CBSE Class–12 Mathematics

NCERT solution

Chapter - 9

Differential Equations - Exercise 9.6

In each of the following differential equations given in each Questions 1 to 4, find the general solution:

1. $\frac{dy}{dx}+2y=\sin x$

Ans. Given: Differential equation $\frac{dy}{dx}+2y=\sin x$

Comparing with $\frac{dy}{dx}+Py=Q$, we have P = 2 and Q = sin x.

$\therefore \int P\,dx=\int 2\,dx=2\int 1\,dx=2x$

=> I.F. = $e^{\int P\,dx}=e^{2x}$

Solution is y (I.F.) = $\int Q(\text{I.F.})\,dx+c$

$\Rightarrow ye^{2x}=\int e^{2x}\sin x\,dx+c$

$\Rightarrow ye^{2x}=I+c$ ....(i)

Applying product rule,

=> I = $e^{2x}(-\cos x)-\int 2e^{2x}(-\cos x)\,dx$

$\Rightarrow -e^{2x}\cos x+2\int e^{2x}\cos x\,dx$

Again applying product rule,

=> I = $-e^{2x}\cos x+2\left[e^{2x}\sin x-\int 2e^{2x}\sin x\,dx\right]$

$\Rightarrow$ I = $-e^{2x}\cos x+2e^{2x}\sin x-4\int e^{2x}\sin x\,dx$

$\Rightarrow$ I = $e^{2x}\left(-\cos x+2\sin x\right)-4\text{I}$

$\Rightarrow 5\text{I}=e^{2x}\left(-\cos x+2\sin x\right)$

$\Rightarrow$ I = $\dfrac{e^{2x}}{5}\left(2\sin x-\cos x\right)$

Putting the value of I in eq. (i),

=> $ye^{2x}=\dfrac{e^{2x}}{5}\left(2\sin x-\cos x\right)+c$

$\Rightarrow y=\dfrac{1}{5}\left(2\sin x-\cos x\right)+\dfrac{c}{e^{2x}}$

$\Rightarrow y=\dfrac{1}{5}\left(2\sin x-\cos x\right)+ce^{-2x}$

2. $\dfrac{dy}{dx}+3y=e^{-2x}$

Ans. Given: Differential equation $\dfrac{dy}{dx}+3y=e^{-2x}$

Comparing with $\dfrac{dy}{dx}+\text{P}y=\text{Q}$,

we have P = 2 and Q = e^{-2x} .

$\therefore \int \text{P}\,dx=\int 3\,dx=3\int 1\,dx=3x$

=> I.F. = $e^{\int \text{P}\,dx}=e^{3x}$

Solution is

=> y (I.F.) = $\int Q(\text{I.F.})\ dx + c$

$\Rightarrow\ ye^{3x} = \int e^{-2x} e^{3x}\ dx + c$

$\Rightarrow\ \int e^{-2x+3x}\ dx + c = \int e^{x}\ dx + c$

$\Rightarrow\ ye^{3x} = e^{x} + c$

$\Rightarrow\ y = \dfrac{e^{x}}{e^{3x}} + \dfrac{c}{e^{3x}}$

$\Rightarrow\ y = e^{-2x} + ce^{-3x}$

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3. $\dfrac{dy}{dx} + \dfrac{y}{x} = x^2$

Ans. Given: Differential equation $\dfrac{dy}{dx} + \dfrac{y}{x} = x^2$

Comparing with $\dfrac{dy}{dx} + Py = Q$,

we have P = $\dfrac{1}{x}$ and Q = x^2.

$\therefore\ \int P\ dx = \int \dfrac{1}{x}\ dx = \log x$

=> I.F. = $e^{\int P\, dx} = e^{\log x}$ = x

Solution is

=> y (I.F.) = $\int Q(\text{I.F.})\ dx + c$

$\Rightarrow \quad yx=\int x^2 . x\ dx+c$

$\Rightarrow \ yx=\int x^3\ dx+c$

$\Rightarrow \ xy=\dfrac{x^4}{4}+c$

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4. $\dfrac{dy}{dx}+(\sec x)y=\tan x\left(0\le y<\dfrac{\pi}{2}\right)$

Ans. Given: Differential equation $\dfrac{dy}{dx}+(\sec x)y=\tan x$

Comparing with $\dfrac{dy}{dx}+\mathrm{P}y=\mathrm{Q}$, we have P = $\sec x$ and Q = $\tan x$.

$\therefore \ \int \mathrm{P}\ dx=\int \sec x\ dx=\log(\sec x+\tan x)$

I.F. = $e^{\int \mathrm{P}\,dx}=e^{\log(\sec x+\tan x)}=\sec x+\tan x$

Solution is

=> y (I.F.) = $\int \mathrm{Q}(\mathrm{I.F.})\ dx+c$

$\Rightarrow \quad y(\sec x+\tan x)=\int \tan x(\sec x+\tan x)\ dx+c$

$\Rightarrow \ y(\sec x+\tan x)=\int\left(\sec x\tan x+\tan^2 x\right)\ dx+c$

$=> y(secx+tanx)=\int(secx.tanx+sec^2x-1)dx+c$

$\Rightarrow \ y(\sec x+\tan x)=\sec x+\tan x-x+c$

For each of the following differential equations given in Question 5 to 8, find the general solution:

5. $\cos^2 x \frac{dy}{dx} + y = \tan x \left(0 \le y < \frac{\pi}{2}\right)$

Ans. Given: Differential equation $\cos^2 x \frac{dy}{dx} + y = \tan x$

$=> \frac{dy}{dx} + \frac{y}{cos^2 x} = \frac{tan x}{cos^2 x}$

$\Rightarrow \frac{dy}{dx} + \left(\sec^2 x\right) y = \sec^2 x \tan x$

Comparing with $\frac{dy}{dx} + \mathrm{P}y = \mathrm{Q}$,

we have P = $\sec^2 x$ and Q = $\sec^2 x \tan x$.

$\therefore \int \mathrm{P}\, dx = \int \sec^2 x\, dx = \tan x$

=> I.F. = $e^{\int \mathrm{P}\, dx} = e^{\tan x}$

Solution is

=> y (I.F.) = $\int \mathrm{Q}(\mathrm{I.F.})\, dx + c$

$\Rightarrow ye^{\tan x} = \int \sec^2 x \tan x e^{\tan x}\, dx + c$ ....(i)

Putting $\tan x = t$ and differentiating $\sec^2 x\, dx = dt$

$\Rightarrow \int \sec^2 x \tan x e^{\tan x}\, dx = \int te^t\, dt$

Applying product rule,

$\Rightarrow \int \sec^2 x \tan x e^{\tan x}\, dx = t.e^t - \int 1.e^t\, dt = t.e^t - e^t = (t-1)e^t = (\tan x - 1)e^{\tan x}$

Putting this value in eq. (i),

=> $ye^{\tan x} = (\tan x - 1)e^{\tan x} + c$

divide by $e^{\tan x}$, we get

$\Rightarrow$ y = (tanx -1) + $ce^{-\tan x}$

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6. $x\dfrac{dy}{dx}+2y=x^2\log x$

Ans. Given: Differential equation $x\dfrac{dy}{dx}+2y=x^2\log x$

$$\Rightarrow \frac{dy}{dx}+\frac{2}{x}y=x\log x$$

Comparing with $\dfrac{dy}{dx}+\text{P}y=\text{Q}$,

we have P = $\dfrac{2}{x}$ and Q = $x\log x$.

$$\therefore \int \text{P}\,dx=2\int\frac{1}{x}\,dx=2\log x$$

=> I.F. = $e^{\int \text{P}\,dx}=e^{2\log x}=e^{\log x^2}=x^2$

Solution is

=> y (I.F.) = $\int \text{Q}(\text{I.F.})\,dx+c$

$$\Rightarrow yx^2=\int(x\log x).x^2\,dx+c$$

$$\Rightarrow yx^2=\int\log x.x^3\,dx+c$$

$$\Rightarrow yx^2=\log x.\frac{x^4}{4}-\int\frac{1}{x}.\frac{x^4}{4}\,dx+c$$

$$\Rightarrow yx^2=\frac{x^4}{4}\log x-\frac{1}{4}\int x^3\,dx+c$$

$\Rightarrow \; yx^2 = \frac{x^4}{4}\log x - \frac{x^4}{16} + c$

$\Rightarrow \; y = \frac{x^2}{4}\log x - \frac{x^2}{16} + \frac{c}{x^2}$

$\Rightarrow \; y = \frac{x^2}{16}(4\log x - 1) + \frac{c}{x^2}$

7. $x\log x\frac{dy}{dx} + y = \frac{2}{x}\log x$

Ans. Given: Differential equation $x\log x\frac{dy}{dx} + y = \frac{2}{x}\log x$

$\Rightarrow \; \frac{dy}{dx} + \frac{1}{x\log x}y = \frac{2}{x^2}$

Comparing with $\frac{dy}{dx} + Py = Q$,

we have P = $\frac{1}{x\log x}$ and Q = $\frac{2}{x^2}$.

$\therefore \; \int P\,dx = \int \frac{1}{x\log x}\,dx = \int \frac{1/x}{\log x}\,dx = \log(\log x)$

=> I.F. = $e^{\int P\,dx} = e^{\log(\log x)} = \log x$

Solution is

=> y (I.F.) = $\int Q(\text{I.F.})\,dx + c$

$\Rightarrow \; y\log x = \int \frac{2}{x^2}\log x\,dx = 2\int(\log x)x^{-2}\,dx + c$

Applying Product rule of Integration,

$$=> ylogx = 2\left[(logx)\frac{x^{-1}}{-1} - \int \frac{1}{x}.\frac{x^{-1}}{-1}dx\right] + c$$

$$\Rightarrow \ y\log x = 2\left[\frac{-\log x}{x} + \int x^{-2}\ dx\right] + c$$

$$\Rightarrow \ y\log x = 2\left[\frac{-\log x}{x} + \frac{x^{-1}}{-1}\right] + c$$

$$\Rightarrow \ y\log x = \frac{-2}{x}(1 + \log x) + c$$

8. $\left(1+x^2\right)\ dy + 2xy\ dx = \cot x\ dx \left(x \neq 0\right)$

Ans. Given: Differential equation $\left(1+x^2\right)\ dy + 2xy\ dx = \cot x\ dx$

$$\Rightarrow \ \left(1+x^2\right)\frac{dy}{dx} + 2xy = \cot x$$

$$\Rightarrow \ \frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{\cot x}{1+x^2}$$

Comparing with $\frac{dy}{dx} + \mathrm{P}y = \mathrm{Q}$,

we have P = $\frac{2x}{1+x^2}$ and Q = $\frac{\cot x}{1+x^2}$.

$$\therefore \ \int \mathrm{P}\ dx = \int \frac{2x}{1+x^2}\ dx = \log\left|1+x^2\right| = \log\left(1+x^2\right)$$

=> I.F. = $e^{\int \mathrm{P}\ dx} = e^{\log\left(1+x^2\right)} = 1 + x^2$

Solution is

=> y (I.F.) = $\int Q(\text{I.F.})\ dx + c$

$$\Rightarrow\ y(1+x^2) = \int \frac{\cot x}{1+x^2}(1+x^2)\ dx + c$$

$$\Rightarrow\ y(1+x^2) = \int \cot x\ dx + c$$

$$\Rightarrow\ y(1+x^2) = \log|\sin x| + c$$

$$\Rightarrow\ y = \frac{\log|\sin x|}{1+x^2} + \frac{c}{1+x^2}$$

$$\Rightarrow\ y = (1+x^2)^{-1}\log|\sin x| + c(1+x^2)^{-1}$$

For each of the following differential equations given in Question 9 to 12, find the general solution:

9. $x\frac{dy}{dx} + y - x + xy\cot x = 0\ (x \neq 0)$

Ans. Given: Differential equation $x\frac{dy}{dx} + y - x + xy\cot x = 0$

$$\Rightarrow\ x\frac{dy}{dx} + y + xy\cot x = x$$

$$\Rightarrow\ x\frac{dy}{dx} + (1 + x\cot x)\,y = x$$

$$\Rightarrow\ \frac{dy}{dx} + \frac{(1+x\cot x)}{x}\,y = 1$$

Comparing with $\frac{dy}{dx} + Py = Q$,

we have P = $\frac{1+x\cot x}{x}$ and Q = 1.

$\therefore \quad \int \mathrm{P}\ dx = \int \frac{1+x\cot x}{x}\ dx = \int \left(\frac{1}{x} + \cot x\right)\ dx$

= $\log x + \log \sin x = \log\left(x \sin x\right)$

=> I.F. = $e^{\int \mathrm{P}\ dx} = e^{\log(x\sin x)} = x\sin x$

Solution is

=> y (I.F.) = $\int \mathrm{Q}(\mathrm{I.F.})\ dx + c$

$\Rightarrow \quad y\left(x\sin x\right) = \int 1.x\sin x\ dx + c$

Applying product rule of Integration,

$\Rightarrow \quad y\left(x\sin x\right) = x\left(-\cos x\right) - \int 1\left(-\cos x\right)\ dx + c$ = $-x\cos x + \int \cos x\ dx + c$

$\Rightarrow \quad y\left(x\sin x\right) = -x\cos x + \sin x + c$

$\Rightarrow \quad y = \frac{-x\cos x}{x\sin x} + \frac{\sin x}{x\sin x} + \frac{c}{x\sin x}$

$\Rightarrow \quad y = -\cot x + \frac{1}{x} + \frac{c}{x\sin x}$

10. $\left(x+y\right)\frac{dy}{dx} = 1$

Ans. Given: Differential equation $\left(x+y\right)\frac{dy}{dx} = 1$

$\Rightarrow \quad \frac{dx}{dy} = x + y$

$\Rightarrow \quad \frac{dx}{dy} - x = y$

Comparing with $\frac{dx}{dy}+\mathrm{P}x=\mathrm{Q}$, we have P = -1 and Q = y .

$\therefore \int \mathrm{P}\ dy=\int -1\ dy=-\int 1\ dt=-y$ I.F. = $e^{\int \mathrm{P}\,dy}=e^{-y}$

Solution is

=> x(I.F.) = $\int Q(I.F.)dy+c$

$\Rightarrow xe^{-y}=\int ye^{-y}\ dy+c$

Applying product rule of Integration,

$\Rightarrow xe^{-y}=y\frac{e^{-y}}{-1}-\int 1.\frac{e^{-y}}{-1}\ dy+c$

=> xe^{-y}= $-ye^{-y}+\int e^{-y}\ dy+c$

=> xe^{-y} = $-ye^{-y}+\frac{e^{-y}}{-1}+c$

$\Rightarrow xe^{-y}=-ye^{-y}-e^{-y}+c$

$\Rightarrow x=-y-1+\frac{c}{e^{-y}}$

$\Rightarrow$ x+y+1 = ce^{y}

11. $y\ dx+\left(x-y^2\right)\ dy=0$

Ans. Given: Differential equation $y\ dx+\left(x-y^2\right)\ dy=0$

$\Rightarrow y\frac{dx}{dy}+x-y^2=0$

$\Rightarrow \quad y\dfrac{dx}{dy}+x=y^2$

$\Rightarrow \frac{dx}{dy}+\frac{1}{y}x=y$

Comparing with $\dfrac{dx}{dy}+\mathrm{P}x=\mathrm{Q}$, w

e have P = $\dfrac{1}{y}$ and Q = y .

$\therefore \quad \int \mathrm{P}\,dy=\int \dfrac{1}{y}\,dy=\log y$

I.F. = $e^{\int \mathrm{P}\,dy}=e^{\log y}=y$

Solution is x(I.F.) = $\int Q(I.F.)dy+c$

$\Rightarrow \quad x.y=\int yy\,dy+c$

$\Rightarrow \quad xy=\int y^2\,dy+c$

$\Rightarrow \quad xy=\dfrac{y^3}{3}+c$

$\Rightarrow \quad x=\dfrac{y^2}{3}+\dfrac{c}{y}$

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12. $\left(x+3y^2\right)\dfrac{dy}{dx}=y\left(y>0\right)$

Ans. Given: Differential equation $\left(x+3y^2\right)\dfrac{dy}{dx}=y$

$\Rightarrow \quad y\dfrac{dx}{dy}=x+3y^2$

$\Rightarrow \quad y\dfrac{dx}{dy} - x = 3y^2$

$\Rightarrow \quad \dfrac{dx}{dy} - \dfrac{1}{y}x = 3y$

Comparing with $\dfrac{dx}{dy} + \mathrm{P}x = \mathrm{Q}$,

we have P = $\dfrac{-1}{y}$ and Q = $3y$.

$\therefore \quad \int \mathrm{P}\, dy = -\int \dfrac{1}{y}\, dy = -\log y = \log y^{-1}$

=> I.F. = $e^{\int \mathrm{P}\, dy} = e^{\log y^{-1}} = y^{-1} = \dfrac{1}{y}$

Solution is x(I.F.) = $\int Q(I.F.)dy + c$

$\Rightarrow \quad x.\dfrac{1}{y} = \int 3y.\dfrac{1}{y}\, dy + c$

$\Rightarrow \quad \dfrac{x}{y} = 3\int 1\, dy + c = 3y + c$

$\Rightarrow \quad x = 3y^2 + cy$

For each of the differential equations given in Questions 13 to 15, find a particular solution satisfying the given condition:

13. $\dfrac{dy}{dx} + 2y\tan x = \sin x; y = 0$ **when** $x = \frac{\pi}{3}$

Ans. Given: Differential equation $\dfrac{dy}{dx} + 2y\tan x = \sin x; y = 0$ when $x = \dfrac{\pi}{3}$

Comparing with $\frac{dy}{dx}+Py=Q$,

we have P = $2\tan x$ and Q = $\sin x$.

$\therefore \int P\, dx = 2\int \tan x\, dx = 2\log\sec x = \log(\sec x)^2$

=> I.F.= $e^{\int P\, dx} = e^{\log(\sec x)^2} = (\sec x)^2 = \sec^2 x$

Solution is y (I.F.) = $\int Q(\text{I.F.})\, dx + c$

$\Rightarrow y\sec^2 x = \int \sin x.\sec^2 x\, dx + c$

$\Rightarrow y\sec^2 x = \int \frac{\sin x}{\cos^2 x}\, dx + c$

$\Rightarrow y\sec^2 x = \int \frac{\sin x}{\cos x.\cos x}\, dx + c$

$\Rightarrow y\sec^2 x = \int \tan x.\sec x\, dx + c = \sec x + c$

$\Rightarrow \frac{y}{\cos^2 x} = \frac{1}{\cos x} + c$ .... (1)

put value of x and y, we get

$=> \frac{0}{cos^2\frac{\pi}{3}} = \frac{1}{cos\frac{\pi}{3}} + c$

=> $0 = 2 + c$

=> c = -2 put value of c in (1), we get

$=> \frac{y}{cos^2 x} = \frac{1}{cosx} - 2$

$=> y = cosx - 2cos^2 x$

14. $\left(1+x^2\right)\frac{dy}{dx}+2xy=\frac{1}{1+x^2}; y=0$ **when** $x=1$

Ans. Given: Differential equation $\left(1+x^2\right)\frac{dy}{dx}+2xy=\frac{1}{1+x^2}; y=0$ when $x=1$

$$\Rightarrow \frac{dy}{dx}+\frac{2x}{1+x^2}y=\frac{1}{\left(1+x^2\right)^2}$$

Comparing with $\frac{dy}{dx}+\text{P}y=\text{Q}$,

we have P = $\frac{2x}{1+x^2}$ and Q = $\frac{1}{\left(1+x^2\right)^2}$.

$$\therefore \int \text{P}\ dx=\int\frac{2x}{1+x^2}\ dx=\log\left(1+x^2\right)$$

=> I.F.= $e^{\int \text{P}\ dx}=e^{\log\left(1+x^2\right)}=1+x^2$

Solution is y (I.F.) = $\int \text{Q}(\text{I.F.})\ dx+c$

$$\Rightarrow y\left(1+x^2\right)=\int\frac{1}{\left(1+x^2\right)^2}\left(1+x^2\right)\ dx+c$$

$$\Rightarrow y\left(1+x^2\right)=\int\frac{1}{\left(1+x^2\right)}\ dx+c=\tan^{-1}x+c \quad \text{..........(i)}$$

Now putting $y=0, x=1$ $\quad 0=\tan^{-1}x+c$

$$\Rightarrow 0=\frac{\pi}{4}+c$$

$$\Rightarrow c=-\frac{\pi}{4}$$

Putting the value of c in eq. (i),

=> $y\left(1+x^2\right)=\tan^{-1}x-\dfrac{\pi}{4}$

15. $\dfrac{dy}{dx}-3y\cot x=\sin 2x; y=2$ **when** $x=\dfrac{\pi}{2}$

Ans. Given: Differential equation $\dfrac{dy}{dx}-3y\cot x=\sin 2x$

Comparing with $\dfrac{dy}{dx}+\text{P}y=\text{Q}$,

we have P = $-3\cot x$ and Q = $\sin 2x$.

$\therefore \int \text{P}\,dx=-3\int \cot x\,dx=-3\log\sin x=\log(\sin x)^{-3}$

I.F.= $e^{\int \text{P}\,dx}=e^{\log(\sin x)^{-3}}=(\sin x)^{-3}=\dfrac{1}{\sin^3 x}$

Solution is y (I.F.) = $\int \text{Q(I.F.)}\,dx+c$

$\Rightarrow y\dfrac{1}{\sin^3 x}=\int \sin 2x.\dfrac{1}{\sin^3 x}\,dx+c$

$\Rightarrow \dfrac{y}{\sin^3 x}=\int \dfrac{2\sin x\cos x}{\sin^3 x}\,dx+c=2\int \dfrac{\cos x}{\sin^2 x}\,dx+c$

$\Rightarrow \dfrac{y}{\sin^3 x}=2\int \dfrac{\cos x}{\sin x.\sin x}\,dx+c$

$\Rightarrow \dfrac{y}{\sin^3 x}=2\int \operatorname{cosec} x\cot x\,dx+c$

$\Rightarrow \dfrac{y}{\sin^3 x}=-2\operatorname{cosec} x+c$

$\Rightarrow \dfrac{y}{\sin^3 x} = -\dfrac{2}{\sin x} + c$

$\Rightarrow y = -2\sin^2 x + c\sin^3 x$ ....(i)

Now putting $y = 2, x = \dfrac{\pi}{2}$ in eq. (i),

$2 = -2\sin^2 \dfrac{\pi}{2} + c\sin^3 \dfrac{\pi}{2}$

$\Rightarrow 2 = -2 + c$

$\Rightarrow c = 4$

Putting $c = 4$ in eq. (i),

=> $y = -2\sin^2 x + 4\sin^3 x$

=> y = $4\sin^3$x - $2\sin^2$x

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16. Find the equation of the curve passing through the origin, given that the slope of the tangent to the curve at any point (x,y) is equal to the sum of coordinates of that point.

Ans. Slope of the tangent to the curve at any point (x,y) = Sum of coordinates of the point (x, y)

$\Rightarrow \dfrac{dy}{dx} = x + y$

$\Rightarrow \dfrac{dy}{dx} - y = x$

Comparing with $\dfrac{dy}{dx} + \mathrm{P}y = \mathrm{Q}$,

we have P = -1 and Q = x.

$\therefore \quad \int \mathrm{P}\ dx = -\int 1\ dx = -x$

=> I.F.= $e^{\int \mathrm{P}\, dx} = e^{-x}$

Solution is

=> y(I.F.) = $\int \mathrm{Q}(\mathrm{I.F.})\ dx + c$

$\Rightarrow \quad ye^{-x} = \int x.e^{-x}\ dx + c$

Applying Product rule of Integration,

=> $\quad ye^{-x} = x\dfrac{e^{-x}}{-1} - \int 1.\dfrac{e^{-x}}{-1}\ dx + c$

$\Rightarrow \quad ye^{-x} = -xe^{-x} + \dfrac{e^{-x}}{-1} + c$

$\Rightarrow \quad ye^{-x} = -xe^{-x} - e^{-x} + c$

$\Rightarrow \quad \dfrac{y}{e^{x}} = -\dfrac{x}{e^{x}} - \dfrac{1}{e^{x}} + c$

$\Rightarrow \quad y = -x - 1 + ce^{x}$ ....(i)

Now, since curve (i) passes through the origin (0, 0),

therefore putting $x = 0,\ y = 0$ in eq. (i)

$\Rightarrow \quad -c = -1$

$\Rightarrow \quad c = 1$

Putting $c = 1$ in eq. (i),

$y = -x - 1 + e^{x}$

$\Rightarrow \quad y + x + 1 = e^{x}$

17. Find the equation of the curve passing through the point (0, 2) given that the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangents to the curve at that point by 5.

Ans. According to the question, Sum of the coordinates of any point say (x, y) on the curve

= Magnitude of the slope of the tangent to the curve + 5

$$\Rightarrow x+y=\frac{dy}{dx}+5$$

$$\Rightarrow \frac{dy}{dx}-y=x-5$$

Comparing with $\frac{dy}{dx}+\mathrm{P}y=\mathrm{Q}$,

we have P = -1 and Q = $x-5$.

$$\therefore \int \mathrm{P}\, dx=-\int 1\, dx=-x$$

=> I.F.= $e^{\int \mathrm{P}\, dx}=e^{-x}$

Solution is

=> y(I.F.) = $\int \mathrm{Q}(\mathrm{I.F.})\, dx+c$

$$\Rightarrow ye^{-x}=\int (x-5).e^{-x}\, dx+c$$

Applying Product rule of Integration,

$$\Rightarrow ye^{-x}=(x-5)\frac{e^{-x}}{-1}-\int 1.\frac{e^{-x}}{-1}\, dx+c$$

$$\Rightarrow ye^{-x}=-(x-5)e^{-x}+\int e^{-x}\, dx+c$$

$\Rightarrow\ ye^{-x} = -(x-5)e^{-x} + \frac{e^{-x}}{-1} + c$

$\Rightarrow\ \frac{y}{e^x} = -\frac{x-5}{e^x} - \frac{1}{e^x} + c$

$\Rightarrow\ y = -x + 5 - 1 + ce^x$

$\Rightarrow\ x + y = 4 + ce^x$ ....(i)

Now, since curve (i) passes through the point (0, 2),

therefore putting $x=0,\ y=2$ in eq. (i)

$\Rightarrow$ $0+2 = 4+ce^0$

$\Rightarrow\ c = -2$

Putting $c=-2$ in eq. (i),

$\Rightarrow\ x + y = 4 - 2e^x$

$\Rightarrow\ y = 4 - x - 2e^x$

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18. Choose the correct answer:

The integrating factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$ **is:**

(A) e^{-x}

(B) e^{-y}

(C) $\frac{1}{x}$

(D) x

Ans. Given: Differential equation $x\frac{dy}{dx}-y=2x^2$

$$\Rightarrow \frac{dy}{dx}-\frac{1}{x}y=2x$$

Comparing with $\frac{dy}{dx}+\mathrm{P}y=\mathrm{Q}$,

we have P = $\frac{-1}{x}$ and Q = $2x$.

$$\therefore \int \mathrm{P}\,dx=-\int\frac{1}{x}\,dx=-\log x=\log x^{-1}$$

=> I.F.= $e^{\int \mathrm{P}\,dx}=e^{\log x^{-1}}=x^{-1}=\frac{1}{x}$

Therefore, option (C) is correct.

19. Choose the correct answer:

The integrating factor of the differential equation $(1-y^2)\frac{dx}{dy}+yx=ay(-1<y<1) is$

(A) $\frac{1}{y^2-1}$

(B) $\frac{1}{\sqrt{y^2-1}}$

(C) $\frac{1}{1-y^2}$

(D) $\frac{1}{\sqrt{1-y^2}}$

Ans. Given: Differential equation $(1-y^2)\frac{dx}{dy}+yx=ay(-1<y<1)$

$$\Rightarrow \frac{dx}{dy}+\frac{y}{1-y^2}x=\frac{ay}{1-y^2}$$

Comparing with $\frac{dx}{dy}+\mathrm{P}x=\mathrm{Q}$,

we have P = $\frac{y}{1-y^2}$ and Q = $\frac{ay}{1-y^2}$

$$\therefore \int \mathrm{P}\,dy=\int\frac{y}{1-y^2}\,dy=\frac{-1}{2}\int\frac{-2y}{1-y^2}\,dy$$

= $\frac{-1}{2}\log\left(1-y^2\right)=\log\left(1-y^2\right)^{-1/2}$

=> I.F. = $e^{\int \mathrm{P}\,dy}=e^{\log\left(1-y^2\right)^{-1/2}}=\left(1-y^2\right)^{-1/2}=\frac{1}{\sqrt{1-y^2}}$

Therefore, option (D) is correct.