CBSE Class–12 Mathematics

NCERT solution

Chapter - 7

Integrals - Exercise 7.6

Integrate the functions in Exercises 1 to 8.

1. $x\sin x$

Ans. $\int x\sin x\,dx$

$= x\int \sin x\,dx - \int\left(\frac{d}{dx}x\int \sin x\,dx\right)dx$

[Applying product rule]

$= x(-\cos x) - \int 1(-\cos x)\,dx$

$= -x\cos x - \int -\cos x\,dx$

$= -x\cos x + \int \cos x\,dx$

$= -x\cos x + \sin x + c$ Ans.

2. $x\sin 3x$

Ans. $\int x\sin 3x\,dx$

$= x\int \sin 3x\,dx - \int\left(\frac{d}{dx}x\int \sin 3x\,dx\right)dx$

[Applying product rule]

$= x\left(\frac{-\cos 3x}{3}\right) - \int 1\left(\frac{-\cos 3x}{3}\right)dx + c$

= $\frac{-1}{3}x\cos 3x+\frac{1}{3}\int \cos 3x\ dx+c$

= $\frac{-1}{3}x\cos 3x+\frac{1}{3}\frac{\sin 3x}{3}+c$

= $\frac{-1}{3}x\cos 3x+\frac{1}{9}\sin 3x+c$

= $-\frac{x}{3}xcos3x+\frac{1}{9}sin3x+c$ Ans.

3. x^2e^x

Ans. $\int x^2\ e^x\ dx$

= $x^2\int e^x\ dx-\int\left[\frac{d}{dx}x^2\int e^x\ dx\right]dx$

[Applying product rule]

= $x^2\ e^x-\int 2xe^x\ dx$

= $x^2\ e^x-2\int x\ e^x\ dx$

= $x^2\ e^x-2\left[x\int e^x\ dx-\int\left\{\frac{d}{dx}x\int e^x\ dx\right\}\ dx\right]$

[Again applying product rule]

= $x^2\ e^x-2\left(x\ e^x-\int 1.e^x\ dx\right)$

= $x^2\ e^x-2\left(x\ e^x-\int e^x\ dx\right)$

= $x^2\ e^x-2x\ e^x+2\int e^x\ dx$

= $x^2\ e^x-2x\ e^x+2e^x+c$

= $e^x\left(x^2-2x+2\right)+c$ Ans.

4. $x\log x$

Ans. $\int x\log x\,dx$

= $\int(\log x).x\,dx$

[Applying product rule]

= $(\log x)\int x\,dx-\int\left[\frac{d}{dx}\log x\int x\,dx\right]dx$

= $(\log x)\frac{x^2}{2}-\int\frac{1}{x}.\frac{x^2}{2}\,dx$

= $\frac{1}{2}x^2\log x-\frac{1}{2}\int x\,dx$

= $\frac{1}{2}x^2\log x-\frac{1}{2}\frac{x^2}{2}+c$

= $\frac{x^2}{2}\log x-\frac{x^2}{4}+c$ Ans.

5. $x\log 2x$

Ans. $\int x\log 2x\,dx$

= $\int(\log 2x).x\,dx$

= $(\log 2x)\int x\,dx-\int\left[\frac{d}{dx}\log 2x\int x\,dx\right]dx$

[Applying product rule]

$= (\log 2x)\frac{x^2}{2} - \int \frac{1}{2x}.2.\frac{x^2}{2}\ dx$

$= \frac{1}{2}x^2 \log 2x - \frac{1}{2}\int x\ dx$

$= \frac{1}{2}x^2 \log 2x - \frac{1}{2}\frac{x^2}{2} + c$

$= \frac{x^2}{2}\log 2x - \frac{x^4}{4} + c$ Ans.

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6. $x^2 \log x$

Ans. $\int x^2 \log x\ dx$

$= \int (\log x) x^2\ dx$

$= \log x \int x^2\ dx - \int \left(\frac{d}{dx}\log x \int x^2\ dx\right) dx$

[Applying product rule]

$= (\log x)\frac{x^3}{3} - \int \frac{1}{x}.\frac{x^3}{3}\ dx$

$= \frac{x^3}{3}\log x - \frac{1}{3}\int x^2\ dx$

$= \frac{x^3}{3}\log x - \frac{1}{3}\frac{x^3}{3} + c$

$= \frac{x^3}{3}\log x - \frac{x^3}{9} + c$ Ans.

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7. $x \sin^{-1} x$

Ans. Let I = $\int x\sin^{-1}x\,dx$

Putting $x=\sin\theta$

$\Rightarrow\ dx=\cos\theta\ d\theta$

$\therefore$ I = $\int \sin\theta.\theta.\cos\theta\ d\theta$

= $\frac{1}{2}\int\theta.2\sin\theta\cos\theta\ d\theta$

$=\frac{1}{2}\int\theta.\,sin2\theta\ d\theta$

= $\frac{1}{2}\left[\theta\left(\frac{-\cos2\theta}{2}\right)-\int 1.\left(\frac{-\cos2\theta}{2}\right)d\theta\right]$

[Integrating by parts]

= $\frac{1}{4}\left[-\theta\cos2\theta+\int\cos2\theta\ d\theta\right]$

= $\frac{1}{4}\left[-\theta\cos2\theta+\frac{\sin2\theta}{2}\right]+c$

= $\frac{1}{4}\left[-\theta\left(1-2\sin^2\theta\right)+\sin\theta\cos\theta\right]+c$

= $\frac{1}{4}\left[-\sin^{-1}x.\left(1-2x^2\right)+x\sqrt{1-x^2}\right]+c$

= $\frac{1}{4}\left(2x^2-1\right)\sin^{-1}x+\frac{x\sqrt{1-x^2}}{4}+c$ Ans.

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8. $x\tan^{-1}x$

Ans. Let I = $\int x\tan^{-1}x\,dx$

= $\int \left(\tan^{-1} x\right).x\, dx$

= $\left(\tan^{-1} x\right).\frac{x^2}{2} - \int \frac{1}{1+x^2}.\frac{x^2}{2}\ dx$

= $\frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int \frac{x^2}{1+x^2}\ dx$

$= \frac{x^2}{2}tan^{-1}x - \frac{1}{2}\int \frac{x^2+1-1}{x^2+1}dx$

$= \frac{x^2}{2}tan^{-1}x - \frac{1}{2}\int (1 - \frac{1}{x^2+1})dx$

= $\frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\left(x - \tan^{-1} x\right) + c$

= $\frac{1}{2}\left[x^2 \tan^{-1} x - x + \tan^{-1} x\right] + c$

$= \frac{x^2}{2}tan^{-1}x - \frac{x}{2} + \frac{1}{2}tan^{-1}x + c\ Ans.$

Integrate the functions in Exercises 9 to 15.

9. $x\cos^{-1} x$

Ans. Let I = $\int x\cos^{-1} x\, dx$ ....(i)

Putting $\cos^{-1} x = \theta$

$\Rightarrow\ x = \cos\theta$

$\Rightarrow\ \frac{dx}{d\theta} = -\sin\theta$

$\Rightarrow dx = -\sin\theta\ d\theta$

$\therefore$ From eq. (i),

I = $\int (\cos\theta).\theta.(-\sin\theta\ d\theta)$

= $\dfrac{-1}{2}\int \theta.(2\sin\theta\cos\theta)\ d\theta$

$= -\frac{1}{2}\int \theta.\, sin2\theta\ d\theta$

[Applying product rule]

= $\dfrac{-1}{2}\left[\theta\left(\dfrac{-\cos 2\theta}{2}\right) - \int 1\left(\dfrac{-\cos 2\theta}{2}\right)\ d\theta\right]$

= $\dfrac{-1}{2}\left[\dfrac{-1}{2}\theta.\cos 2\theta + \dfrac{1}{2}\int \cos 2\theta\ d\theta\right]$

= $\dfrac{1}{4}\theta.\cos 2\theta - \dfrac{1}{4}\left(\dfrac{\sin 2\theta}{2}\right) + c$

= $\dfrac{1}{4}\theta.\cos 2\theta - \dfrac{1}{8}(2\sin\theta\cos\theta) + c$

= $\dfrac{1}{4}\theta.\left(2\cos^2\theta - 1\right) - \dfrac{1}{4}\sqrt{1-\cos^2\theta}.\cos\theta + c$

Putting $\cos\theta = x$ and $\theta = \cos^{-1} x$

= $\dfrac{1}{4}\left(\cos^{-1} x\right).\left(2x^2 - 1\right) - \dfrac{1}{4}\sqrt{1-x^2}.x + c$

= $\left(2x^2 - 1\right)\dfrac{\cos^{-1} x}{4} - \dfrac{x}{4}\sqrt{1-x^2} + c$ Ans.

10. $\left(\sin^{-1} x\right)^2$

Ans. Putting $x = \sin\theta \Rightarrow dx = \cos\theta\ d\theta$

$\therefore \int \left(\sin^{-1} x\right)^2\ dx$

= $\int \theta^2 \cos\theta \, d\theta$

[Applying product rule]

= $\theta^2 \sin\theta - \int 2\theta \sin\theta \, d\theta$

= $\theta^2 \sin\theta - 2\int \theta \sin\theta \, d\theta$

[Again applying product rule]

= $\theta^2 \sin\theta - 2\left[\theta(-\cos\theta) - \int 1.(-\cos\theta)\, d\theta\right]$

= $\theta^2 \sin\theta + 2\theta\cos\theta - 2\int \cos\theta \, d\theta$

= $\theta^2 \sin\theta + 2\theta\cos\theta - 2\sin\theta + c$

= $x\left(\sin^{-1} x\right)^2 + 2\sqrt{1-x^2}\sin^{-1} x - 2x + c$ Ans.

11. $\dfrac{x\cos^{-1} x}{\sqrt{1-x^2}}$

Ans. Let I = $\int \dfrac{x\cos^{-1} x}{\sqrt{1-x^2}}\, dx$ ....(i)

Putting $\cos^{-1} x = \theta$

$\Rightarrow x = \cos\theta$

$\Rightarrow \dfrac{dx}{d\theta} = -\sin\theta$

$\Rightarrow dx = -\sin\theta \, d\theta$

$\therefore$ From eq. (i),

I = $\int \frac{(\cos\theta)\theta}{\sqrt{1-\cos^2\theta}}(-\sin\theta\ d\theta)$

$= -\int \frac{\theta.cos\theta.sin\theta}{sin\theta}d\theta$

$= -\int \theta.\, cos\theta\ d\theta$

[Applying product rule]

= $-\left[\theta.\sin\theta - \int 1.\sin\theta\ d\theta\right]$

= $-\theta\sin\theta + \int \sin\theta\ d\theta$

= $-\theta\sin\theta - \cos\theta + c$

= $-\theta\sqrt{1-\cos^2\theta} - \cos\theta + c$

= $-\left(\cos^{-1}x\right)\sqrt{1-x^2} - x + c$

= $-\left[\sqrt{1-x^2}\cos^{-1}x + x\right] + c$ Ans.

12. $x\sec^2 x$

Ans. $\int x\sec^2 x\ dx$

[Applying product rule]

= $x\int \sec^2 x\ dx - \int\left[\frac{d}{dx}x\int \sec^2 x\ dx\right]dx$

= $x\tan x - \int 1.\tan x\ dx$

= $x\tan x - \int \tan x\ dx$

= $x\tan x - \left(-\log\left|\cos x\right|\right) + c$

= $x\tan x+\log|\cos x|+c$ Ans.

13. $\tan^{-1}x$

Ans. Let I = $\tan^{-1}x\,dx$

= $\int\left(\tan^{-1}x\right).1\,dx$

= $\tan^{-1}x.x-\int\frac{1}{1+x^2}.x\,dx$

= $x\tan^{-1}x-\frac{1}{2}\int\frac{2x}{1+x^2}\,dx$

= $x\tan^{-1}x-\frac{1}{2}\log\left|\left(1+x^2\right)\right|+c$

$\left[\because\int\frac{f'(x)}{f(x)}\,dx=\log|f(x)|\right]$

= $x\tan^{-1}x-\frac{1}{2}\log\left(1+x^2\right)+c$

14. $x(\log x)^2$

Ans. $\int x(\log x)^2\,dx$

= $\int(\log x)^2.x\,dx$

= $(\log x)^2\int x\,dx-\int\left[\frac{d}{dx}(\log x)^2\int x\,dx\right]dx$

= $(\log x)^2\frac{x^2}{2}-\int 2(\log x)\frac{d}{dx}(\log x).\frac{x^2}{2}\,dx$

= $(\log x)^2 \frac{x^2}{2} - \int \frac{2(\log x)}{x} \cdot \frac{x^2}{2}\ dx$

= $\frac{x^2}{2}(\log x)^2 - \int (\log x) x\ dx$

= $\frac{x^2}{2}(\log x)^2 - \left[(\log x)\frac{x^2}{2} - \int \left(\frac{1}{x}\frac{x^2}{2}\right)\ dx\right] + c$

= $\frac{x^2}{2}(\log x)^2 - \frac{x^2}{2}\log x + \frac{1}{2}\int x\ dx + c$

= $\frac{x^2}{2}(\log x)^2 - \frac{x^2}{2}\log x + \frac{x^2}{4} + c$ Ans.

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15. $(x^2+1)\log x$

Ans. $\int (x^2+1)\log x\ dx$

= $\int (\log x)(x^2+1)\ dx$

[Applying product rule]

= $\log x\left(\frac{x^3}{3} + x\right) - \int \frac{1}{x}\left(\frac{x^3}{3} + x\right)\ dx$

$= (\frac{x^3}{3} + x)logx - \int(\frac{x^2}{3} + 1)dx$

= $\left(\frac{x^3}{3} + x\right)\log x - \frac{1}{3}\int x^2\ dx - \int 1\ dx$

= $\left(\frac{x^3}{3} + x\right)\log x - \frac{1}{3}\frac{x^3}{3} - x + c$

= $\left(\frac{x^3}{3}+x\right)\log x-\frac{x^3}{9}-x+c$ Ans.

Integrate the functions in Exercises 16 to 22.

16. $e^x\left(\sin x+\cos x\right)$

Ans. Let I = $\int e^x\left(\sin x+\cos x\right)\,dx$

$\left[\int e^x\left\{f\left(x\right)+f'\left(x\right)\right\}\,dx\right]$

It is in the form of $\int e^x\left\{f\left(x\right)+f'\left(x\right)\right\}\,dx$ since here $f\left(x\right)=\sin x$ and $f'\left(x\right)=\cos x$

$\therefore$ I = $e^x\sin x+c$

$\left[\because \int e^x\left\{f\left(x\right)+f'\left(x\right)\right\}\,dx=e^x f\left(x\right)+c\right]$

17. $\frac{xe^x}{\left(1+x\right)^2}$

Ans. Let I = $\int \frac{xe^x}{\left(x+1\right)^2}\,dx$

= $\int \frac{\left(x+1\right)-1}{\left(x+1\right)^2}e^x\,dx$

= $\int e^x\left[\frac{x+1}{\left(x+1\right)^2}-\frac{1}{\left(x+1\right)^2}\right]dx$

I = $\int e^x\left[\frac{1}{x+1}+\frac{-1}{\left(x+1\right)^2}\right]dx$

$\left[\int e^x\{f(x)+f'(x)\}\,dx\right]$

It is in the form of $\int e^x\{f(x)+f'(x)\}\,dx$ since here $f(x)=\frac{1}{x+1}$ and $f'(x)=\frac{-1}{(x+1)^2}$

I = $\frac{e^x}{x+1}+c$

$\left[\because \int e^x\{f(x)+f'(x)\}\,dx=e^x f(x)+c\right]$

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18. $e^x\left(\frac{1+\sin x}{1+\cos x}\right)$

Ans. Let I = $\int e^x.\frac{1+\sin x}{1+\cos x}\,dx$

= $\int e^x.\frac{1+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\,dx$

= $\int e^x.\left[\frac{1}{2\cos^2\frac{x}{2}}+\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right]dx$

= $\int e^x\left(\frac{1}{2}\sec^2\frac{x}{2}+\tan\frac{x}{2}\right)dx$

= $\int e^x(tan\frac{x}{2}+\frac{1}{2}sec^2\frac{x}{2})dx$

$\left[\int e^x\{f(x)+f'(x)\}\,dx\right]$

It is in the form of $\int e^x\{f(x)+f'(x)\}\,dx$ since here $f(x)=\tan\frac{x}{2}$ and $f'(x)=\frac{1}{2}\sec^2\frac{x}{2}$

= $e^x\tan\frac{x}{2}+c$

$\left[\because \int e^x\{f(x)+f'(x)\}\,dx=e^x f(x)+c\right]$

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19. $e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)$

Ans. Let I = $\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$

$\left[\int e^x\{f(x)+f'(x)\}\,dx\right]$

It is in the form of $\int e^x\{f(x)+f'(x)\}\,dx$ since here $f(x)=\frac{1}{x}=x^{-1}$ and $f'(x)=\frac{-1}{x^2}$

$\Rightarrow$ I = $e^x\frac{1}{x}+c$

= $\frac{e^x}{x}+c$

$\left[\because \int e^x\{f(x)+f'(x)\}\,dx=e^x f(x)+c\right]$

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20. $\frac{(x-3)e^x}{(x-1)^3}$

Ans. Let I = $\int\frac{(x-3)e^x}{(x-1)^3}\,dx$

= $\int \frac{(x-1)-2}{(x-1)^3} e^x \, dx$

= $\int e^x \left[\frac{x-1}{(x-1)^3} - \frac{2}{(x-1)^3} \right] dx$

$\Rightarrow$ I = $\int e^x \left[\frac{1}{(x-1)^2} + \frac{-2}{(x-1)^3} \right] dx$

$\left[\int e^x \{ f(x) + f'(x) \} \, dx \right]$

It is in the form of $\int e^x \{ f(x) + f'(x) \} \, dx$ since here $f(x) = \frac{1}{(x-1)^2}$ and $f'(x) = \frac{d}{dx}\{(x-1)^{-2}\}$

= $-2(x-1)^{-3}$

= $\frac{-2}{(x-1)^3}$.

$\Rightarrow$ I = $\frac{e^x}{(x-1)^2} + c$ Ans.

$\left[\because \int e^x \{ f(x) + f'(x) \} \, dx = e^x f(x) + c \right]$

21. $e^{2x} \sin x$

Ans. Let I = $\int e^{2x} \sin x \, dx$

[Applying product rule]

= $e^{2x}(-\cos x) - \int e^{2x}.2.(-\cos x) \, dx$

$\Rightarrow$ I = $-e^{2x}\cos x + 2\int e^{2x}\cos x\,dx$

[Again applying product rule]

$\Rightarrow$ I = $-e^{2x}\cos x + 2\left[e^{2x}\sin x - \int 2e^{2x}\sin x\,dx\right]$

$\Rightarrow$ I = $-e^{2x}\cos x + 2e^{2x}\sin x - 4\int e^{2x}\sin x\,dx$

= $e^{2x}(-\cos x + 2\sin x) - 4\mathrm{I}$

$\Rightarrow$ 5I = $e^{2x}(-\cos x + 2\sin x)$

$\Rightarrow$ I = $\dfrac{e^{2x}}{5}(2\sin x - \cos x) + c$ Ans.

22. $\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$

Ans. Putting $x = \tan\theta$

$\Rightarrow dx = \sec^2\theta\,d\theta$

$\therefore \int \sin^{-1}\left(\dfrac{2x}{1+x^2}\right)dx$

= $\int \sin^{-1}\left(\dfrac{2\tan\theta}{1+\tan^2\theta}\right).\sec^2\theta\,d\theta$

= $\int \sin^{-1}(\sin 2\theta).\sec^2\theta\,d\theta$

= $\int 2\theta\sec^2\theta\,d\theta$

= $2\int \theta\sec^2\theta\,d\theta$

[Applying product rule]

= $2\left[\theta.\tan\theta - \int 1.\tan\theta \ d\theta\right]$

= $2\left[\theta\tan\theta - \int \tan\theta \ d\theta\right]$

= $2\left[\theta\tan\theta - \log\sec\theta\right] + c$

= $2\left[\tan^{-1}x.x - \log\sqrt{1+x^2}\right] + c$

= $2\left[x\tan^{-1}x - \frac{1}{2}\log\left(1+x^2\right)\right] + c$

= $2x\tan^{-1}x - \log\left(1+x^2\right) + c$ Ans.

Choose the correct answer in Exercise 23 and 24.

23. $\int x^2 e^{x^3} \ dx$ **equals to**

(A) $\frac{1}{3}e^{x^3} + C$

(B) $\frac{1}{3}e^{x^2} + C$

(C) $\frac{1}{2}e^{x^3} + C$

(D) $\frac{1}{2}e^{x^2} + C$

Ans. Let I = $\int x^2 e^{x^3} \ dx$

= $\frac{1}{3}\int e^{\left(x^3\right)}\left(3x^2\right) \ dx \left[\because \frac{d}{dx}x^3 = 3x^2\right]$ ....(i)

Putting $x^3 = t$

$\Rightarrow 3x^2 = \frac{dt}{dx}$

$\Rightarrow 3x^2\ dx = dt$

$\therefore$ From eq. (i), I = $\frac{1}{3}\int e^t\ dt$

= $\frac{1}{3}e^t + C$

= $\frac{1}{3}e^{x^3} + C$

Therefore, option (A) is correct.

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24. $\int e^x \sec x(1+\tan x)\ dx$ **equals:**

(A) $e^x \cos x + C$

(B) $e^x \sec x + C$

(C) $e^x \sin x + C$

(D) $e^x \tan x + C$

Ans. Let I = $\int e^x \sec x(1+\tan x)\ dx$

= $\int e^x(\sec x + \sec x \tan x)\ dx$

It is in the form of $\int e^x\{f(x)+f'(x)\}\ dx$ since here $f(x) = \sec x$ and $f'(x) = \sec x \tan x$

$\therefore$ I = $e^x \sec x + c$

$\left[\because \int e^x\{f(x)+f'(x)\}\ dx = e^x f(x) + c\right]$

Therefore, option (B) is correct.