}$ are the coefficients of $\hat{i}, \hat{j}, \hat{k}$ in $\overrightarrow{PQ}$, i.e.,

$(x_2 - x_1), (y_2 - y_1), (z_2 - z_1)$

And magnitude of vector $\overrightarrow{PQ}$ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

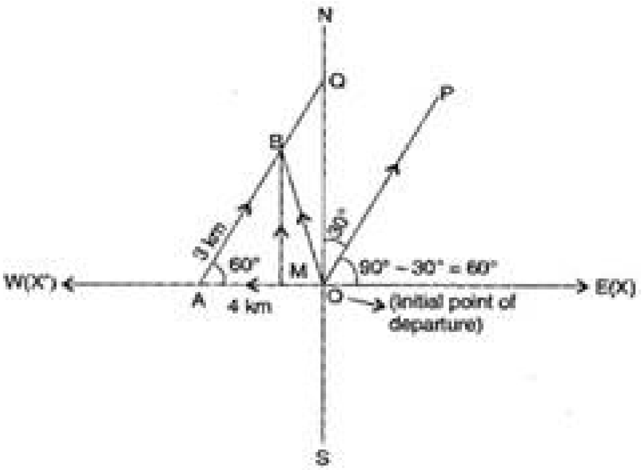
3. A girl walks 4 km towards west, then she walks 3 km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure.

Ans. Let the initial point of departure is origin (0, 0) and the girl walks a distance OA = 4 km towards west.

Through the point A, draw a line AQ parallel to a line OP, which is 30° East of North, i.e., in East-North quadrant making an angle of 30° with North.

Again, let the girl walks a distance AB = 3 km along this direction $\overrightarrow{OQ}$

$\therefore\ \overrightarrow{OA} = 4\left(-\hat{i}\right) = -4\hat{i}$ ....(i) [$\because$ Vector $\overrightarrow{OA}$ is along OX']



Now, draw BM perpendicular to $x -$ axis.

In ΔAMB, by Triangle Law of Addition of vectors,

$$\overrightarrow{AB} = \overrightarrow{AM} + \overrightarrow{MB} = (AM)\hat{i} + (MB)\hat{i}$$

Dividing and multiplying by AB in R.H.S.,

$$\overrightarrow{AB} = AB\,\frac{AM}{AB}\hat{i} + AB\,\frac{MB}{AB}\hat{j} = 3\cos 60^\circ\hat{i} + 3\sin 60^\circ\hat{j}$$

$$\Rightarrow\ \overrightarrow{AB} = 3\frac{1}{2}\hat{i} + 3\frac{\sqrt{3}}{2}\hat{i} = \frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j} \quad \text{..........(ii)}$$

$\therefore$ Girl's displacement from her initial point O of departure to final point B,

$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = -4\hat{i} + \left(\frac{3}{2}\hat{i} + \frac{3\sqrt{2}}{2}\hat{j}\right) = \left(-4 + \frac{3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

$$\Rightarrow\ \overrightarrow{OB} = \frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

4. If $\vec{a} = \vec{b} + \vec{c}$, then is it true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$? Justify your answer.

Ans. Given: $\vec{a} = \vec{b} + \vec{c}$

$\therefore$ Either the vectors $\vec{a}, \vec{b}, \vec{c}$ are collinear or form the sides of a triangle.

Case I: Vectors $\vec{a}, \vec{b}, \vec{c}$ are collinear.

Let $\vec{a} = \overrightarrow{AC}, \vec{b} = \overrightarrow{AB}$ and $\vec{c} = \overrightarrow{BC}$

Then $\vec{a} = \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \vec{b} + \vec{c}$

Also, $|\vec{a}|$ = AC = AB + BC = $|\vec{b}| + |\vec{c}|$

Case II: Vectors $\vec{a}, \vec{b}, \vec{c}$ form a triangle.

Here also by Triangle Law of vectors, $\vec{a} = \vec{b} + \vec{c}$

But $|\vec{a}| < |\vec{b}| + |\vec{c}|$ [$\because$ Each side of a triangle is less than sum of the other two sides]

$\therefore$ $|\vec{a} = \vec{b} + \vec{c}| = |\vec{b}| + |\vec{c}|$ is true only when vectors $\vec{b}$ and $\vec{c}$ are collinear vectors.

5. Find the value of x for which $x\left(\hat{i} + \hat{j} + \hat{k}\right)$ is a unit vector.

Ans. Since $x\left(\hat{i} + \hat{j} + \hat{k}\right) = x\hat{i} + x\hat{j} + x\hat{k}$ is a unit vector,

Therefore, $\left|x\hat{i} + x\hat{j} + x\hat{k}\right| = 1$

$\therefore \sqrt{x^2 + x^2 + x^2} = 1 \Rightarrow \sqrt{3x^2} = 1$

Squaring both sides, $3x^2 = 1$

$\Rightarrow x^2 = \frac{1}{3}$

$\Rightarrow x = \pm\frac{1}{\sqrt{3}}$

6. Find a vector of magnitude 5 units and parallel to the resultant of the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ **and** $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

Ans. Given: Vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

Let vector $\vec{c}$ be the resultant vector of $\vec{a}$ and $\vec{b}$.

$\therefore \vec{c} = \vec{a} + \vec{b}$ = $2\hat{i} + 3\hat{j} - \hat{k}$ + $\hat{i} - 2\hat{j} + \hat{k}$

= $3\hat{i} + \hat{j} + 0\hat{k}$

$\therefore$ Required vector pf magnitude 5 units and parallel (or collinear) to resultant vector $\vec{c} = \vec{a} + \vec{b}$ is

$$5\hat{c} = 5\frac{\vec{c}}{\left|\vec{c}\right|} = 5\left(\frac{3\hat{i} + \hat{j} + 0\hat{k}}{\sqrt{9+1+0}}\right)$$

= $\frac{5}{\sqrt{10}}\left(3\hat{i} + \hat{j}\right)$

= $\frac{5}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}\left(3\hat{i} + \hat{j}\right)$

= $\frac{5\sqrt{10}}{10}\left(3\hat{i} + \hat{j}\right)$

$\Rightarrow \hat{c} = \frac{\sqrt{10}}{2}\left(3\hat{i} + \hat{j}\right)$

= $\frac{3\sqrt{10}}{2}\hat{i} + \frac{\sqrt{10}}{2}\hat{j}$

7. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+\hat{k},$ find a unit vector parallel to the vector $2\vec{a}-\vec{b}+3\vec{c}.$

Ans. Given: Vectors $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+\hat{k}$

Let $\vec{d}=2\vec{a}-\vec{b}+3\vec{c}$

$= 2\left(\hat{i}+\hat{j}+\hat{k}\right)-\left(2\hat{i}-\hat{j}+3\hat{k}\right)+3\left(\hat{i}-2\hat{j}+\hat{k}\right)$

$= 2\hat{i}+2\hat{j}+2\hat{k}-2\hat{i}+\hat{j}-3\hat{k}+3\hat{i}-6\hat{j}+3\hat{k}$

$= 3\hat{i}-3\hat{j}+2\hat{k}$

$\therefore$ A unit vector parallel to the vector $\vec{d}$ is

$$\hat{d}=\frac{\vec{d}}{\left|\vec{d}\right|}=\frac{3\hat{i}-3\hat{j}+2\hat{k}}{\sqrt{9+9+4}}$$

$$=\frac{3\hat{i}-3\hat{j}+2\hat{k}}{\sqrt{22}}$$

$$=\frac{3}{\sqrt{22}}\hat{i}-\frac{3}{\sqrt{22}}\hat{j}+\frac{2}{\sqrt{22}}\hat{k}$$

8. Show that the points A $(1,-2,-8)$, B $(5,0,-2)$ and C (11, 3, 7) are collinear and find the ratio in which B divides AC.

Ans. Given: Points A $(1,-2,-8)$, B $(5,0,-2)$ and C (11, 3, 7).

$\therefore$ Position vector of point A = $(1,-2,-8)=\hat{i}-2\hat{j}-8\hat{k}$

Position vector of point B = $(5,0,-2)=5\hat{i}+0\hat{j}-2\hat{k}$

Position vector of point C = $(11,3,7)=11\hat{i}+3\hat{j}+7\hat{k}$

Now $\overrightarrow{AB}$ = Position vector of point B – Position vector of point A

= $5\hat{i}-2\hat{k}-\left(\hat{i}-2\hat{j}-8\hat{k}\right)$ = $5\hat{i}-2\hat{k}-\hat{i}+2\hat{j}+8\hat{k}$ = $4\hat{i}+2\hat{j}+6\hat{k}$

$\therefore\ AB=\left|\overrightarrow{AB}\right|=\sqrt{16+4+36}=\sqrt{56}=2\sqrt{14}$

Again $\overrightarrow{BC}$ = Position vector of point C – Position vector of point B

= $11\hat{i}+3\hat{j}+7\hat{k}-\left(5\hat{i}-2\hat{k}\right)$ = $11\hat{i}+3\hat{i}+7\hat{k}-5\hat{i}+2\hat{k}$ = $6\hat{i}+3\hat{j}+9\hat{k}$

$\therefore\ BC=\left|\overrightarrow{BC}\right|=\sqrt{36+9+81}=\sqrt{126}=3\sqrt{14}$

Again $\overrightarrow{AC}$ = Position vector of point C – Position vector of point A

= $11\hat{i}+3\hat{j}+7\hat{k}-\left(\hat{i}-2\hat{j}-8\hat{k}\right)$ = $11\hat{i}+3\hat{j}+7\hat{k}-\hat{i}+2\hat{j}+8\hat{k}$ = $10\hat{i}+5\hat{j}+15\hat{k}$

$\therefore\ AC=\left|\overrightarrow{AC}\right|=\sqrt{100+25+225}=\sqrt{350}=5\sqrt{14}$

Now $\overrightarrow{AB}+\overrightarrow{AC}=4\hat{i}+2\hat{j}+6\hat{k}+6\hat{i}+3\hat{j}+9\hat{k}$ = $10\hat{i}+5\hat{j}+15\hat{k}$ = $\overrightarrow{AC}$

Therefore, points A, B, C are either collinear or are the vertices of a triangle ABC.

Again AB + BC = $2\sqrt{14}+3\sqrt{14}=5\sqrt{14}$ = AC

Now to find ratio in which B divides AC

Let the point B divides AC in the ratio $\lambda:1$.

Therefore, using section formula, Position vector of point B $(5,0,-2)$ is $\dfrac{\lambda\vec{c}+1\vec{a}}{\lambda+1}$

$\Rightarrow\ (5,0,-2)$ = $\dfrac{\lambda(11,3,7)+1(1,-2,-8)}{\lambda+1}$

$\Rightarrow\ (5,0,-2)(\lambda+1)=\lambda(11,3,7)+(1,-2,-8)$

$\Rightarrow (\lambda+1)\left(5\hat{i}+0\hat{j}-2\hat{k}\right)=\lambda\left(11\hat{i}+3\hat{j}+7\hat{k}\right)+\left(\hat{i}-2\hat{j}-8\hat{k}\right)$

$\Rightarrow 5(\lambda+1)\hat{i}-2(\lambda+1)\hat{k}=11\lambda\hat{i}+3\lambda\hat{j}+7\lambda\hat{k}+\hat{i}-2\hat{j}-8\hat{k}$

Comparing coefficients of $\hat{i}, \hat{j}, \hat{k}$ both sides, we get

$5\lambda+5=11\lambda+1, \quad 0=3\lambda-2, \quad -(2\lambda+2)=7\lambda-8$

$\Rightarrow -6\lambda=-4, \quad 3\lambda=2, \quad -2\lambda-2=7\lambda-8$

$\Rightarrow \lambda=\frac{4}{6}=\frac{2}{3}, \quad \lambda=\frac{2}{3}, \quad \lambda=\frac{6}{9}=\frac{2}{3}$

Therefore, required ratio = $\lambda:1$ = $\frac{2}{3}$: 1 = 2 : 3

---

9. Find the position vector of a point R which divides the line joining the two points P and Q whose position vectors are $\left(2\vec{a}+\vec{b}\right)$ and $\left(\vec{a}-3\vec{b}\right)$ externally in the ratio 1 : 2. Also, show that P is the middle point of line segment RQ.

Ans. Since position vector of point R dividing the join of P and Q externally in the ratio 1 : 2 = $m:n$ is given by $\vec{c}=\frac{m\vec{b}-n\vec{a}}{m-n}$

$\therefore \quad \vec{c}=\frac{1\left(\vec{a}-3\vec{b}\right)-2\left(2\vec{a}+\vec{b}\right)}{1-2} \quad \Rightarrow \vec{c}=\frac{\vec{a}-3\vec{b}-4\vec{a}-2\vec{b}}{-1}=\frac{-3\vec{a}-5\vec{b}}{-1}=3\vec{a}+5\vec{b}$

Again position vector of the middle point of the line segment RQ

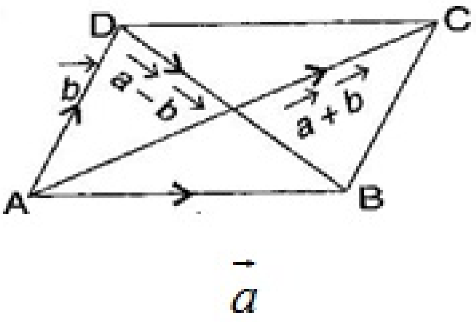
= $\frac{1}{2}$ (Position vector of point R – Position vector of point Q)

= $\frac{1}{2}\left(3\vec{a}+5\vec{b}+\vec{a}-3\vec{b}\right)$ = $\frac{1}{2}\left(4\vec{a}+2\vec{b}\right)$ = $2\vec{a}+\vec{b}$ = Position vector of point P (given)

Therefore, P is the middle point of the line segment RQ.

10. Two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$. Find the unit vector parallel to its diagonal. Also, find its area.

Ans. Let ABCD is a parallelogram.



Given: The vectors representing two adjacent sides of this parallelogram say,

$\vec{a}=2\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}-3\hat{k}$

Now vectors along the diagonals $\overrightarrow{AC}$ and $\overrightarrow{DB}$ of the parallelogram are

$\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$

$\therefore\ \vec{a}+\vec{b}$ = $2\hat{i}-4\hat{j}+5\hat{k}+\hat{i}-2\hat{j}-3\hat{k}$ = $3\hat{i}-6\hat{j}+2\hat{k}$

And $\vec{a}-\vec{b}$ = $2\hat{i}-4\hat{j}+5\hat{k}-\hat{i}+2\hat{j}+3\hat{k}$ = $\hat{i}-2\hat{j}+8\hat{k}$

Therefore, Unit vectors parallel to (or along) diagonals are

$$\frac{\vec{a}+\vec{b}}{\left|\vec{a}+\vec{b}\right|} \text{ and } \frac{\vec{a}-\vec{b}}{\left|\vec{a}-\vec{b}\right|}$$

$$\Rightarrow \frac{3\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{9+36+4}} \text{ and } \frac{\hat{i}-2\hat{j}+8\hat{k}}{\sqrt{1+4+64}} \Rightarrow \frac{3\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{49}} \text{ and } \frac{\hat{i}-2\hat{j}+8\hat{k}}{\sqrt{69}}$$

$$\Rightarrow \frac{3\hat{i}-6\hat{j}+2\hat{k}}{7} \text{ and } \frac{\hat{i}-2\hat{j}+8\hat{k}}{\sqrt{69}}$$

Now Area of parallelogram = $\vec{a}\times\vec{b}$ = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 5 \\ 1 & -2 & -3 \end{vmatrix}$

= $(12+10)\hat{i}-(-6-5)\hat{j}+(-4+4)\hat{k}$ = $22\hat{i}+11\hat{j}+0\hat{k}$

$\Rightarrow \left|\vec{a}\times\vec{b}\right| = \sqrt{(22)^2+(11)^2+(0)^2}$

= $\sqrt{484+121}$ = $\sqrt{605}$ = $11\sqrt{5}$ sq. units

---

11. Show that the direction cosines of a vector equally inclined $\frac{1}{\sqrt{3}}$ to the axes OX, OY and OZ are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$.

Ans. Let l, m, n be the direction cosines of a vector equally inclined to axes OX, OY and OZ respectively.

$\therefore$ A unit vector along the given vector is

$l\hat{i}+m\hat{j}+n\hat{k}$ and $\left|\hat{a}\right|=1$

$\Rightarrow \sqrt{l^2+m^2+n^2}=1 \Rightarrow l^2+m^2+n^2=1$ ....(i)

Let the given vector (for which unit vector is $\hat{a}$) make equal angle (given) θ, θ, θ (say) with OX$(\Rightarrow \hat{i})$, OY$(\Rightarrow \hat{j})$ and OZ$(\Rightarrow \hat{k})$.

$\therefore$ The given vector is in positive octant OXYZ and hence θ is acute. ....(ii)

Now angle θ between $\hat{a}$ and $\hat{i}$ $\cos\theta = \frac{\hat{a}.\hat{i}}{\left|\hat{a}\right|.\left|\hat{i}\right|}$

$$\Rightarrow \cos\theta = \frac{\left(l\hat{i}+m\hat{j}+n\hat{k}\right).\left(\hat{i}+0\hat{j}+0\hat{k}\right)}{(1)(1)}$$

$$\Rightarrow \cos\theta = l(1)+m(0)+n(0)$$

$\Rightarrow l = \cos\theta$ ....(iii)

Similarly, angle θ between $\hat{a}$ and $\hat{j}$, $m = \cos\theta$ ....(iv)

And angle θ between $\hat{a}$ and $\hat{k}$, $n = \cos\theta$ ....(v)

Putting the values of l, m, n in eq. (i), we get

$$\cos^2\theta + \cos^2\theta + \cos^2\theta = 1$$

$$\Rightarrow 3\cos^2\theta = 1$$

$$\Rightarrow \cos^2\theta = \frac{1}{3}$$

$$\Rightarrow \cos\theta = \pm\frac{1}{\sqrt{3}}$$

But $\cos\theta = \frac{1}{\sqrt{3}}$ [$\because$ θ is acute and hence $\cos\theta$ is positive]

Therefore, required vectors l, m, n are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ and $\frac{1}{\sqrt{3}}$.

---

12. Let $\vec{a} = \hat{i}+4\hat{j}+2\hat{k}, \vec{b} = 3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c} = 2\hat{i}-\hat{j}+4\hat{k}$. Find a vector $\vec{b}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 15$.

Ans. Given: Vectors $\vec{a} = \hat{i}+4\hat{j}+2\hat{k}$ and $\vec{b} = 3\hat{i}-2\hat{j}+7\hat{k}$

We know that the cross-product of two vectors, $\vec{a}\times\vec{b}$ is a vector perpendicular to both $\vec{a}$ and $\vec{b}$.

Hence, vector $\vec{a}$ which is also perpendicular to both $\vec{a}$ and $\vec{b}$ is $\vec{d} = \lambda\left(\vec{a}\times\vec{b}\right)$ where $\lambda = 1$ or some other scalar.

Therefore, $\vec{d} = \lambda\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix}$

= $\lambda\left[\hat{i}\,(28+4) - \hat{j}\,(7-6) + \hat{k}\,(-2-12)\right]$

$\Rightarrow\ \vec{d} = \lambda\left[32\hat{i} - \hat{j} - 14\hat{k}\right]$ ....(i)

$\Rightarrow\ \vec{d} = 32\lambda\hat{i} - \lambda\hat{j} - 14\lambda\hat{k}$

Now given $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{c}.\vec{d} = 15$

$\vec{c}.\vec{d} = 15$

$\Rightarrow\ 2(32\lambda) + (-1)(-\lambda) + 4(-14\lambda) = 15$

$\Rightarrow\ 64\lambda + \lambda - 56\lambda = 15$

$\Rightarrow\ 9\lambda = 15$

$\Rightarrow\ \lambda = \frac{15}{9}$

$\Rightarrow\ \lambda = \frac{5}{3}$

Putting $\lambda = \frac{5}{3}$ in eq. (i), we get

$\vec{d} = \frac{5}{3}\left[32\hat{i} - \hat{j} - 14\hat{k}\right]$

$\Rightarrow \vec{d} = \frac{1}{3}\left[160\hat{i} - 5\hat{j} - 70\hat{k}\right]$

13. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ.

Ans. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$

Now $\vec{b} + \vec{c} = \vec{d}$ (say) = $(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}$

$\therefore$ $\hat{d}$ a unit vector along $\vec{b} + \vec{c} = \vec{d}$ is

$$\hat{d} = \frac{\vec{d}}{|\vec{d}|} = \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + (6)^2 + (2)^2}}$$

$$= \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{4+\lambda^2 + 4\lambda + 36 + 4}}$$

$$= \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{4+\lambda^2 + 4\lambda + 40}}$$

$$\Rightarrow \hat{d} = \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{\lambda^2 + 4\lambda + 44}}$$

$$= \frac{(2+\lambda)}{\sqrt{\lambda^2 + 4\lambda + 44}}\hat{i} + \frac{6}{\sqrt{\lambda^2 + 4\lambda + 44}}\hat{j} - \frac{2}{\sqrt{\lambda^2 + 4\lambda + 44}}\hat{k} \quad .....(i)$$

Also given Dot product of $\vec{a}$ and $\vec{d}$ is 1.

$\Rightarrow \vec{a} \cdot \vec{d} = 1$

$$\Rightarrow \frac{1(2+\lambda)}{\sqrt{\lambda^2 + 4\lambda + 44}} + \frac{1(6)}{\sqrt{\lambda^2 + 4\lambda + 44}} + \frac{1(-2)}{\sqrt{\lambda^2 + 4\lambda + 44}} = 1$$

$\Rightarrow 2+\lambda+6-2=\sqrt{\lambda^2+4\lambda+44}$

$\Rightarrow \lambda+6=\sqrt{\lambda^2+4\lambda+44}$

Squaring both sides,

$(\lambda+6)^2=\lambda^2+4\lambda+44$

$\Rightarrow (\lambda+6)^2=\lambda^2+4\lambda+44$

$\Rightarrow \lambda^2+12\lambda+36=\lambda^2+4\lambda+44$

$\Rightarrow 8\lambda=8 \Rightarrow \lambda=1$

14. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}, \vec{b}, \vec{c}$.

Ans. Given: $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitude.

$\therefore \vec{a}.\vec{b}=\vec{b}.\vec{a}=0, \vec{b}.\vec{c}=\vec{c}.\vec{b}=0, \vec{c}.\vec{a}=\vec{a}.\vec{c}=0$ ....(i)

And $|\vec{a}|=|\vec{b}|=|\vec{c}|=\lambda$ (say) ....(ii)

Let vector $\vec{d}=\vec{a}+\vec{b}+\vec{c}$ make angles $\theta_1, \theta_2, \theta_3$ with vectors $\vec{a}, \vec{b}, \vec{c}$ respectively.

$\therefore \cos\theta_1=\dfrac{\vec{d}.\vec{a}}{|\vec{d}|.|\vec{a}|}=\dfrac{(\vec{a}+\vec{b}+\vec{c}).\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}$

$=\dfrac{\vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{a}.\vec{c}}{|\vec{a}+\vec{b}+\vec{c}|.|\vec{a}|}$

$=\dfrac{\vec{a}.\vec{a}+0+0}{|\vec{a}+\vec{b}+\vec{c}|.|\vec{a}|}$ [From eq. (i)]

$\Rightarrow \cos\theta_1 = \dfrac{\left|\vec{a}\right|^2}{\left|\vec{a}+\vec{b}+\vec{c}\right|.\left|\vec{a}\right|} = \dfrac{\left|\vec{a}\right|}{\left|\vec{a}+\vec{b}+\vec{c}\right|}$ ....(iii)

We know that $\left|\vec{a}+\vec{b}+\vec{c}\right|^2 = \left(\vec{a}+\vec{b}+\vec{c}\right)^2$

$= \left(\vec{a}\right)^2 + \left(\vec{b}\right)^2 + \left(\vec{c}\right)^2 + 2\vec{b}.\vec{c} + 2\vec{a}.\vec{b} + 2\vec{a}.\vec{c}$

Putting the values from eq. (i) and (ii),

$\left|\vec{a}+\vec{b}+\vec{c}\right|^2 = (\lambda)^2 + (\lambda)^2 + (\lambda)^2 + 0 + 0 + 0 = 3\lambda^2$

$\Rightarrow \left|\vec{a}+\vec{b}+\vec{c}\right| = \lambda\sqrt{3}$

Now $\cos\theta_1 = \dfrac{\left|\vec{a}\right|}{\left|\vec{a}+\vec{b}+\vec{c}\right|}$

$= \dfrac{\lambda}{\lambda\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \theta_1 = \cos^{-1}\dfrac{1}{\sqrt{3}}$

Similarly, $\theta_2 = \cos^{-1}\dfrac{1}{\sqrt{3}}$ and $\theta_3 = \cos^{-1}\dfrac{1}{\sqrt{3}}$

$\therefore \theta_1 = \theta_2 = \theta_3$

$= \cos^{-1}\dfrac{1}{\sqrt{3}}$

Therefore, $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to the vectors $\vec{a}, \vec{b}$ and $\vec{c}$.

15. Prove that $\left(\vec{a}+\vec{b}\right).\left(\vec{a}+\vec{b}\right)=\left|\vec{a}\right|^2+\left|\vec{b}\right|^2$, if and only if $\vec{a},\vec{b}$ are perpendicular given $\vec{a}\neq\vec{0},\vec{b}\neq\vec{0}.$

Ans. We know that $\left(\vec{a}+\vec{b}\right).\left(\vec{a}+\vec{b}\right)=\vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{b}.\vec{a}+\vec{b}.\vec{b}$ = $\left|\vec{a}\right|^2+\vec{a}.\vec{b}+\vec{a}.\vec{b}+\left|\vec{b}\right|^2$

$\Rightarrow\ \left(\vec{a}+\vec{b}\right).\left(\vec{a}+\vec{b}\right)$

= $\left|\vec{a}\right|^2+\left|\vec{b}\right|^2+2\left|\vec{a}\right|.\left|\vec{b}\right|$ ....(i)

Now if $\vec{a}$ and $\vec{b}$ are perpendicular $\Rightarrow$ $\vec{a}.\vec{b}=0$

Putting $\vec{a}.\vec{b}=0$ in $\left(\vec{a}+\vec{b}\right).\left(\vec{a}+\vec{b}\right)$

= $\left|\vec{a}\right|^2+\left|\vec{b}\right|^2+2\left|\vec{a}\right|.\left|\vec{b}\right|$,

$\Rightarrow\left(\vec{a}+\vec{b}\right).\left(\vec{a}+\vec{b}\right)$

= $\left|\vec{a}\right|^2+\left|\vec{b}\right|^2$ ....(ii)

$\Rightarrow\ \left|\vec{a}\right|^2+\left|\vec{b}\right|^2=\left|\vec{a}\right|^2+\left|\vec{b}\right|^2+2\left|\vec{a}\right|.\left|\vec{b}\right|$

= $\left|\vec{a}\right|^2+\left|\vec{b}\right|^2$ [Putting value of $\left(\vec{a}+\vec{b}\right).\left(\vec{a}+\vec{b}\right)$ in eq. (i)]

$\Rightarrow\ 0=2\vec{a}.\vec{b}$

$\Rightarrow\ \vec{a}.\vec{b}=0$

But $\vec{a}\neq 0,\vec{b}\neq 0$ (given)

Therefore, vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

---

16. Choose the correct answer:

If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a}.\vec{b} \geq 0$ only when:

(A) $0 < \theta < \frac{\pi}{2}$

(B) $0 \leq \theta \leq \frac{\pi}{2}$

(C) $0 < \theta < \pi$

(D) $0 \leq \theta \leq \pi$

Ans. Given: $\vec{a}.\vec{b} \geq 0$

$\Rightarrow \left|\vec{a}\right|.\left|\vec{b}\right|\cos\theta \geq 0$

$\Rightarrow \cos\theta \geq 0$

[$\because \left|\vec{a}\right|$ and $\left|\vec{b}\right|$ being lengths of vectors are always $\geq$ 0]

Therefore, option (B) is correct.

---

17. Choose the correct answer:

Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a}+\vec{b}$ is a unit vector if:

(A) $\theta = \frac{\pi}{4}$

(B) $\theta = \frac{\pi}{3}$

(C) $\theta = \frac{\pi}{2}$

(D) $\theta = \frac{2\pi}{3}$

Ans. Given: $\vec{a}, \vec{b}$ and $\vec{a}+\vec{b}$ are unit vectors.

$\Rightarrow |\vec{a}|=1, |\vec{b}|=1$ and $|\vec{a}+\vec{b}|=1$

Now squaring both sides of $|\vec{a}+\vec{b}|=1$, we have,

$|\vec{a}+\vec{b}|^2=1$

$\Rightarrow (\vec{a}+\vec{b})^2=1$

$\Rightarrow (\vec{a})^2+(\vec{b})^2+2\vec{a}.\vec{b}=1$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+2|\vec{a}|.|\vec{b}|\cos\theta=1$, where θ is the given angle between vectors $\vec{a}$ and $\vec{b}$.

Putting $|\vec{a}|=1, |\vec{b}|=1$, we have, $1+1+2\cos\theta=1$

$\Rightarrow 2\cos\theta=-1$

$\Rightarrow \cos\theta=\frac{-1}{2} = \cos 120^\circ$

$\Rightarrow \theta=120^\circ$

$= 120^\circ \times \frac{\pi}{180^\circ} = \frac{2\pi}{3}$

Therefore, option (D) is correct.

18. Choose the correct answer:

The value of $\hat{i}.(\hat{j}\times\hat{k})+\hat{j}.(\hat{i}\times\hat{k})+\hat{k}.(\hat{i}\times\hat{j})$ **is:**

(A) 0

(B) -1

(C) 1

(D) 3

Ans. $\hat{i}.\left(\hat{j}\times\hat{k}\right)+\hat{j}.\left(\hat{i}\times\hat{k}\right)+\hat{k}.\left(\hat{i}\times\hat{j}\right)=\hat{i}.\hat{i}+\hat{j}.\left(-\hat{j}\right)+\hat{k}.\hat{k}$

Also $\hat{i}\times\hat{k}=-\hat{k}\times\hat{i}=-\hat{j}$ = 1 – 1 + 1 = 1

Therefore, option (C) is correct.

---

19. If θ be the angle between any two vectors $\vec{a}$ and $\vec{b}$, then $\left|\vec{a}.\vec{b}\right|=\left|\vec{a}\times\vec{b}\right|$, when θ is equal to:

(A) 0

(B) $\dfrac{\pi}{4}$

(C) $\dfrac{\pi}{2}$

(D) π

Ans. Given: $\left|\vec{a}.\vec{b}\right|=\left|\vec{a}\times\vec{b}\right|$

$\Rightarrow \left|\vec{a}\right|.\left|\vec{b}\right|\cos\theta=\left|\vec{a}\right|.\left|\vec{b}\right|\sin\theta$

And this equation is true only for option (B) namely $\theta=\dfrac{\pi}{4}$, since $\cos\dfrac{\pi}{4}=\sin\dfrac{\pi}{4}=\dfrac{1}{\sqrt{2}}$

Therefore, option (B) is correct.