CBSE Class–12 Mathematics

NCERT solution

Chapter - 7

Integrals - Exercise 7.1

Find an antiderivative (or integral) of the following functions by the method of inspection in Exercises 1 to 5.

1. $\sin 2x$

Ans. $\because \dfrac{d}{dx}(\cos 2x) = -2\sin 2x$

$\Rightarrow \dfrac{1}{-2}\dfrac{d}{dx}(\cos 2x) = \sin 2x$

$\Rightarrow \dfrac{d}{dx}\left(\dfrac{-1}{2}\cos 2x\right) = \sin 2x$

$\therefore$ An anti-derivative of $\sin 2x$ is $\dfrac{-1}{2}\cos 2x.$

2. $\cos 3x$

Ans. $\because \dfrac{d}{dx}(\sin 3x) = 3\cos 3x$

$\Rightarrow \dfrac{1}{3}\dfrac{d}{dx}(\sin 3x) = \cos 3x$

$\Rightarrow \dfrac{d}{dx}\left(\dfrac{1}{3}\sin 3x\right) = \cos 3x$

$\therefore$ An anti-derivative of $\cos 3x$ is $\dfrac{1}{3}\sin 3x.$

3. e^{2x}

Ans. $\because \dfrac{d}{dx}e^{2x} = e^{2x}\dfrac{d}{dx}(2x) = 2e^{2x}$

$\Rightarrow \dfrac{1}{2}\dfrac{d}{dx}e^{2x} = e^{2x}$

$\Rightarrow \dfrac{d}{dx}\left(\dfrac{1}{2}e^{2x}\right) = e^{2x}$

$\therefore$ An anti-derivative of e^{2x} is $\dfrac{1}{2}e^{2x}$.

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4. $(ax+b)^2$

Ans. $\because \dfrac{d}{dx}(ax+b)^3 = 3(ax+b)^2\dfrac{d}{dx}(ax+b) = 3(ax+b)^2 a$

$\Rightarrow \dfrac{1}{3a}\dfrac{d}{dx}(ax+b)^3 = (ax+b)^2$

$\Rightarrow \dfrac{d}{dx}\left[\dfrac{1}{3a}(ax+b)^3\right] = (ax+b)^2$

$\therefore$ An anti-derivative of $(ax+b)^2$ is $\dfrac{1}{3a}(ax+b)^3$.

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5. $\sin 2x - 4e^{3x}$

Ans. $\because \dfrac{d}{dx}(\cos 2x) = -2\sin 2x$

$\Rightarrow \dfrac{1}{-2}\dfrac{d}{dx}(\cos 2x) = \sin 2x$

$\Rightarrow \frac{d}{dx}\left(\frac{-1}{2}\cos 2x\right) = \sin 2x$ .....(i)

Again $\frac{d}{dx}e^{3x} = 3e^{3x}$

$\Rightarrow \frac{d}{dx}\left(\frac{1}{3}e^{3x}\right) = e^{3x}$

$\frac{d}{dx}\left(\frac{-4}{3}e^{3x}\right) = -4e^{3x}$ [Multiplying both sides by -4] ....(ii)

Adding eq. (i) and (ii), we get

$\Rightarrow \frac{d}{dx}\left(\frac{-1}{2}\cos 2x\right) + \frac{d}{dx}\left(\frac{-4}{3}e^{3x}\right) = \sin 2x + \left(-4e^{3x}\right)$

$\Rightarrow \frac{d}{dx}\left(\frac{-1}{2}\cos 2x - \frac{4}{3}e^{3x}\right) = \sin 2x - 4e^{3x}$

$\therefore$ An anti-derivative of $\sin 2x - 4e^{3x}$ is $\frac{-1}{2}\cos 2x - \frac{4}{3}e^{3x}$.

Evaluate the following integrals in Exercises 6 to 11.

6. $\int\left(4e^{3x}+1\right)\, dx$

Ans. $\int\left(4e^{3x}+1\right)\, dx$

= $\int 4e^{3x}\, dx + \int 1\, dx$

= $4\int e^{3x}\, dx + x$

= $4\frac{e^{3x}}{3} + x + c$ $\left[\because e^{ax}\, dx = \frac{e^{ax}}{x} \text{ and } \int 1\, dx = x\right]$

7. $\int x^2\left(1-\frac{1}{x^2}\right)\,dx$

Ans. $\int x^2\left(1-\frac{1}{x^2}\right)\,dx = \int\left(x^2-\frac{x^2}{x^2}\right)\,dx = \int\left(x^2-1\right)\,dx = \int x^2\,dx-\int 1\,dx$

$\frac{x^3}{3}-x+c\left[\because \int x^n\,dx=\frac{x^{n+1}}{n+1}\text{ if } n\neq -1\right]$

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8. $\int\left(ax^2+bx+c\right)\,dx$

Ans. $\int\left(ax^2+bx+c\right)\,dx$

= $\int ax^2\,dx+\int bx\,dx+\int c\,dx$

= $a\int x^2\,dx+b\int x\,dx+c\int 1\,dx$

= $a\frac{x^3}{3}+b\frac{x^2}{2}+cx+c_1$ where c_1 is the constant of integration.

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9. $\int\left(2x^2+e^x\right)\,dx$

Ans. $\int\left(2x^2+e^x\right)\,dx$

= $\int 2x^2\,dx+\int e^x\,dx$

= $2\int x^2\,dx+\int e^x\,dx$

= $2\frac{x^3}{3}+e^x+c$

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10. $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx$

Ans. $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx$

$= \int\left\{(\sqrt{x})^2+\left(\frac{1}{\sqrt{x}}\right)^2-2\sqrt{x}\,\frac{1}{\sqrt{x}}\right\}dx$

$= \int\left(x+\frac{1}{x}-2\right) dx$

$= \int x\,dx+\int\frac{1}{x}\,dx-\int 2\,dx$

$= \frac{x^2}{2}+\log|x|-2x+c$

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11. $\int\frac{x^3+5x^2-4}{x^2}\,dx$

Ans. $\int\frac{x^3+5x^2-4}{x^2}\,dx$

$= \int\left(\frac{x^3}{x^2}+\frac{5x^2}{x^2}-\frac{4}{x^2}\right) dx$

$= \int\left(x+5-4x^{-2}\right) dx$

$= \int x\,dx+\int 5\,dx-\int 4x^{-2}\,dx$

$= \int x\,dx+5\int 1\,dx-4\int x^{-2}\,dx$

$= \frac{x^2}{2}+5x+\frac{4}{x}+c$

Evaluate the following integrals in Exercises 12 to 16.

12. $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

Ans. $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$= \int \left(\frac{x^3}{x^{\frac{1}{2}}} + \frac{3x}{x^{\frac{1}{2}}} + \frac{4}{x^{\frac{1}{2}}} \right) dx$

$= \int \left(x^{3-\frac{1}{2}} + 3x^{1-\frac{1}{2}} + 4x^{\frac{-1}{2}} \right) dx$

$= \int \left(x^{\frac{5}{2}} + 3x^{\frac{1}{2}} + 4x^{\frac{-1}{2}} \right) dx$

$= \int x^{\frac{5}{2}}\,dx + 3\int x^{\frac{1}{2}}\,dx + 4\int x^{\frac{-1}{2}}\,dx$

$= \frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1} + 3\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + 4\frac{x^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} + c$

$= \frac{x^{\frac{7}{2}}}{\frac{7}{2}} + 3\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 4\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= \frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8x^{\frac{1}{2}} + c$

13. $\int \frac{x^3-x^2+x-1}{x-1}\,dx$

Ans. $\int \frac{x^3 - x^2 + x - 1}{x-1}\ dx$

$= \int \frac{x^2(x-1)+(x-1)}{(x-1)}\ dx$

$= \int \frac{(x-1)(x^2+1)}{(x-1)}\ dx$

$= \int (x^2+1)\ dx$

$= \int x^2\ dx + \int 1\ dx$

$= \frac{x^3}{3} + x + c$

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14. $\int (1-x)\sqrt{x}\ dx$

Ans. $\int (1-x)\sqrt{x}\ dx$

$= \int \left(\sqrt{x} - x\sqrt{x}\right)\ dx$

$= \int \left(x^{\frac{1}{2}} - x^{\frac{3}{2}}\right)\ dx$

$= \int x^{\frac{1}{2}}\ dx - \int x^{\frac{3}{2}}\ dx$

$= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + c$

$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}} = \frac{2}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} + c$

15. $\int \sqrt{x}\left(3x^2 + 2x + 3\right)\, dx$

Ans. $\int \sqrt{x}\left(3x^2 + 2x + 3\right)\, dx$

$= \int x^{\frac{1}{2}}\left(3x^2 + 2x + 3\right)\, dx$

$= \int\left(3x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + 3x^{\frac{1}{2}}\right) dx$

$= \int 3x^{\frac{5}{2}}\, dx + \int 2x^{\frac{3}{2}}\, dx + \int 3x^{\frac{1}{2}}\, dx$

$= 3\frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1} + 2\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + 3\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$

$= 3\frac{x^{\frac{7}{2}}}{\frac{7}{2}} + 2\frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 3\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= \frac{6}{7}x^{\frac{7}{2}} + \frac{4}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + c$

16. $\int\left(2x - 3\cos x + e^x\right)\, dx$

Ans. $\int\left(2x - 3\cos x + e^x\right)\, dx$

$= \int 2x\, dx - \int 3\cos x\, dx + \int e^x\, dx$

= $2\int x\ dx - 3\int \cos x\ dx + \int e^x\ dx$

= $2\dfrac{x^2}{2} - 3\sin x + e^x + c$

= $x^2 - 3\sin x + e^x + c$

Evaluate the following integrals in Exercises 17 to 20.

17. $\int \left(2x^2 - 3\sin x + 5\sqrt{x}\right)\ dx$

Ans. $\int \left(2x^2 - 3\sin x + 5\sqrt{x}\right)\ dx$

= $2\int x^2\ dx - 3\int \sin x\ dx + 5\int x^{\frac{1}{2}}\ dx$

= $2\dfrac{x^3}{3} - 3(-\cos x) + 5\dfrac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$

= $2\dfrac{x^3}{3} + 3\cos x + 5\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

= $2\dfrac{x^3}{3} + 3\cos x + \dfrac{10}{3}x^{\frac{3}{2}} + c$

18. $\int \sec x(\sec x + \tan x)\ dx$

Ans. $\int \sec x(\sec x + \tan x)\ dx$

= $\int \left(\sec^2 x + \sec x \tan x\right)\ dx$

$= \int \sec^2 x\, dx + \int \sec x \tan x\, dx$

$= \tan x + \sec x + c$

19. $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x}\, dx$

Ans. $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x}\, dx$

$= \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}}\, dx$

$= \int \frac{\sin^2 x}{\cos^2 x}\, dx$

$= \int \tan^2 x\, dx$

$= \int (\sec^2 x - 1)\, dx$

$= \int \sec^2 x\, dx - \int 1\, dx$

$= \tan x - x + c$

20. $\int \frac{2 - 3\sin x}{\cos^2 x}\, dx$

Ans. $\int \frac{2 - 3\sin x}{\cos^2 x}\, dx$

$= \int \left(\frac{2}{\cos^2 x} - \frac{3\sin x}{\cos^2 x} \right) dx$

= $\int\left(2\sec^2 x - 3\tan x \sec x\right)\,dx$

= $2\int \sec^2 x\,dx - 3\int \sec x \tan x\,dx$

= $2\tan x - 3\sec x + c$

21. Choose the correct answer:

The anti derivative of $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$ **equals.**

(A) $\frac{1}{3}x^{1/3}+2x^{1/2}+C$

(B) $\frac{2}{3}x^{2/3}+\frac{1}{2}x^2+C$

(C) $\frac{2}{3}x^{3/2}+2x^{1/2}+C$

(D) $\frac{3}{2}x^{3/2}+\frac{1}{2}x^{1/2}+C$

Ans. $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)\,dx$

= $\int\left(x^{\frac{1}{2}}+x^{\frac{-1}{2}}\right)\,dx$

= $\int x^{\frac{1}{2}}\,dx+\int x^{\frac{-1}{2}}\,dx$

= $\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+\frac{x^{\frac{-1}{2}+1}}{\frac{-1}{2}+1}+C$

= $\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+C$

= $\frac{2}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+C$

Therefore, option (C) is correct.

22. Choose the correct answer:

If $\frac{d}{dx}f(x)=4x^3-\frac{3}{x^4}$ **such that** $f(2)=0$. **Then** $f(x)$ **is:**

(A) $x^4+\frac{1}{x^3}-\frac{129}{8}$

(B) $x^3+\frac{1}{x^4}+\frac{129}{8}$

(C) $x^4+\frac{1}{x^3}+\frac{129}{8}$

(D) $x^3+\frac{1}{x^4}-\frac{129}{8}$

Ans. $f(x)=\int\left(4x^3-\frac{3}{x^4}\right)dx$

= $4\int x^3\,dx-3\int\frac{1}{x^4}\,dx$

= $4.\frac{x^4}{4}-3\int x^{-4}\,dx$

= $x^4-3\frac{x^{-3}}{-3}+c$

$\Rightarrow \ f(x) = x^4 + \frac{1}{x^3} + c$ ....(i)

$\Rightarrow f(2) = 16 + \frac{1}{8} + c$

$\Rightarrow 0 = \frac{128+1}{8} + c$

$\Rightarrow c + \frac{129}{8} = 0$

$\Rightarrow c = \frac{-129}{8}$

Putting $c = \frac{-129}{8}$ in eq. (i),

$f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$

Therefore, option (A) is correct.