CBSE Class–12 Mathematics

NCERT solution

Chapter - 7

Integrals - Exercise 7.8

Evaluate the following definite integrals as limit of sums:

1. $\int_a^b x\,dx$

Ans. We know that

$$\int_a^b f(x)dx = \lim_{h\to 0}[f(a) + f(a+h) + f(a+2h)+\ldots+f(a+(n-1)h)]$$

where $nh = b - a$

Here, $a = a, b = b$ and $f(x) = x$

$$\therefore \int_a^b x\,dx = \lim_{h\to 0} h\left[a+(a+h)+(a+2h)+\ldots\ldots+(a+(n-1)h)\right]$$

$$\Rightarrow \int_a^b x\,dx = \lim_{h\to 0} h\left[na+h(1+2+3+\ldots\ldots\ldots+(n-1))\right]$$

$$\Rightarrow \int_a^b x\,dx = \lim_{h\to 0}\left[anh+h\frac{n(n-1)}{2}\right]$$

$$= \lim_{h\to 0}\left[anh+\frac{nh(nh-h)}{2}\right]$$

$$= \lim_{h\to 0}\left[a(b-a)+\frac{(b-a)(b-a-h)}{2}\right]\left[\because nh = b-a\right]$$

= $\left[a(b-a)+\frac{(b-a)(b-a)}{2}\right]$

= $(b-a)\left[a+\frac{b-a}{2}\right]$

= $(b-a)\left[\frac{2a+b-a}{2}\right]$

= $\frac{(b-a)(b+a)}{2}$

= $\frac{b^2-a^2}{2}$

2. $\int_0^5 (x+1)\ dx$

Ans. We know that

$$\int_a^b f(x)dx = \lim_{h\to 0} h[f(a)+f(a+h)+f(a+2h)+\ldots+f(a+(n-1)h)]$$

where $nh = b-a$

Here, $a=0, b=5, nh=5$ and $f(x)=x+1$

$$= \int_0^5 (x+1)dx = \lim_{h\to 0} h[1+(h+1)+(2h+1)+\ldots+((n-1)h+1)]$$

$$= \lim_{h\to 0} h[n+h(1+2+3+\ldots+(n-1))]$$

$$= \lim_{h\to 0}[nh+h^2\frac{n(n-1)}{2}]$$

$$= \lim_{h\to 0}[nh+\frac{nh(nh-h)}{2}]$$

$= \lim_{h\to 0}\left[5+\frac{5(5-h)}{2}\right]$

$= \left[5+\frac{5(5-0)}{2}\right]$

$= 5+\frac{25}{2} = \frac{35}{2}$

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3. $\int_2^3 x^2\, dx$

Ans. We know that

$$\int_a^b f(x)dx = \lim_{h\to 0} h[f(a)+f(a+h)+f(a+2h)+\ldots+f(a+(n-1)h)]$$

where $nh = b-a$

Here, $a=2, b=3, nh=1$ and $f(x)=x^2$

$$= \int_2^3 x^2dx = \lim_{h\to 0} h[4+(4+4h+h^2)+(4+8h+2^2h^2)+\ldots+(4+4(n-1)h+(n-1)$$

$$= \lim_{h\to 0} h[(4+4+\ldots n\ times)+4h(1+2+\ldots+(n-1))+h^2(1^2+2^2+\ldots.+(n-1)^2)]$$

$$= \lim_{h\to 0} h[4n+4h\frac{n(n-1)}{2}+h^2\frac{n(n-1)(2n-1)}{6}]$$

$$= \lim_{h\to 0}[4nh+2nh(nh-h)+\frac{nh(nh-h)(2nh-h)}{6}]$$

$= \lim_{h\to 0}\left[4+2(1-h)+\frac{(1-h)(2-h)}{6}\right]$

$= \left[4+2(1-0)+\frac{(1-0)(2-0)}{6}\right]$

= $6+\frac{1}{3}$ = $\frac{19}{3}$

4. $\int_1^4 \left(x^2-x\right)\,dx$

Ans. We know that

$$\int_a^b f(x)dx = \lim_{h\to 0} h[f(a)+f(a+h)+f(a+2h)+\ldots+f(a+(n-1)h)]$$

where $nh=b-a$

Here, $a=1, b=4, nh=3$ and $f(x)=x^2-x$

$$=\int_1^4 (x^2-x)dx = \lim_{h\to 0} h[0+h+h^2+2h+2^2h^2+\ldots..+(n-1)h+(n-1)^2.h^2)]$$

$$=\lim_{h\to 0} h[h(1+2+3+..+(n-1))+h^2(1++2^2+3^2\ldots..+(n-1)^2)]$$

$$=\lim_{h\to 0} h[h\tfrac{n(n-1)}{2}+h^2\tfrac{n(n-1)(2n-1)}{6}]$$

$$=\lim_{h\to 0}[\tfrac{nh(nh-h)}{2}+\tfrac{nh(nh-h)(2nh-h)}{6}]$$

= $\lim_{h\to 0}\left[3\frac{(3-h)}{2}+\frac{3(3-h)(2.3-h)}{6}\right]$

= $\left[3\frac{(3-0)}{2}+\frac{3(3-0)(6-0)}{6}\right]$

= $\left[\frac{9}{2}+9\right]$ = $\frac{27}{2}$

5. $\int_{-1}^{1} e^x\,dx$

Ans. We know that

$$\int_a^b f(x)dx = \lim_{h\to 0} h[f(a) + f(a+h) + f(a+2h)+\ldots+f(a+(n-1)h)]$$

where $nh = b - a$

Here, $a = -1$, $b = 1$, $nh = 2$ and $f(x) = e^x$

$$= \int_{-1}^{1} e^x dx = \lim_{h\to 0} h[e^{-1} + e^{-1}e^h + e^{-1}e^{2h}+\ldots\ldots\ldots+e^{-1}e^{(n-1)h}]$$

[$\because$ The series within brackets is a G.P. and $S_n = a\frac{r^n - 1}{r - 1}$]

So, We get

$$= \lim_{h\to 0} he^{-1}\frac{[(e^h)^n - 1]}{e^h - 1}$$

$$= \lim_{h\to 0} he^{-1}\frac{(e^{nh} - 1)}{e^h - 1}$$

$$= \lim_{h\to 0} he^{-1}\frac{(e^2 - 1)}{e^h - 1}$$

$$= e^{-1}(e^2 - 1)\lim_{h\to 0}\frac{h}{e^h - 1}$$

$$= e^{-1}(e^2 - 1)\times 1\left[\because \lim_{x\to 0}\frac{x}{e^x - 1} = 1\right]$$

$$= e^{-1+2} - e^{-1} = e - e^{-1}$$

$$= e - \frac{1}{e}$$

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6. $\int_0^4 (x + e^{2x})\, dx$

Ans. We know that

$$\int_a^b f(x)dx = \lim_{h\to 0} h[f(a) + f(a+h) + f(a+2h) + \ldots + f(a+(n-1)h)]$$

where $nh = b - a$

Here, $a = 0, b = 4, nh = 4$ and $f(x) = x + e^{2x}$

$$= \int_0^4 (x + e^{2x})dx = \lim_{h\to 0} h[1 + (h + e^{2h}) + (2h + e^{4h}) + \ldots\ldots\ldots + ((n-1)h + e^{2(n-1)h}]$$

$$= \lim_{h\to 0} h[(h + 2h + 3h + \ldots\ldots + (n-1)h) + (1 + e^{2h} + e^{4h} + \ldots\ldots + e^{2(n-1)h}]$$

$$= \lim_{h\to 0} h[h(1 + 2 + 3 + \ldots\ldots + (n-1)) + (1 + e^{2h} + e^{4h} + \ldots\ldots + e^{2(n-1)h}]$$

[$\because$ The series within brackets is a G.P. and $S_n = a\dfrac{r^n - 1}{r - 1}$]

$$= \lim_{h\to 0} h[h\frac{n(n-1)}{2} + 1\frac{((e^{2h})^n - 1)}{e^{2h} - 1}]$$

$$= \lim_{h\to 0}[\frac{nh(nh-1)}{2} + h\frac{e^{2nh} - 1}{e^{2h} - 1}]$$

$$= \lim_{h\to 0}\left[\frac{4(4-h)}{2} + \frac{h\left(\left(e^{2\cdot 4}\right) - 1\right)}{e^{2h} - 1}\right]$$

$$= \frac{4(4-0)}{2} + \left(e^8 - 1\right)\lim_{h\to 0}\frac{h}{e^{2h} - 1}$$

$$= 8 + \left(e^8 - 1\right)\frac{1}{2}\lim_{h\to 0}\frac{2h}{e^{2h} - 1}$$

$$= 8 + \frac{\left(e^8 - 1\right)}{2} \quad \left[\because \lim_{x\to 0}\frac{x}{e^x - 1} = 1\right]$$