CBSE Class–12 Mathematics

NCERT solution

Chapter - 9

Differential Equations - Exercise 9.3

In each of the questions 1 to 5, form a differential equation representing the given family of curves by eliminating arbitrary constants a and b

1. $\frac{x}{a}+\frac{y}{b}=1$

Ans. Given: Equation of the family of curves $\frac{x}{a}+\frac{y}{b}=1$ ....(i)

Here there are two arbitrary constants a and b, therefore we will differentiate both sides two times w.r.t. x,

$$\Rightarrow \frac{1}{a}.1+\frac{1}{b}.\frac{dy}{dx}=0$$

$$\Rightarrow \frac{1}{a}=\frac{-1}{b}.\frac{dy}{dx} \quad \text{..........(ii)}$$

Again differentiating w.r.t. x,

=> $0=\frac{-1}{b}.\frac{d^2y}{dx^2}$

Multiplying both sides by $-b$,

$\frac{d^2y}{dx^2}=0$, which the required differential equation.

2. $y^2=a\left(b^2-x^2\right)$

Ans. Given: Equation of the family of curves $y^2=a\left(b^2-x^2\right)$ ....(i)

Here there are two arbitrary constants a and b, therefore we will differentiate both sides two times w.r.t. x,

$$\Rightarrow \ 2y.\frac{dy}{dx} = a(0-2x) = -2ax$$

$$\Rightarrow \ y.\frac{dy}{dx} = -ax \text{ ....(ii)}$$

Again differentiating w.r.t. x,

$$y.\frac{d^2y}{dx^2} + \frac{dy}{dx}.\frac{dy}{dx} = -a$$

$$\Rightarrow \ y.\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -a \text{ ....(iii)}$$

Putting this value of -a from eq (i) in eq. (ii), we get

$$\Rightarrow \ y.\frac{dy}{dx} = \left\{y.\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right\}x$$

$$\Rightarrow \ xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 = y.\frac{dy}{dx}$$

$$\Rightarrow \ xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y.\frac{dy}{dx} = 0$$

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3. $y = ae^{3x} + be^{-2x}$

Ans. Given: Equation of the family of curves $y = ae^{3x} + be^{-2x}$ ....(i)

Here there are two arbitrary constants a and b, therefore we will differentiate both sides two times w.r.t. x,

$\Rightarrow \dfrac{dy}{dx} = 3ae^{3x} - 2be^{-2x}$ ....(ii)

Again differentiating w.r.t. x

$=> \frac{d^2y}{dx^2} = 9ae^{3x} + 4be^{-2x}$ ....(iii)

Multiplying eq. (i) by 3 and subtracting eq. (ii) from it, we get

$\dfrac{dy}{dx} - 3y = -5be^{-2x}$ ....(iv)

Again multiplying eq. (ii) by 3 and subtracting it from eq. (iii), we get

$\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} = 10be^{-2x}$ ....(v)

Now, eq. (v) + 2 eq. (iv) gives,

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2\left(\frac{dy}{dx} - 3y\right) = 10be^{-2x} - 10be^{-2x}$$

$$\Rightarrow \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2\frac{dy}{dx} - 6y = 0$$

$\Rightarrow \dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} - 6y = 0$, which is required differential equation.

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4. $y = e^{2x}(a+bx)$

Ans. Given: Equation of the family of curves $y = e^{2x}(a+bx)$ ....(i)

Here there are two arbitrary constants a and b, therefore we will differentiate both sides two times w.r.t. x

$$\Rightarrow \frac{dy}{dx} = \left(\frac{d}{dx}e^{2x}\right)(a+bx) + e^{2x}\frac{d}{dx}(a+bx)$$

$\Rightarrow \dfrac{dy}{dx} = 2e^{2x}(a+bx) + e^{2x}.b$

$\Rightarrow \dfrac{dy}{dx} = 2y + be^{2x}$ [By eq. (i)]..........(ii)

Again differentiating w.r.t. x,

=> $\dfrac{d^2y}{dx^2} = 2\dfrac{dy}{dx} + 2be^{2x}$ ....(iii)

Now from eq. (ii), $\dfrac{dy}{dx} - 2y = be^{2x}$

Putting this value of be^{2x} in eq. (iii),

=> $\dfrac{d^2y}{dx^2} = 2\dfrac{dy}{dx} + 2\left(\dfrac{dy}{dx} - 2y\right)$

$\Rightarrow \dfrac{d^2y}{dx^2} = 2\dfrac{dy}{dx} + 2\dfrac{dy}{dx} - 4y$

$\Rightarrow \dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y = 0$

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5. $y = e^x(a\cos x + b\sin x)$

Ans. Given: Equation of the family of curves $y = e^x(a\cos x + b\sin x)$ ....(i)

Here there are two arbitrary constants a and b, therefore we will differentiate both sides two times w.r.t. x

$\Rightarrow \dfrac{dy}{dx} = \left(\dfrac{d}{dx}e^x\right)(a\cos x + b\sin x) + e^x(-a\sin x + b\cos x)$

$\Rightarrow \dfrac{dy}{dx} = e^x(a\cos x + b\sin x) + e^x(-a\sin x + b\cos x)$

$\Rightarrow \dfrac{dy}{dx} = y + e^x(-a\sin x + b\cos x)$ [By eq. (i)]..........(ii)

Again differentiating w.r.t. x,

=> $\dfrac{d^2y}{dx^2} = \dfrac{dy}{dx} + e^x(-a\sin x + b\cos x) + e^x(-a\cos x - b\sin x)$

$\Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{dy}{dx} + \left(\dfrac{dy}{dx} - y\right) - e^x(a\cos x + b\sin x)$

$\Rightarrow \dfrac{d^2y}{dx^2} = 2\dfrac{dy}{dx} - y - y$ [By eq. (i)]

$\Rightarrow \dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 2y = 0$

6. Form the differential equation of the family of circles touching the $y-$axis at the origin.

Ans. It is clear that if a circle touches y-axis at the origin must have its centre on x-axis, because x-axis being at right angles to y-axis is the normal or line of radius of the circle.

Therefore, the centre of the circle is (r,0) where r is the radius of the circle.

$\therefore$ Equation of the required circle is $(x-r)^2 + (y-0)^2 = r^2$

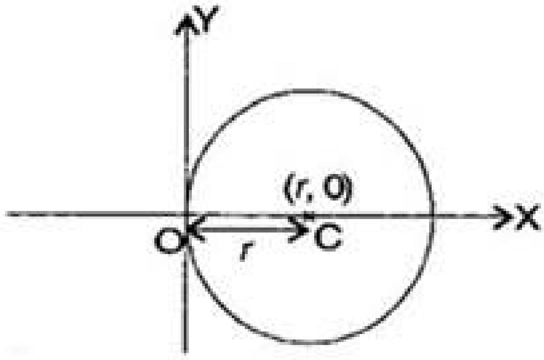
$\Rightarrow x^2 + r^2 + 2rx + y^2 = r^2$

$\Rightarrow x^2 + y^2 = 2rx$ ....(i)

Here r is the only arbitrary constant.

$\therefore$ differentiating w.r.t. x, we get

$2x + 2y\dfrac{dy}{dx} = 2r$ ....(ii)



Putting the value of 2r from eq. (ii) in eq. (i), we get

$$x^2+y^2=\left(2x+2y\frac{dy}{dx}\right)x$$

$$\Rightarrow\ x^2+y^2=2x^2+2xy\frac{dy}{dx}$$

$$\Rightarrow\ -2xy\frac{dy}{dx}-x^2+y^2=0$$

$$\Rightarrow\ 2xy\frac{dy}{dx}+x^2-y^2=0$$

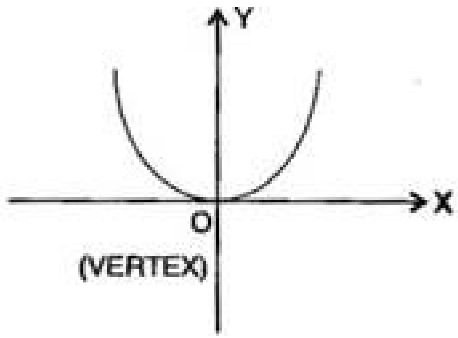
$\Rightarrow\ 2xy\dfrac{dy}{dx}+x^2=y^2$, which is the required differential equation.

7. Find the differential equation of the family of parabolas having vertex at origin and axis along positive y-axis.

Ans. We know that equation of parabolas having vertex at origin and axis along positive $y-$axis is $x^2=4ay$

$\Rightarrow\ 4a=\dfrac{x^2}{y}$ ....(i)

Here a is the only arbitrary constant. Therefore differentiating w.r.t. x, we get



$2x = 4a\frac{dy}{dx}$ ....(ii)

$\Rightarrow\ 2x = \frac{x^2}{y}.\frac{dy}{dx}$ [From eq. (i)]

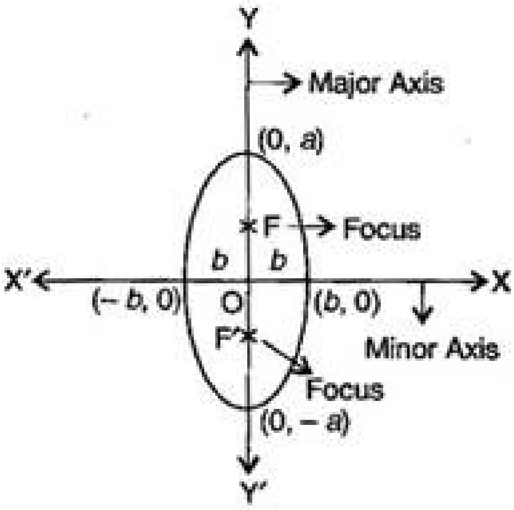
$\Rightarrow\ 2xy = x^2\frac{dy}{dx}$

$\Rightarrow\ 2y = x\frac{dy}{dx}$

$\Rightarrow\ x\frac{dy}{dx} - 2y = 0$, which is the required differential equation.

8. Form the differential equation of family of ellipse having foci on y-axis and centre at the origin.

Ans. We know that equation of ellipse having foci on y-axis i.e., vertical ellipse with major axis as y-axis is $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$ ....(i)



Here there are two arbitrary constants a and b, therefore we will differentiate both sides two times w.r.t. x,

$$\Rightarrow \frac{1}{a^2} 2y\frac{dy}{dx} + \frac{1}{b^2} 2x = 0$$

$$\Rightarrow \frac{2}{a^2} y\frac{dy}{dx} = -\frac{2}{b^2} x$$

$$\Rightarrow \frac{1}{a^2} y\frac{dy}{dx} = -\frac{1}{b^2} x \text{ ....(ii)}$$

Again differentiating w.r.t. x,

$$=> \frac{1}{a^2}\left[y\frac{d^2y}{dx^2} + \frac{dy}{dx}.\frac{dy}{dx}\right] = \frac{-1}{b^2} \text{ ....(iii)}$$

Putting the value of $\frac{-1}{b^2}$ from eq. (iii), in eq. (ii), we get

$$\frac{1}{a^2} y\frac{dy}{dx} = \frac{1}{a^2}\left[y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right]x$$

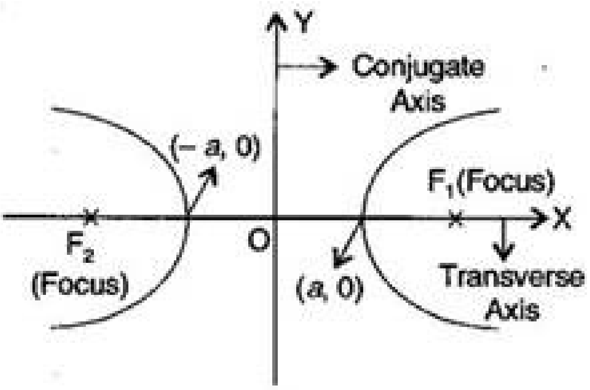
$$\Rightarrow y\frac{dy}{dx} = xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$$

9. Form the differential equation of the family of hyperbolas having foci on x-axis and centre at the origin.

Ans. We know that equation of hyperbolas having foci on x-axis and centre at origin is

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ ....(i)



Here there are two arbitrary constants a and b, therefore we will differentiate both sides two times w.r.t. x

$$\Rightarrow \frac{1}{a^2}2x-\frac{1}{b^2}2y\frac{dy}{dx}=0$$

$$\Rightarrow \frac{2}{a^2}x=\frac{2}{b^2}y\frac{dy}{dx}$$

$\Rightarrow \frac{1}{a^2}x=\frac{1}{b^2}y\frac{dy}{dx}$ ....(ii)

Again differentiating w.r.t. x,

=> $\frac{1}{a^2}.1=\frac{1}{b^2}\left[y\frac{d^2y}{dx^2}+\frac{dy}{dx}.\frac{dy}{dx}\right]$

$$\Rightarrow \frac{1}{a^2}=\frac{1}{b^2}\left[y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right]$$ ....(iii)

Dividing eq. (iii) by eq. (ii), we get

$$\frac{1}{x}=\frac{y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2}{y\frac{dy}{dx}}$$

$$\Rightarrow x\left\{y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right\}=y\frac{dy}{dx}$$

$\Rightarrow xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$, which is required differential equation.

10. Form the differential equation of the family of circles having centres on y-axis and radius 3 units.

Ans. We know that on y-axis, $x=0$.

$\therefore$ Centre of the circle on y- axis is $(0,\beta)$.

$\therefore$ Equation of the circle having centre on y-axis an radius β unit is

$$(x-0)^2+(y-\beta)^2=(3)^2$$

$\Rightarrow x^2+(y-\beta)^2=9$ ....(i)

Here β is the only arbitrary constant, therefore we will differentiate only once.

$$\Rightarrow 2x+2(y-\beta)\frac{d}{dx}(y-\beta)=0$$

$$\Rightarrow 2x+2(y-\beta)\frac{dy}{dx}=0$$

$\Rightarrow 2(y-\beta)\frac{dy}{dx} = -2x$

$\Rightarrow y-\beta = \frac{-2x}{2\frac{dy}{dx}} = \frac{-x}{\frac{dy}{dx}}$

Putting this value of $(y-\beta)$ in eq. (i), we get

=> $x^2 + \frac{x^2}{\left(\frac{dy}{dx}\right)^2} = 9$

$\Rightarrow x^2\left(\frac{dy}{dx}\right)^2 + x^2 = 9\left(\frac{dy}{dx}\right)^2$

$\Rightarrow x^2\left(\frac{dy}{dx}\right)^2 - 9\left(\frac{dy}{dx}\right)^2 + x^2 = 0$

$\Rightarrow (x^2-9)\left(\frac{dy}{dx}\right)^2 + x^2 = 0$

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11. Which of the following differential equation has $y = c_1e^x + c_2e^{-x}$ as the general solution:

(A) $\frac{d^2y}{dx^2} + y = 0$

(B) $\frac{d^2y}{dx^2} - y = 0$

(C) $\frac{d^2y}{dx^2} + 1 = 0$

(D) $\frac{d^2y}{dx^2}-1=0$

Ans. Given: $y=c_1e^x+c_2e^{-x}$ ....(i)

$\therefore \frac{dy}{dx}=c_1e^x+c_2e^{-x}(-1)=c_1e^x-c_2e^{-x}$

$\Rightarrow \frac{d^2y}{dx^2}=c_1e^x-c_2e^{-x}(-1)=c_1e^x+c_2e^{-x}$

$\Rightarrow \frac{d^2y}{dx^2}=y$ [From eq. (i)]

$\Rightarrow \frac{d^2y}{dx^2}-y=0$

Therefore, option (B) is correct.

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12. Which of the following differential equations has $y=x$ as one of its particular solutions:

(A) $\frac{d^2y}{dx^2}-x^2\frac{dy}{dx}+xy=x$

(B) $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+xy=x$

(C) $\frac{d^2y}{dx^2}-x^2\frac{dy}{dx}+xy=0$

(D) $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+xy=0$

Ans. Given: $y=x$

$\therefore \quad \frac{dy}{dx} = 1$

$\Rightarrow \frac{d^2y}{dx^2} = 0$

On putting these values in the given option, we get the correct answer in option (C).

L.H.S. of differential equation of option (C) = $\frac{d^2y}{dx^2} - x\frac{dy}{dx} + xy$

= $0 - x^2(1) + x(x)$

= $-x^2 + x^2 = 0$

= R.H.S. of option (C)

Therefore, option (C) is correct.