CBSE Class–12 Mathematics

NCERT solution

Chapter - 9

Differential Equations - Exercise 9.2

In each of the Questions 1 to 6 verify that the given functions (explicit) is a solution of the corresponding differential equation:

1. $y = e^x + 1 : y'' - y' = 0$

Ans. Given: $y = e^x + 1$ ....(i)

To prove: y is a solution of the differential equation $y'' - y' = 0$ ....(ii)

Proof: From eq. (i), $y' = e^x + 0 = e^x$ and $y'' = e^x$

$\therefore$L.H.S. of eq. (ii), $y'' - y' = e^x - e^x = 0$ = R.H.S.

Hence, y given by eq. (i) is a solution of $y'' - y' = 0$.

2. $y = x^2 + 2x + C : y' - 2x - 2 = 0$

Ans. Given: $y = x^2 + 2x + C$ ....(i)

To prove: y is a solution of the differential equation $y' - 2x - 2 = 0$ ....(ii)

Proof:From, eq. (i),

$y' = 2x + 2$

L.H.S. of eq. (ii),

= $y' - 2x - 2$

= $(2x+2) - 2x - 2$

= $2x + 2 - 2x - 2 = 0$ = R.H.S.

Hence, y given by eq. (i) is a solution of $y' - 2x - 2 = 0$.

3. $y = \cos x + C : y' + \sin x = 0$

Ans. Given: $y = \cos x + C$ ....(i)

To prove: y is a solution of the differential equation $y' + \sin x = 0$ ....(ii)

Proof: From eq. (i),

$y' = -\sin x$

L.H.S. of eq. (ii),

$y' + \sin x = -\sin x + \sin x = 0$ = R.H.S.

Hence, y given by eq. (i) is a solution of $y' + \sin x = 0$.

4. $y = \sqrt{1+x^2} : y' = \dfrac{xy}{1+x^2}$

Ans. Given: $y = \sqrt{1+x^2}$ ....(i)

To prove:y is a solution of the differential equation $y' = \dfrac{xy}{1+x^2}$ ....(ii)

Proof: From eq. (i),

$$y' = \frac{d}{dx}\sqrt{1+x^2} = \frac{d}{dx}\left(1+x^2\right)^{1/2}$$

$$= \frac{1}{2}\left(1+x^2\right)^{-1/2}\frac{d}{dx}\left(1+x^2\right) = \frac{1}{2}\left(1+x^2\right)^{-1/2}.2x$$

$$= \frac{x}{\sqrt{1+x^2}} \text{ ....(iii)}$$

Now R.H.S. of eq. (ii)

= $\frac{xy}{1+x^2}$

= $\frac{x}{1+x^2}\sqrt{1+x^2}$ [From eq. (i)]

= $\frac{x}{\sqrt{1+x^2}}$ = y'

$\therefore$ L.H.S. = R.H.S

Hence, y given by eq. (i) is a solution of $y' = \frac{xy}{1+x^2}$.

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5. $y = Ax : xy' = y\left(x \neq 0\right)$

Ans. Given: $y =$ Ax ....(i)

To prove:y given by eq. (i) is a solution of differential equation $xy' = y\left(x \neq 0\right)$ ....(ii)

Proof: From eq. (i)

$y' = A\left(1\right) = A$

L.H.S. of eq. (ii)

= $xy' = xA$ = Ax = y = R.H.S. of eq. (ii)

$\therefore$ y given by eq. (i) is a solution of differential equation $xy' = y\left(x \neq 0\right)$.

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6. $y = x\sin x : xy' = y + x\sqrt{x^2 - y^2}\left(x \neq 0 \text{ and } x > y \text{ or } x < -y\right)$

Ans. Given: $y = x\sin x$ ....(i)

To prove:y given by eq. (i) is a solution of differential equation $xy' = y + x\sqrt{x^2 - y^2}$ ..(ii)

Proof: From eq. (i),

$\frac{dy}{dx}(=y')=x\frac{d}{dx}\sin x+\sin x\frac{d}{dx}x$

= $x\cos x+\sin x$

L.H.S. of eq. (ii)

= $xy'=x(x\cos x+\sin x)=x^2\cos x+x\sin x$

R.H.S. of eq. (ii)

= $y+x\sqrt{x^2-y^2}$

= $x\sin x+x\sqrt{x^2-x^2\sin^2 x}$ [From eq. (i)]

= $x\sin x+x\sqrt{x^2\left(1-\sin^2 x\right)}$

= $x\sin x+x\sqrt{x^2\cos^2 x}$

= $x\sin x+x.x\cos x$

= $x\sin x+x^2\cos x$

= $x^2\cos x+x\sin x$

$\therefore$ L.H.S. = R.H.S

Hence, y given by eq. (i) is a solution of $xy'=y+x\sqrt{x^2-y^2}$.

In each of the questions 7 to 10, verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation:

7. $xy=\log y+C : y'=\frac{y^2}{1-xy}(xy\neq 1)$

Ans. Given: $xy=\log y+C$ ....(i)

To prove:y given by eq. (i) is a solution of differential equation $y' = \frac{y^2}{1-xy}$ ....(ii)

Proof: Differentiating both sides of eq. (i) w.r.t x, we have

$$xy' + y(1) = \frac{1}{y}y' + 0$$

$$\Rightarrow xy' - \frac{y'}{y} = -y$$

$$\Rightarrow y'\left(x - \frac{1}{y}\right) = -y$$

$$\Rightarrow y'\left(\frac{xy-1}{y}\right) = -y$$

$$\Rightarrow y'(xy-1) = -y^2$$

$$\implies y' = \frac{-y^2}{xy-1}$$

$$\Rightarrow y' = \frac{-y^2}{-(1-xy)} = \frac{y^2}{1-xy}$$

Hence, Function (implicit) given by eq. (i) is a solution of $y' = \frac{y^2}{1-xy}$.

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8. $y - \cos y = x : (y\sin y + \cos y + x)y' = y$

Ans. Given: $y - \cos y = x$ ....(i)

To prove: y given by eq. (i) is a solution of differential equation

$(y\sin y + \cos y + x)y' = y$ ....(ii)

Proof: Differentiating both sides of eq. (i) w.r.t x, we have

$y'+(\sin y)y'=1$

$\Rightarrow y'(1+\sin y)=1$

$\Rightarrow y'=\dfrac{1}{1+\sin y}$ ....(iii)

Putting the value of x from eq. (i) and value of y' from eq. (iii) in L.H.S. of eq. (ii),

$(y\sin y+\cos y+x)y'$

$\Rightarrow (y\sin y+\cos y+y-\cos y)\dfrac{1}{1+\sin y}$

$\Rightarrow (y\sin y+y)\dfrac{1}{1+\sin y}$

$\Rightarrow y(\sin y+1)\dfrac{1}{1+\sin y}=y$ = R.H.S. of (ii)

Hence, Function given by eq. (i) is a solution of $(y\sin y+\cos y+x)y'=y$.

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9. $x+y=\tan^{-1}y : y^2y'+y^2+1=0$

Ans. Given: $x+y=\tan^{-1}y$ ....(i)

To prove:y given by eq. (i) is a solution of differential equation $y^2y'+y^2+1=0$ ....(ii)

Proof: Differentiating both sides of eq. (i) w.r.t x we have

$1+y'=\dfrac{1}{1+y^2}y'$

$\Rightarrow (1+y')(1+y^2)=y'$

$\Rightarrow 1+y^2+y'+y'y^2=y'$

$\Rightarrow y^2 y' + y^2 + 1 = 0$ = eq. (ii)

Hence, Function given by eq. (i) is a solution of $y^2 y' + y^2 + 1 = 0$

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10. $y = \sqrt{a^2 - x^2}, \in (-a, a) : x + y\frac{dy}{dx} = 0 \, (y \neq 0)$

Ans. Given: $y = \sqrt{a^2 - x^2}, x \in (-a, a)$ ....(i)

To prove:y given by eq. (i) is a solution of differential equation $x + y\frac{dy}{dx} = 0$ ......(ii)

Proof: From eq. (i),

$\frac{dy}{dx} = \frac{1}{2\sqrt{a^2 - x^2}}(-2x)$

$= \frac{-x}{\sqrt{a^2 - x^2}}$ ....(iii)

Putting the values of y and $\frac{dy}{dx}$ from eq. (i) and (iii) in L.H.S. of eq. (ii),

$= x + y\frac{dy}{dx}$

$= x + \sqrt{a^2 - x^2}\left(\frac{-x}{\sqrt{a^2 - x^2}}\right)$

$= x - x = 0$ = R.H.S. of eq. (ii)

Hence, Function given by eq. (i) is a solution of $x + y\frac{dy}{dx} = 0$.

Choose the correct answer:

11. The number of arbitrary constants in the general solution of a differential equation

of fourth order are:

(A) 0

(B) 2

(C) 3

(D) 4

Ans. Option (D) is correct.

The number of arbitrary constants ($c_1, c_2, c_3,$ etc.) in the general solution of a differential equation of n^{th} order is n.

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12. The number of arbitrary constants in the particular solution of a differential equation of third order are:

(A) 3

(B) 2

(C) 1

(D) 0

Ans. The number of arbitrary constants in a particular solution of a differential equation of any order is zero (0) as a particular solution is a solution which contains no arbitrary constant.

Therefore, option (D) is correct.