**CBSE Class–12 Mathematics**

**NCERT solution**

**Chapter - 7**

**Integrals - Exercise 7.4**

**Integrate the following functions in Exercises 1 to 9.**

**1.** $\frac{3x^2}{x^6+1}$

**Ans.** Let I = $\int \frac{3x^2}{x^6+1}\, dx$

= $\int \frac{3x^2}{\left(x^3\right)^2+1}\, dx$ ..........(i)

Putting $x^3 = t$

$\Rightarrow\ 3x^2 = \frac{dt}{dx}$

$\Rightarrow\ 3x^2\ dx = dt$

$\therefore$ From eq. (i),

I = $\int \frac{dt}{t^2+1}$

= $\frac{1}{1}\tan^{-1}\frac{t}{1}+c$ using $\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}+c$

= $\tan^{-1}x^3+c$ Ans.

**2.** $\frac{1}{\sqrt{1+4x^2}}$

**Ans.** $\int \frac{1}{\sqrt{1+4x^2}}\ dx$

= $\int \frac{1}{\sqrt{(2x)^2+1^2}}\ dx$

= $\frac{\log\left|(2x)+\sqrt{(2x)^2+1^2}\right|}{2\rightarrow \text{Coeff. of } x}+c$

$\left[\because \int \frac{1}{\sqrt{x^2+a^2}}\ dx=\log\left|x+\sqrt{x^2+a^2}\right|\right]$

= $\frac{1}{2}\log\left|2x+\sqrt{4x^2+1}\right|+c$

**3.** $\frac{1}{\sqrt{(2-x)^2+1}}$

**Ans.** $\int \frac{1}{\sqrt{(2-x)^2+1}}\ dx$

= $\frac{\log\left|(2-x)+\sqrt{(2-x)^2+1^2}\right|}{-1\rightarrow \text{Coeff. of } x}+c$

$\left[\because \int \frac{1}{\sqrt{x^2+a^2}}\ dx=\log\left|x+\sqrt{x^2+a^2}\right|\right]$

= $-\log\left|2-x+\sqrt{4+x^2-4x+1}\right|+c$

= $= \log\left|\frac{1}{2-x+\sqrt{x^2-4x+5}}\right|+c$

**4.** $\frac{1}{\sqrt{9-25x^2}}$

**Ans.** $\int \frac{1}{\sqrt{9-25x^2}}\,dx$

= $\int \frac{1}{\sqrt{(3)^2-(5x)^2}}\,dx$

= $\frac{\sin^{-1}\frac{5x}{3}}{5 \to \text{Coeff. of } x}+c$

$\left[\because \int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}\right]$

= $\frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right)+c$

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**5.** $\frac{3x}{1+2x^4}$

**Ans.** Let I = $\int \frac{3x}{1+2x^4}\,dx$

= $\frac{3}{2}\int \frac{2x}{1+2\left(x^2\right)^2}\,dx$ ..........(i)

Putting $x^2 = t$

$\Rightarrow\ 2x = \frac{dt}{dx}$

$\Rightarrow 2x\,dx = dt$

$\therefore$ From eq. (i),

I = $\frac{3}{2}\int \frac{dt}{1+2t^2}$

= $\frac{3}{2}\int \frac{1}{\left(\sqrt{2}t\right)^2+1^2}\ dt$

= $\frac{3}{2}\frac{\frac{1}{1}\tan^{-1}\frac{\sqrt{2}t}{1}}{\sqrt{2}\rightarrow \text{Coeff. of } t}+c$

= $= \frac{3}{2\sqrt{2}}\tan^{-1}\sqrt{2}t+c$

= $= \frac{3}{2\sqrt{2}}\tan^{-1}\sqrt{2}x^2+c$ Ans.

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**6.** $\frac{x^2}{1-x^6}$

**Ans.** Let I = $\int \frac{x^2}{1-x^6}\ dx$

= $\int \frac{x^2}{1-\left(x^3\right)^2}\ dx$

= $\frac{1}{3}\int \frac{3x^2}{1-\left(x^3\right)^2}\ dx$ ..........(i)

Putting $x^3=t$

$\Rightarrow 3x^2=\frac{dt}{dx}$

$\Rightarrow 3x^2\ dx=dt$

$\therefore$ From eq. (i),

I = $\frac{1}{3}\int \frac{dt}{1-t^2}$

= $= \frac{1}{3}\int \frac{1}{(1)^2-(t)^2}dt$

= $\frac{1}{3}\cdot\frac{1}{2\times 1}\log\left|\frac{1+t}{1-t}\right|+c$

$\left[\because \int \frac{1}{a^2-x^2}\ dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|\right]$

= $\frac{1}{6}\log\left|\frac{1+x^3}{1-x^3}\right|+c$

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**7.** $\frac{x-1}{\sqrt{x^2-1}}$

**Ans.** Let I = $\int \frac{x-1}{\sqrt{x^2-1}}\ dx$

= $\int\left(\frac{x}{\sqrt{x^2-1}}-\frac{1}{\sqrt{x^2-1}}\right)\ dx$

= $\int \frac{x}{\sqrt{x^2-1}}\ dx - \int \frac{1}{\sqrt{x^2-1}}\ dx$

$= \frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx - \log\left|x+\sqrt{(x^2-(1)^2)}\right|$

Let $I_1$ = $\int \frac{2x}{\sqrt{x^2-1}}\ dx$

Putting $x^2-1=t$

$\Rightarrow 2x = \frac{dt}{dx}$

$\Rightarrow\ 2x\,dx = dt$

$\therefore$ $I_1 = \int \frac{dt}{\sqrt{t}} = \int t^{\frac{-1}{2}}\,dt = \frac{t^{1/2}}{1/2} = 2\sqrt{t}$

$\Rightarrow$ I = $\frac{1}{2}\left(2\sqrt{x^2-1}+c\right)-\log\left|x+\sqrt{x^2-1}\right|$

= $\sqrt{x^2-1}+\frac{c}{2}-\log\left|x+\sqrt{x^2-1}\right|$

$\Rightarrow$ I = $\sqrt{x^2-1}-\log\left|x+\sqrt{x^2-1}\right|+c_1$ where $c_1 = \frac{c}{2}$

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**8.** $\frac{x^2}{\sqrt{x^6+a^6}}$

**Ans.** Let I = $\int \frac{x^2}{\sqrt{x^6+a^6}}\,dx$

= $\frac{1}{3}\int \frac{x^2}{\sqrt{\left(x^3\right)^2+a^6}}\,dx$ ..........(i)

Putting $x^3 = t$

$\Rightarrow\ 3x^2 = \frac{dt}{dx}$

$\Rightarrow\ 3x^2\,dx = dt$

$\therefore$ From eq. (i),

I = $\frac{1}{3}\int \frac{dt}{t^2+a^6}$

$= \frac{1}{3}\int \frac{1}{t^2+\left(a^3\right)^2}\,dt$

$= \frac{1}{3}\log\left|t+\sqrt{t^2+\left(a^3\right)^2}\right|+c$

$= \frac{1}{3}\log\left|x^3+\sqrt{x^6+a^6}\right|+c$

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**9.** $\frac{\sec^2 x}{\sqrt{\tan^2 x+4}}$

**Ans.** Let I = $\int \frac{\sec^2 x}{\sqrt{\tan^2 x+4}}\,dx$ ..........(i)

Putting $\tan x = t$

$\Rightarrow \sec^2 x = \frac{dt}{dx}$

$\Rightarrow \sec^2 x\,dx = dt$

$\therefore$ From eq. (i),

I = $\int \frac{dt}{\sqrt{t^2+4}}$

$= \int \frac{1}{\sqrt{t^2+(2)^2}}\,dt$

$= \log\left|t+\sqrt{t^2+(2)^2}\right|+c$

$= \log\left|\tan x+\sqrt{\tan^2 x+4}\right|+c$

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**Integrate the following functions in Exercises 10 to 18.**

**10.** $\frac{1}{\sqrt{x^2+2x+2}}$

**Ans.** $\int \frac{1}{\sqrt{x^2+2x+2}}dx$

$= \int \frac{1}{\sqrt{x^2+2x+1+1}}dx$

$= \int \frac{1}{\sqrt{(x+1)^2+(1)^2}}dx$

$= \log\left|x+1+\sqrt{(x+1)^2+(1)^2}\right|+c$

$= \log\left|x+1+\sqrt{x^2+2x+2}\right|+c$

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**11.** $\frac{1}{9x^2+6x+5}$

**Ans.** $\int \frac{1}{9x^2+6x+5}\,dx$

$= \int \frac{1}{9\left(x^2+\frac{6x}{9}+\frac{5}{9}\right)}\,dx$

$= \int \frac{1}{9\left(x^2+\frac{2x}{3}+\frac{5}{9}\right)}\,dx$

$= \int \frac{1}{9\left(x^2+\frac{2x}{3}+\left(\frac{1}{3}\right)^2-\left(\frac{1}{3}\right)^2+\frac{5}{9}\right)}\,dx$

[For making completing the squares]

$= \int \frac{1}{9\left\{\left(x+\frac{1}{3}\right)^2+\frac{4}{9}\right\}}\,dx$

$= \int \frac{1}{9\left\{\left(x+\frac{1}{3}\right)^2+\left(\frac{2}{3}\right)^2\right\}}\,dx$

$= \frac{1}{9}\int \frac{1}{\left\{\left(x+\frac{1}{3}\right)^2+\left(\frac{2}{3}\right)^2\right\}}\,dx$

$= \frac{1}{9}\cdot\frac{1}{\left(\frac{2}{3}\right)}\tan^{-1}\frac{x+\frac{1}{3}}{\frac{2}{3}}+c$

$\left[\because \int \frac{1}{x^2+a^2}\,dx=\frac{1}{a}\tan^{-1}\frac{x}{a}\right]$

$= \frac{1}{9}\cdot\frac{3}{2}\tan^{-1}\left(\frac{\frac{3x+1}{3}}{\frac{2}{3}}\right)+c$

$= \frac{1}{6}\tan^{-1}\left(\frac{3x+1}{2}\right)+c$

**12.** $\frac{1}{\sqrt{7-6x-x^2}}$

**Ans.** $\int \frac{1}{\sqrt{7-6x-x^2}}\,dx$

$= \int \frac{1}{\sqrt{-x^2-6x+7}}\,dx$

$= \int \frac{1}{\sqrt{-\left(x^2+6x-7\right)}}\,dx$

$= \int \frac{1}{\sqrt{-\left(x^2+6x+9-9-7\right)}}\,dx$

$= \int \frac{1}{\sqrt{-\left\{\left(x+3\right)^2-16\right\}}}\,dx$

$= \int \frac{1}{\sqrt{-(x+3)^2+16}}dx$

$= \int \frac{1}{\sqrt{(4)^2-(x+3)^2}}dx$

$= \sin^{-1}\left(\frac{x+3}{4}\right)+c$

$\left[\because \int \frac{1}{a^2-x^2}\,dx = \sin^{-1}\frac{x}{a}\right]$

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**13.** $\frac{1}{\sqrt{(x-1)(x-2)}}$

**Ans.** $\int \frac{1}{\sqrt{(x-1)(x-2)}}\,dx$

$= \int \frac{1}{\sqrt{x^2-2x-x+2}}\,dx$

$= \int \frac{1}{\sqrt{x^2-3x+2}}\,dx$

$= \int \frac{1}{\sqrt{x^2 - 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 2}} \, dx$

$= \int \frac{1}{\sqrt{\left(x - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}} \, dx$

$= \log \left| x - \frac{3}{2} + \sqrt{\left(x - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2} \right| + c$

$= \log \left| x - \frac{3}{2} + \sqrt{x^2 - 3x + 2} \right| + c$

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**14.** $\frac{1}{\sqrt{8 + 3x - x^2}}$

**Ans.** $\int \frac{1}{\sqrt{8 + 3x - x^2}} \, dx$

$= \int \frac{1}{\sqrt{-x^2 + 3x + 8}} \, dx$

$= \int \frac{1}{\sqrt{-\left(x^2 - 3x - 8\right)}} \, dx$

$= \int \frac{1}{\sqrt{-\left\{x^2 - 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 8\right\}}} \, dx$

$= \int \frac{1}{\sqrt{-\left\{\left(x-\frac{3}{2}\right)^2-\frac{41}{4}\right\}}}\,dx$

$= \int \frac{1}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}}\,dx$

$= \sin^{-1}\frac{x-\frac{3}{2}}{\frac{\sqrt{41}}{2}}+c$

$= \sin^{-1}\left(\frac{2x-3}{\sqrt{41}}\right)+c$

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**15.** $\frac{1}{\sqrt{(x-a)(x-b)}}$

**Ans.** $\int \frac{1}{\sqrt{(x-a)(x-b)}}\,dx$

$= \int \frac{1}{\sqrt{x^2-bx-ax+ab}}\,dx$

$= \int \frac{1}{\sqrt{x^2-x(a+b)+ab}}\,dx$

$= \int \frac{1}{\sqrt{x^2-x(a+b)+\left(\frac{a+b}{2}\right)^2-\left(\frac{a+b}{2}\right)^2+ab}}\,dx$

$$= \int \frac{1}{\sqrt{\left[\left\{x-\left(\frac{a+b}{2}\right)\right\}^2 - \left\{\frac{(a+b)^2}{4} - ab\right\}\right]}}\, dx$$

$$= \int \frac{1}{\sqrt{\left[\left\{x-\left(\frac{a+b}{2}\right)\right\}^2 - \left\{\frac{(a+b)^2 - 4ab}{4}\right\}\right]}}\, dx$$

$$= \int \frac{1}{\sqrt{\left[\left\{x-\left(\frac{a+b}{2}\right)\right\}^2 - \left\{\frac{(a-b)^2}{4}\right\}\right]}}\, dx$$

$$= \int \frac{1}{\sqrt{\left[\left\{x-\left(\frac{a+b}{2}\right)\right\}^2 - \left\{\left(\frac{a-b}{2}\right)^2\right\}\right]}}\, dx$$

$$= \log\left|x-\left(\frac{a+b}{2}\right) + \sqrt{\left\{\left(x-\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2\right\}}\right| + c$$

$$= \log\left|x-\left(\frac{a+b}{2}\right) + \sqrt{x^2 - x(a+b) + ab}\right| + c$$

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**16.** $\frac{4x+1}{\sqrt{2x^2+x-3}}$

**Ans.** Let I = $\int \frac{4x+1}{\sqrt{2x^2+x-3}}\, dx$ ..........(i)

Putting $2x^2+x-3=t$

$\Rightarrow\ 4x+1=\dfrac{dt}{dx}$

$\Rightarrow\ (4x+1)\ dx=dt$

$\therefore$ From eq. (i),

I = $\int \dfrac{dt}{\sqrt{t}}$

= $\int t^{\frac{-1}{2}}\ dt$

= $\dfrac{t^{1/2}}{1/2}+c$

= $2\sqrt{t}+c$

= $2\sqrt{2x^2+x-3}+c$

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**17.** $\dfrac{x+2}{\sqrt{x^2-1}}$

**Ans.** Let I = $\int \dfrac{x+2}{\sqrt{x^2-1}}\ dx$

= $\int\left(\dfrac{x}{\sqrt{x^2-1}}+\dfrac{2}{\sqrt{x^2-1}}\right)\ dx$

= $\int \dfrac{x}{\sqrt{x^2-1}}\ dx+2\int \dfrac{1}{\sqrt{x^2-1}}\ dx$

= $\int \dfrac{x}{\sqrt{x^2-1}}\ dx+2\log\left|x+\sqrt{x^2-1^2}\right|+c$ .......(i)

Let $I_1 = \int \frac{x}{\sqrt{x^2-1}}\, dx$

$= \frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}\, dx$

Putting $x^2 - 1 = t$

$\Rightarrow 2x = \frac{dt}{dx}$

$\Rightarrow 2x\, dx = dt$

$\therefore I_1 = \frac{1}{2}\int \frac{dt}{\sqrt{t}}$

$= \frac{1}{2}\int t^{\frac{-1}{2}}\, dt$

$= \frac{1}{2}.\frac{t^{1/2}}{1/2} + c$

$= \sqrt{t}$

$= \sqrt{x^2-1}$

Putting this value in eq. (i),

$\sqrt{x^2-1} + 2\log\left|x + \sqrt{x^2-1^2}\right| + c$

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**18.** $\frac{5x-2}{1+2x+3x^2}$

**Ans.** Let I = $\int \frac{5x-2}{1+2x+3x^2}\, dx$ ..........(i)

Let $numerator = A\frac{d}{dx}\left(deno\min ator\right) + B$

$\Rightarrow 5x-2 = A\frac{d}{dx}\left(1+2x+3x^2\right)+B$

$\Rightarrow 5x-2 = A\left(2+6x\right)+B$ ......(ii)

$\Rightarrow 5x-2 = 2A+6Ax+B$

Comparing coefficients of $x$, 6A = 5

$\Rightarrow$ A = $\frac{5}{6}$

Comparing constants,

2A + B = $-2$

On solving, we get A = $\frac{5}{6}$, B = $\frac{-11}{3}$

Putting the values of A and B in eq. (ii),

$5x-2 = \frac{5}{6}\left(2+6x\right)-\frac{11}{3}$

Putting this value of $5x-2$ in eq. (i),

I = $\int \frac{\frac{5}{6}\left(2+6x\right)-\frac{11}{3}}{1+2x+3x^2}\,dx$

I = $\frac{5}{6}\int \frac{2+6x}{1+2x+3x^2}\,dx - \frac{11}{3}\int \frac{1}{1+2x+3x^2}\,dx$

$\Rightarrow$ I = $\frac{5}{6}I_1 - \frac{11}{3}I_2$ ..........(iii)

Now $I_1$ = $\int \frac{2+6x}{1+2x+3x^2}\,dx$

Putting $1+2x+3x^2=t$

$\Rightarrow\ 2+6x=\frac{dt}{dx}$

$\Rightarrow\ (2+6x)\ dx=dt$

$\therefore I_1 = \int\frac{dt}{t} = \log|t| = \log\left|1+2x+3x^2\right|$ ..........(iv)

Again $I_2 = \int\frac{1}{1+2x+3x^2}\ dx$

$= \int\frac{1}{3x^2+2x+1}\ dx$

$= \int\frac{1}{3\left(x^2+\frac{2}{3}x+\frac{1}{3}\right)}\ dx$

$= \int\frac{1}{3\left[x^2+\frac{2}{3}x+\left(\frac{1}{3}\right)^2-\left(\frac{1}{3}\right)^2+\frac{1}{3}\right]}\ dx$

$= \int\frac{1}{3\left[\left(\left(x+\frac{1}{3}\right)^2+\frac{2}{9}\right)\right]}\ dx$

$= \frac{1}{3}\int\frac{1}{\left[\left(\left(x+\frac{1}{3}\right)^2+\frac{2}{9}\right)\right]}\ dx$

$\frac{1}{3}\int\frac{1}{\left[\left(x+\frac{1}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2\right]}dx$

$$\left[\because \int \frac{1}{x^2+a^2}\, dx = \frac{1}{a}\tan^{-1}\frac{x}{a}\right]$$

$$= \frac{1}{3}\cdot\frac{1}{\frac{\sqrt{2}}{3}}\tan^{-1}\frac{x+\frac{1}{3}}{\frac{\sqrt{2}}{3}}$$

$$= \frac{1}{3}\cdot\frac{3}{\sqrt{2}}\tan^{-1}\frac{3x+1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right) \text{..........(v)}$$

Putting values of $I_1$ and $I_2$ in eq. (iii),

$$I = \frac{5}{6}\log\left|1+2x+3x^2\right| - \frac{11}{3\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)+c$$

**Integrate the following functions in Exercises 19 to 23.**

**19.** $\frac{6x+7}{\sqrt{(x-5)(x-4)}}$

**Ans.** Let I = $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}\, dx$

$$= \int \frac{6x+7}{\sqrt{x^2-9x+20}}\, dx \text{..........(i)}$$

Let Linear = A $\frac{d}{dx}$ (Quadratic) + B

$$\Rightarrow 6x+7 = A\frac{d}{dx}\left(x^2-9x+20\right)+B$$

$\Rightarrow 6x+7=A(2x-9)+B$ ......(ii)

$\Rightarrow 6x+7=2Ax-9A+B$

Comparing coefficients of $x$, 2A = 6 $\Rightarrow$ A = 3

Comparing constants, –9A + B = 7

On solving, we get A = 3, B = 34

Putting the values of A and B in eq. (ii),

$6x+7=3(2x-9)+34$

Putting this value of $6x+7$ in eq. (i),

I = $\int \frac{3(2x-9)+34}{\sqrt{x^2-9x+20}}\,dx$

I = $3\int \frac{2x-9}{\sqrt{x^2-9x+20}}\,dx+34\int \frac{1}{\sqrt{x^2-9x+20}}\,dx$

$\Rightarrow$ I = $3I_1+34I_2$ ..........(iii)

Now $I_1$ = $\int \frac{2x-9}{\sqrt{x^2-9x+20}}\,dx$

Putting $x^2-9x+20=t$

$\Rightarrow 2x-9=\frac{dt}{dx}$

$\Rightarrow (2x-9)\,dx=dt$

$\therefore I_1$ = $\int \frac{dt}{\sqrt{t}}$

= $\int t^{\frac{-1}{2}}\,dt$

$= \frac{t^{1/2}}{1/2} = 2\sqrt{t}$

$= 2\sqrt{x^2 - 9x + 20}$ ..........(iv)

Again $I_2 = \int \frac{1}{\sqrt{x^2 - 9x + 20}} dx$

$= \int \frac{1}{\sqrt{x^2 - 9x + \left(\frac{9}{2}\right)^2 - \left(\frac{9}{2}\right)^2 + 20}} dx$

$= \int \frac{1}{\sqrt{\left(x - \frac{9}{2}\right)^2 - \frac{1}{4}}} dx$

$= \int \frac{1}{\sqrt{\left(x + \frac{1}{3}\right)^2 - \left(\frac{1}{2}\right)^2}} dx$

$= \log\left| x - \frac{9}{2} + \sqrt{\left(x - \frac{9}{2}\right)^2 - \left(\frac{1}{2}\right)^2} \right|$

$= \log\left| x - \frac{9}{2} + \sqrt{x^2 - 9x + 20} \right|$ ..........(v)

Putting values of $I_1$ and $I_2$ in eq. (iii),

$I = 6\sqrt{x^2 - 9x + 20} + 34 \log\left| x - \frac{9}{2} + \sqrt{x^2 - 9x + 20} \right| + c$

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**20.** $\frac{x+2}{\sqrt{4x - x^2}}$

**Ans.** Let I = $\int \frac{x+2}{\sqrt{4x-x^2}}\,dx$ ..........(i)

Let Linear = A $\frac{d}{dx}$ (Quadratic) + B

$\Rightarrow x+2 = A\frac{d}{dx}\left(4x-x^2\right)+B$

$\Rightarrow x+2 = A(4-2x)+B$ ......(ii)

$\Rightarrow x+2 = 4A-2Ax+B$

Comparing coefficients of $x$, –2A = 1 $\Rightarrow$ A = $\frac{-1}{2}$

Comparing constants, 4A + B = 2

On solving, we get A = $\frac{-1}{2}$, B = 4

Putting the values of A and B in eq. (ii),

$x+2 = \frac{-1}{2}(4-2x)+4$

Putting this value of $x+2$ in eq. (i),

I = $\int \frac{\frac{-1}{2}(4-2x)+4}{\sqrt{4x-x^2}}\,dx$

I = $\frac{-1}{2}\int \frac{4-2x}{\sqrt{4x-x^2}}\,dx + 4\int \frac{1}{\sqrt{4x-x^2}}\,dx$

$\Rightarrow$ I = $\frac{-1}{2}I_1+4I_2$ ..........(iii)

Now $I_1 = \int \frac{4-2x}{\sqrt{4x-x^2}}\ dx$

Putting $4x - x^2 = t$

$\Rightarrow 4 - 2x = \frac{dt}{dx}$

$\Rightarrow (4-2x)\ dx = dt$

$\therefore I_1 = \int \frac{dt}{\sqrt{t}} = \int t^{\frac{-1}{2}}\ dt$

$= \frac{t^{1/2}}{1/2} = 2\sqrt{t}$

$= 2\sqrt{4x-x^2}$ .......(iv)

Again $I_2 = \int \frac{1}{\sqrt{4x-x^2}}\ dx$

$= \int \frac{1}{\sqrt{-x^2+4x}} dx$

$= \int \frac{1}{\sqrt{-(x^2-4x)}} dx$

$= \int \frac{1}{\sqrt{-(x^2-4x+4-4)}} dx$

$= \int \frac{1}{\sqrt{-\left[(x-2)^2-(2)^2\right]}} dx$

$= \int \frac{1}{\sqrt{(2)^2-(x-2)^2}} dx$

$= \sin^{-1}\frac{x-2}{2}$ ..........(v)

Putting values of $I_1$ and $I_2$ in eq. (iii), I = $-\sqrt{4x-x^2}+4\sin^{-1}\frac{x-2}{2}+c$

**21.** $\frac{x+2}{\sqrt{x^2+2x+3}}$

**Ans.** Let I = $\int\frac{x+2}{\sqrt{x^2+2x+3}}\,dx$

Let Linear = A $\frac{d}{dx}$ (Quadratic) + B

$\Rightarrow x+2=\mathrm{A}\frac{d}{dx}\left(x^2+2x+3\right)+\mathrm{B}$

$\Rightarrow\ x+2=\mathrm{A}\left(2x+2\right)+\mathrm{B}$ ......(ii)

$\Rightarrow\ x+2=2\mathrm{A}x+2\mathrm{A}+\mathrm{B}$

Comparing coefficients of $x$, 2A = 1 $\Rightarrow$ A = $\frac{1}{2}$

Comparing constants, 2A + B = 2

On solving, we get A = $\frac{1}{2}$, B = 1

Putting the values of A and B in eq. (ii),

$x+2=\frac{1}{2}\left(2x+2\right)+1$

Putting this value of $x+2$ in eq. (i),

I = $\int\frac{\frac{1}{2}\left(2x+2\right)+1}{\sqrt{x^2+2x+3}}\,dx$

I = $\frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}}\ dx + \int \frac{1}{\sqrt{x^2+2x+3}}\ dx$

$\Rightarrow$ I = $\frac{1}{2}I_1 + I_2$ ..........(iii)

Now $I_1$ = $\int \frac{2x+2}{\sqrt{x^2+2x+3}}\ dx$

Putting $x^2+2x+3=t$

$\Rightarrow\ 2x+2=\frac{dt}{dx}$

$\Rightarrow\ (2x+2)\ dx = dt$

$\therefore I_1$ = $\int \frac{dt}{\sqrt{t}}$ = $\int t^{\frac{-1}{2}}\ dt$

= $\frac{t^{1/2}}{1/2}$ = $2\sqrt{t}$

= $2\sqrt{x^2+2x+3}$ ..........(iv)

Again $I_2$ = $\int \frac{1}{\sqrt{x^2+2x+3}}\ dx$

= $\int \frac{1}{\sqrt{x^2+2x+1+2}}$

= $\int \frac{1}{\sqrt{(x+1)^2+\left(\sqrt{2}\right)^2}}\ dx$

= $\log\left|x+1+\sqrt{(x+1)^2+\left(\sqrt{2}\right)^2}\right|$

$= \log\left|x+1+\sqrt{x^2+2x+3}\right|$ ..........(v)

Putting values of $I_1$ and $I_2$ in eq. (iii),

I = $\sqrt{x^2+2x+3}+\log\left|x+1+\sqrt{x^2+2x+3}\right|+c$

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**22.** $\dfrac{x+3}{x^2-2x-5}$

**Ans.** Let I = $\int \dfrac{x+3}{x^2-2x-5}\,dx$ ..........(i)

Let Linear = A $\dfrac{d}{dx}$ (Quadratic) + B

$\Rightarrow x+3 = \text{A}\dfrac{d}{dx}\left(x^2-2x-5\right)+\text{B}$

$\Rightarrow x+3 = \text{A}\left(2x-2\right)+\text{B}$ ......(ii)

$\Rightarrow x+3 = 2\text{A}x-2\text{A}+\text{B}$

Comparing coefficients of $x$, 2A = 1

$\Rightarrow$ A = $\dfrac{1}{2}$

Comparing constants, –2A + B = 3

On solving, we get A = $\dfrac{1}{2}$, B = 4

Putting the values of A and B in eq. (ii), $x+3=\dfrac{1}{2}\left(2x-2\right)+4$

Putting this value of $x+3$ in eq. (i),

I = $\int \frac{\frac{1}{2}(2x+2)+4}{x^2-2x-5}\ dx$

I = $\frac{1}{2}\int \frac{2x-2}{x^2-2x-5}\ dx + 4\int \frac{1}{x^2-2x-5}\ dx$

$\Rightarrow$ I = ..........(iii)

Now $I_1$ = $\int \frac{2x-2}{x^2-2x-5}\ dx$

Putting $x^2-2x-5=t$

$\Rightarrow 2x-2=\frac{dt}{dx}$

$\Rightarrow (2x-2)\ dx = dt$

$\therefore I_1$ = $\int \frac{dt}{t}$ = $\log|t|$

= $\log\left|x^2-2x-5\right|$ ..........(iv)

Again $I_2$ = $\int \frac{1}{x^2-2x-5}\ dx$

= $\int \frac{1}{\sqrt{x^2-2x+1-1-5}}$

= $\int \frac{1}{\sqrt{(x-1)^2+\left(\sqrt{6}\right)^2}}\ dx$

= $\frac{1}{2\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right|$ ..........(v)

Putting values of $I_1$ and $I_2$ in eq. (iii),

I = $\frac{1}{2}\log\left|x^2-2x-5\right|+\frac{2}{\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right|+c$

---

**23.** $\frac{5x+3}{\sqrt{x^2+4x+10}}$

**Ans.** Let I = $\int\frac{5x+3}{\sqrt{x^2+4x+10}}\,dx$ ..........(i)

Let Linear = A $\frac{d}{dx}$ (Quadratic) + B

$\Rightarrow\ 5x+3=A\frac{d}{dx}\left(x^2+4x+10\right)+B$

$\Rightarrow\ 5x+3=A\left(2x+4\right)+B$ ......(ii)

$\Rightarrow\ 5x+3=2Ax+4A+B$

Comparing coefficients of $x$, 2A = 5 $\Rightarrow$ A = $\frac{5}{2}$

Comparing constants, 4A + B = 3

On solving, we get A = $\frac{1}{2}$, B = $-7$

Putting the values of A and B in eq. (ii),

$5x+3=\frac{5}{2}\left(2x+4\right)-7$

Putting this value of $5x+3$ in eq. (i), I = $\int \frac{\frac{5}{2}(2x+4)-7}{\sqrt{x^2+4x+10}}\ dx$

I = $\frac{5}{2}\int \frac{2x+4}{\sqrt{x^2+4x+10}}\ dx - 7\int \frac{1}{\sqrt{x^2+4x+10}}\ dx$

$\Rightarrow$ I = $\frac{5}{2}I_1 - 7I_2$..........(iii)

Now $I_1$ = $\int \frac{2x+4}{\sqrt{x^2+4x+10}}\ dx$

Putting $x^2+4x+10=t$

$\Rightarrow\ 2x+4=\frac{dt}{dx}$

$\Rightarrow\ (2x+4)\ dx = dt$

$\therefore I_1$ = $\int \frac{dt}{\sqrt{t}}$

= $\int t^{\frac{-1}{2}}\ dt$

= $\frac{t^{1/2}}{1/2}$ = $2\sqrt{t}$

= $2\sqrt{x^2+4x+10}$ ..........(iv)

Again $I_2$ = $\int \frac{1}{\sqrt{x^2+4x+10}}\ dx$

= $\int \frac{1}{\sqrt{x^2+4x+4+6}}$

= $\int \frac{1}{\sqrt{(x+2)^2+(\sqrt{6})^2}}\,dx$

= $\log\left|x+2+\sqrt{(x+2)^2+(\sqrt{6})^2}\right|$

= $\log\left|x+2+\sqrt{x^2+4x+10}\right|$ ..........(v)

Putting values of $I_1$ and $I_2$ in eq. (iii),

I = $5\sqrt{x^2+4x+10}-7\log\left|x+2+\sqrt{x^2+4x+10}\right|+c$

**Choose the correct answer in Exercise 24 and 25.**

**24.** $\int \frac{dx}{x^2+2x+2}$ **equals**

**(A)** $x\tan^{-1}(x+1)+C$

**(B)** $\tan^{-1}(x+1)+C$

**(C)** $(x+1)\tan^{-1}x+C$

**(D)** $\tan^{-1}x+C$

**Ans.** $\int \frac{dx}{x^2+2x+2}$

= $\int \frac{1}{x^2+2x+1+1}\,dx$

= $\int \frac{1}{(x+1)^2+(1)^2}\,dx$

= $\tan^{-1}(x+1)+C$

Therefore, option (B) is correct.

**25.** $\int \frac{dx}{\sqrt{9x-4x^2}}$ **equals**

**(A)** $\frac{1}{9}\sin^{-1}\left(\frac{9x-8}{8}\right)+C$

**(B)** $\frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right)+C$

**(C)** $\frac{1}{3}\sin^{-1}\left(\frac{9x-8}{8}\right)+C$

**(D)** $\frac{1}{2}\sin^{-1}\left(\frac{9x-8}{8}\right)+C$

**Ans.** Let I = $\int \frac{dx}{\sqrt{9x-4x^2}}$

= $\int \frac{1}{\sqrt{-4x^2+9x}}dx$

= $\int \frac{1}{\sqrt{-4\left(x^2-\frac{9}{4}x\right)}}dx$

= $\int \frac{1}{\sqrt{-4\left(x^2-\frac{9}{4}x+\left(\frac{9}{8}\right)^2-\left(\frac{9}{8}\right)^2\right)}}dx$

= $\int \frac{1}{\sqrt{-4\left[\left(x-\frac{9}{8}\right)^2+\left(\frac{9}{8}\right)^2\right]}}dx$

= $\int \frac{1}{\sqrt{4\left[\left(\frac{9}{8}\right)^2-\left(x-\frac{9}{8}\right)^2\right]}}dx$

= $\frac{1}{2}\int \frac{1}{\sqrt{\left[\left(\frac{9}{8}\right)^2-\left(x-\frac{9}{8}\right)^2\right]}}dx$

$$= \frac{1}{2}\sin^{-1}\frac{x-\frac{9}{8}}{\frac{9}{8}}+C$$

$$= \frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right)+C$$

Therefore, option (B) is correct.