CBSE Class–12 Mathematics

NCERT solution

Chapter - 7

Integrals - Exercise 7.11

By using the properties of definite integrals, evaluate the integrals in Exercises 1 to 6.

1. $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$

Ans. Let I = $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$ ....(i)

= $\int_0^{\frac{\pi}{2}} \cos^2\left(\frac{\pi}{2} - x\right) \, dx$

$\left[\because \int_0^a f(x) \, dx = \int_0^a f(a-x) \, dx = \right]$

$\Rightarrow \int_0^{\frac{\pi}{2}} \sin^2 x \, dx$ ....(ii)

Adding eq. (i) and (ii),

2I = $\int_0^{\frac{\pi}{2}} \left(\cos^2 x + \sin^2 x\right) \, dx$

= $\int_0^{\frac{\pi}{2}} 1 \, dx$

$=> 2I = (x)_0^{\frac{\pi}{2}}$

$\Rightarrow 2I = \frac{\pi}{2}$

$\Rightarrow I = \frac{\pi}{4}$ Ans.

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2. $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$

Ans. Let I = $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ ....(i)

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}\,dx$

$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx =\right]$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$ ....(ii)

Adding eq. (i) and (ii),

$2I = \int_0^{\frac{\pi}{2}} \left(\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\right) dx$

= $\int_0^{\frac{\pi}{2}} \left(\frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} \right) dx$

$\Rightarrow$ 2I = $\int_0^{\frac{\pi}{2}} 1 \, dx$

$=> 2I = (x)_0^{\frac{\pi}{2}}$

$\Rightarrow$ 2I = $\frac{\pi}{2}$

$\Rightarrow$ I = $\frac{\pi}{4}$ Ans.

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3. $\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x \, dx}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}$

Ans. Let I = $\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} \, dx$ ....(i)

$\Rightarrow$ I = $\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} \left(\frac{\pi}{2} - x \right)}{\sin^{\frac{3}{2}} \left(\frac{\pi}{2} - x \right) + \cos^{\frac{3}{2}} \left(\frac{\pi}{2} - x \right)} \, dx$

$\left[\because \int_0^a f(x) \, dx = \int_0^a f(a-x) \, dx = \right]$

$\Rightarrow$ I = $\int_0^{\frac{\pi}{2}} \frac{\cos^{\frac{3}{2}} x}{\cos^{\frac{3}{2}} x + \sin^{\frac{3}{2}} x} \, dx$ ....(ii)

Adding eq. (i) and (ii),

$$2I = \int_0^{\frac{\pi}{2}} \left(\frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} + \frac{\cos^{\frac{3}{2}} x}{\cos^{\frac{3}{2}} x + \sin^{\frac{3}{2}} x} \right) dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} \right) dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \, dx$$

$$=> 2I = (x)_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4} \text{ Ans.}$$

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4. $\int_0^{\frac{\pi}{2}} \frac{\cos^5 x \, dx}{\sin^5 x + \cos^5 x}$

Ans. Let $I = \int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x + \cos^5 x} dx$ ....(i)

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^5\left(\frac{\pi}{2} - x\right)}{\sin^5\left(\frac{\pi}{2} - x\right) + \cos^5\left(\frac{\pi}{2} - x\right)} dx$$

$$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx = \right]$$

$\Rightarrow$ I = $\int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\cos^5 x + \sin^5 x}\,dx$ ....(ii)

Adding eq. (i) and (ii),

2I = $\int_0^{\frac{\pi}{2}} \left(\frac{\cos^5 x}{\sin^5 x + \cos^5 x} + \frac{\sin^5 x}{\cos^5 x + \sin^5 x} \right) dx$

= $\int_0^{\frac{\pi}{2}} \left(\frac{\cos^5 x + \sin^5 x}{\sin^5 x + \cos^5 x} \right) dx$

$\Rightarrow$ 2I = $\int_0^{\frac{\pi}{2}} 1\,dx$

$=> 2I = (x)_0^{\frac{\pi}{2}}$

$\Rightarrow$ 2I = $\frac{\pi}{2}$

$\Rightarrow$ I = $\frac{\pi}{4}$ Ans.

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5. $\int_{-5}^{5} |x+2|\,dx$

Ans. Let I = $\int_{-5}^{5} |x+2|\,dx$ ....(i)

Putting $x+2=0$

$\Rightarrow x = -2 \in (-5, 5)$

$\therefore$ From eq. (i),

I = $\int_{-5}^{-2} |x+2| \, dx + \int_{-2}^{5} |x+2| \, dx$

= $\int_{-5}^{-2} -(x+2) \, dx + \int_{-2}^{5} (x+2) \, dx$

= $-\left(\frac{x^2}{2} + 2x\right)_{-5}^{-2} + \left(\frac{x^2}{2} + 2x\right)_{-2}^{5}$

$= -\left[\left(\frac{4}{2} - 4\right) - \left(\frac{25}{2} - 10\right)\right] + \left[\left(\frac{25}{2} + 10\right) - \left(\frac{4}{2} - 4\right)\right]$

= $-\left(-2 - \frac{5}{2}\right) + \left(\frac{45}{2} + 2\right)$

= $2 + \frac{5}{2} + \frac{45}{2} + 2$

= $4 + 25 = 29$ Ans.

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6. $\int_{2}^{8} |x-5| \, dx$

Ans. Let I = $\int_{2}^{8} |x-5| \, dx$ ....(i)

Putting $x - 5 = 0$

$\Rightarrow x = 5 \in (2, 8)$

$\therefore$ From eq. (i),

I = $\int_2^5 |x-5|\ dx + \int_5^8 |x-5|\ dx$

= $\int_2^5 -(x-5)\ dx + \int_5^8 (x-5)\ dx$

$= -\left(\frac{x^2}{2} - 5x\right)_2^5 + \left(\frac{x^2}{2} - 5x\right)_5^8$

$= -\left[\left(\frac{25}{2} - 25\right) - (2-10)\right] + \left[(32-40) - \left(\frac{25}{2} - 25\right)\right]$

$= -(-\frac{25}{2} + 8) + (-8 + \frac{25}{2})$

$= \frac{25}{2} - 8 - 8 + \frac{25}{2}$

= 25 - 16

= 9 Ans.

By using the properties of definite integrals, evaluate the integrals in Exercises 7 to 11.

7. $\int_0^1 x(1-x)^n\ dx$

Ans. Let I = $\int_0^1 x(1-x)^n\ dx$

= $\int_0^1 (1-x)\{1-(1-x)\}^n\ dx$

$\left[\because \int_0^a f(x)\ dx = \int_0^a f(a-x)\ dx = \right]$

$\Rightarrow$ I = $\int_0^1 (1-x)(1-1+x)^n\ dx$

= $\int_0^1 (1-x)x^n \, dx$

= $\int_0^1 \left(x^n - x^{n+1}\right) \, dx$

$\Rightarrow$ I = $\left(\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right)_0^1$

= $\frac{1}{n+1} - \frac{1}{n+2} - (0-0)$

= $\frac{n+2-n-1}{(n+1)(n+2)} = \frac{1}{(n+1)(n+2)}$ Ans.

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8. $\int_0^{\frac{\pi}{4}} \log(1+\tan x) \, dx$

Ans. Let I = $\int_0^{\frac{\pi}{4}} \log(1+\tan x) \, dx$ ....(i)

$\Rightarrow$ I = $\int_0^{\frac{\pi}{4}} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right] \, dx$

$\left[\because \int_0^a f(x) \, dx = \int_0^a f(a-x) \, dx = \right]$

$\Rightarrow$ I = $\int_0^{\frac{\pi}{4}} \log\left[1+\frac{1-\tan x}{1+\tan x}\right] \, dx$

= $\int_0^{\frac{\pi}{4}} \log\left[\frac{1+\tan x+1-\tan x}{1+\tan x}\right] dx$

= $\int_0^{\frac{\pi}{4}} \log\left[\frac{2}{1+\tan x}\right] dx$ .....(ii)

Adding eq. (i) and (ii),

2I = $\int_0^{\frac{\pi}{4}}\left[\log(1+\tan x)+\log\left(\frac{2}{1+\tan x}\right)\right] dx$

= $\int_0^{\frac{\pi}{4}}\left[\log(1+\tan x)\left(\frac{2}{1+\tan x}\right)\right] dx$

$\Rightarrow$ 2I = $\int_0^{\frac{\pi}{4}}[\log 2]\ dx$

= $(\log 2)(x)_0^{\frac{\pi}{4}}$

= $\frac{\pi}{4}\log 2$

$\Rightarrow$ I = $\frac{\pi}{8}\log 2$ Ans

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9. $\int_0^2 x\sqrt{2-x}\ dx$

Ans. Let I = $\int_0^2 x\sqrt{2-x}\ dx$

$= \int_0^2 (2-x)\sqrt{2-(2-x)}\ dx$

$\left[\because \int_0^a f(x)\ dx = \int_0^a f(a-x)\ dx = \right]$

$\Rightarrow$ I = $\int_0^2 (2-x)\sqrt{x}\ dx$

$= \int_0^2 \left(2x^{\frac{1}{2}} - x^{\frac{3}{2}}\right) dx$

$= \left[2.\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^2$

$= \left(\frac{4}{3}.2^{\frac{3}{2}} - \frac{2}{5}.2^{\frac{5}{2}}\right) - (0-0)$

$\Rightarrow$ I = $\frac{4}{3} \times 2\sqrt{2} - \frac{2}{5} \times 4\sqrt{2}$

$= \left(\frac{8}{3} - \frac{8}{5}\right)\sqrt{2}$

$= \frac{16\sqrt{2}}{15}$ Ans.

10. $\int_0^{\frac{\pi}{2}} (2\log \sin x - \log \sin 2x)\ dx$

Ans. Let I = $\int_0^{\frac{\pi}{2}} (2\log \sin x - \log \sin 2x)\ dx$

= $\int_0^{\frac{\pi}{2}} (\log \sin^2 x - \log \sin 2x)\ dx$

= $\int_0^{\frac{\pi}{2}} \log\left(\frac{\sin^2 x}{\sin 2x}\right)\ dx$

= $\int_0^{\frac{\pi}{2}} \log\left(\frac{\sin^2 x}{2\sin x \cos x}\right)\ dx$

= $\int_0^{\frac{\pi}{2}} \log\left(\frac{1}{2}\tan x\right)\ dx$ ....(i)

$\Rightarrow$ I = $\int_0^{\frac{\pi}{2}} \log\left(\frac{1}{2}\tan\left(\frac{\pi}{2} - x\right)\right)\ dx$

$\left[\because \int_0^a f(x)\ dx = \int_0^a f(a-x)\ dx = \right]$

$\Rightarrow$ I = $\int_0^{\frac{\pi}{2}} \log\left(\frac{1}{2}\cot x\right)\ dx$ ....(ii)

Adding eq. (i) and (ii),

2I = $\int_0^{\frac{\pi}{2}} \left[\log\left(\frac{1}{2}\tan x\right) + \log\left(\frac{1}{2}\cot x\right)\right]\ dx$

= $\int_0^{\frac{\pi}{2}} \left[\log\left(\frac{1}{2}\tan x\right)\left(\frac{1}{2}\cot x\right)\right] dx$

$\Rightarrow$ 2I = $\int_0^{\frac{\pi}{2}} \left[\log\frac{1}{4}\right] dx$

= $\log\frac{1}{4}(x)_0^{\frac{\pi}{2}}$

= $(\log 1 - \log 4)\frac{\pi}{2}$

= $-\frac{\pi}{2}\log 4$

$\Rightarrow$ I = $-\frac{\pi}{4}\log 2^2$

= $-\frac{2\pi}{4}\log 2$

= $-\frac{\pi}{2}\log 2$

= $\frac{\pi}{2} log 2^{-1}$

= $\frac{\pi}{2} log\frac{1}{2}$ Ans.

11. $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^2 x\, dx$

Ans. Let I = $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^2 x\, dx$

= $2\int_0^{\frac{\pi}{2}} \sin^2 x \, dx$ ...(i)

$\left[\because \int_{-a}^{a} f(x)\, dx = 2\int_0^{a} f(x)\, dx, \text{when } f(x) \text{is even function}\right]$

$\Rightarrow$ I = $2\int_0^{\frac{\pi}{2}} \sin^2\left(\frac{\pi}{2}-x\right) dx$

$\left[\because \int_0^{a} f(x)\, dx = \int_0^{a} f(a-x)\, dx =\right]$

$\Rightarrow$ I = $2\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$ ....(ii)

Adding eq. (i) and (ii),

2I = $2\int_0^{\frac{\pi}{2}} \left(\sin^2 x + \cos^2 x\right) dx$

= $2\int_0^{\frac{\pi}{2}} 1 \, dx$

= $2(x)_0^{\frac{\pi}{2}}$

= $2.\frac{\pi}{2} = \pi$

$\Rightarrow$ I = $\frac{\pi}{2}$ Ans.

Using properties of definite integrals, evaluate the following integrals in Exercise 12 to 18.

12. $\int_0^{\pi} \frac{x\,dx}{1+\sin x}$

Ans. Let I = $\int_0^{\pi} \frac{x}{1+\sin x}\,dx$ ....(i)

$\Rightarrow$ I = $\int_0^{\pi} \frac{\pi - x}{1+\sin(\pi - x)}\,dx$

= $\int_0^{\pi} \frac{\pi - x}{1+\sin x}\,dx$ ....(ii)

Adding eq. (i) and (ii),

2I = $\int_0^{\pi} \left(\frac{x}{1+\sin x} + \frac{\pi - x}{1+\sin x}\right) dx$

= $\int_0^{\pi} \left(\frac{x+\pi - x}{1+\sin x}\right) dx$

= $\int_0^{\pi} \left(\frac{\pi}{1+\sin x}\right) dx$

= $\pi \int_0^{\pi} \left(\frac{1}{1+\sin x}\right) dx$

$\Rightarrow$ 2I = $2\pi \int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x}$

$\left[\because \int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx, \text{if } f(2a-x) = f(x)\right]$

$$\Rightarrow 2I = 2\pi\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin\left(\frac{\pi}{2}-x\right)}$$

$$\left[\because \int_0^a f(x)\ dx = \int_0^a f(a-x)\ dx = \right]$$

$$\Rightarrow 2I = 2\pi\int_0^{\frac{\pi}{2}} \frac{dx}{1+\cos x}$$

$$\Rightarrow I = \pi\int_0^{\frac{\pi}{2}} \frac{dx}{2\cos^2\frac{x}{2}}$$

$$= \frac{\pi}{2}\int_0^{\frac{\pi}{2}} \sec^2\frac{x}{2}\ dx$$

$$= \frac{\pi}{2}\left[\frac{\tan\frac{x}{2}}{\frac{1}{2}}\right]_0^{\frac{\pi}{2}}$$

$$= \pi\left(\tan\frac{\pi}{4} - \tan 0°\right)$$

$$= \pi(1-0) = \pi \text{ Ans.}$$

13. $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\ dx$

Ans. Let I = $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\, dx$

Here $f(x) = \sin^7 x$

$\therefore\ f(-x) = \sin^7(-x)$

$= (-\sin x)^7$

$= -\sin^7 x = -f(x)$

$\therefore\ f(x)$ is an odd function of x.

$\therefore$ I = $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\, dx$ = 0

$\left[\because \int_{-a}^{a} f(x)\, dx = 0, \text{ when } f(x) \text{ is an odd function}\right]$

14. $\int_0^{2\pi} \cos^5 x\, dx$

Ans. $\int_0^{2\pi} \cos^5 x\, dx$

$= 2\int_0^{\pi} \cos^5 x\, dx$

$\left[\because \int_0^{2a} f(x)\, dx = 2\int_0^{a} f(x)\, dx, \text{ if } f(2a-x) = f(x)\right]$

Here $f(x)=\cos^5 x$

$\therefore\ f(2\pi-x)=\cos^5(2\pi-x)=\cos^5 x$

$\Rightarrow f(x)=2(0)=0$

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15. $\int_0^{\frac{\pi}{2}} \frac{\sin x-\cos x}{1+\sin x\cos x}\ dx$

Ans. Let I = $\int_0^{\frac{\pi}{2}} \frac{\sin x-\cos x}{1+\sin x\cos x}\ dx$ ....(i)

$\Rightarrow$ I = $\int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2}-x\right)-\cos\left(\frac{\pi}{2}-x\right)}{1+\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}\ dx$

= $\int_0^{\frac{\pi}{2}} \frac{\cos x-\sin x}{1+\cos x\sin x}\ dx$

= $-\int_0^{\frac{\pi}{2}} \frac{\sin x-\cos x}{1+\sin x\cos x}\ dx$ ....(ii)

Adding eq. (i) and (ii), we have 2I = 0 $\Rightarrow$ I = 0 Ans.

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16. $\int_0^{\pi} \log(1+\cos x)\ dx$R

Ans. Let I = $\int_0^{\pi} \log(1+\cos x)\ dx$ ....(i)

$\Rightarrow$ I = $\int_0^{\pi} \log\left(1+\cos\left(\pi-x\right)\right)\, dx$

= $\int_0^{\pi} \log\left(1-\cos x\right)\, dx$ ....(ii)

Adding eq. (i) and (ii),

2I = $\int_0^{\pi} \left[\log\left(1+\cos x\right)+\log\left(1-\cos x\right)\right] dx$

= $\int_0^{\pi} \left[\log\left(1+\cos x\right)\left(1-\cos x\right)\right] dx$

$\Rightarrow$ 2I = $\int_0^{\pi} \left[\log\left(1-\cos^2 x\right)\right] dx$

= $\int_0^{\pi} \left[\log \sin^2 x\right] dx$

= $2\int_0^{\pi} \left[\log \sin x\right] dx$

$\Rightarrow$ I = $2\int_0^{\frac{\pi}{2}} \log \sin x\, dx$ ....(iii)

$\Rightarrow$ I = $2\int_0^{\frac{\pi}{2}} \log \sin\left(\frac{\pi}{2}-x\right) dx$

= $2\int_0^{\frac{\pi}{2}} \log \cos x\, dx$ ....(iv)

Adding eq. (i) and (ii),

2I = $2\int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x)\ dx$

= $2\int_0^{\frac{\pi}{2}} (\log \sin x \cos x)\ dx$

$\Rightarrow$ I = $\int_0^{\frac{\pi}{2}} \log\left(\frac{2\sin x \cos x}{2}\right)\ dx$

= $\int_0^{\frac{\pi}{2}} \log\left(\frac{\sin 2x}{2}\right)\ dx$

= $\int_0^{\frac{\pi}{2}} (\log\ \sin 2x - \log\ 2)\ dx$

$\Rightarrow$ I = $\int_0^{\frac{\pi}{2}} \log \sin 2x\ dx - \int_0^{\frac{\pi}{2}} \log\ 2\ dx$

= $\int_0^{\frac{\pi}{2}} \log \sin 2x\ dx - \log\ 2 (x)_0^{\frac{\pi}{2}}$

= $\int_0^{\frac{\pi}{2}} \log \sin 2x\ dx - \frac{\pi}{2} \log 2$

$\Rightarrow$ I = $I_1 - \frac{\pi}{2}\log 2$ ....(v)

Where I_1 = $\int_0^{\frac{\pi}{2}} \log \sin 2x\ dx$ ....(vi)

Putting $2x = t$ in eq. (vi),

$\Rightarrow 2 = \frac{dt}{dx}$

$\Rightarrow dx = \frac{dt}{2}$

Limits of integration when $x = 0, t = 0$ and $x = \frac{\pi}{2}, t = \pi$

$\therefore$ From eq. (vi),

$I_1 = \int_0^{\pi} \log \sin t \ \frac{dt}{2}$

$= \frac{1}{2}\int_0^{\pi} \log \sin t \ dt$

$= \frac{1}{2} \times 2\int_0^{\frac{\pi}{2}} \log \sin t \ dt$

$\Rightarrow I_1 = \int_0^{\frac{\pi}{2}} \log \sin t \ dt$

$= \int_0^{\frac{\pi}{2}} \log \sin x \ dx$

$\left[\because \int_a^b f(t)\ dt = \int_a^b f(x)\ dx\right]$

$\Rightarrow I_1 = \frac{1}{2}$ [From eq. (iii)]

Putting this value in eq. (v), I = $\frac{1}{2} - \frac{\pi}{2}\log 2$

$\Rightarrow$ 2I = $\mathrm{I} - \pi \log 2$

$\Rightarrow$ 2I – I = $-\pi \log 2$

$\Rightarrow$ I = $-\pi \log 2$

17. $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\ dx$

Ans. Let I = $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\ dx$ ....(i)

$\Rightarrow$ I = $\int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{a-(a-x)}}\ dx$

$\left[\because \int_0^a f(x)\ dx = \int_0^a f(a-x)\ dx = \right]$

= $\int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}\ dx$ ....(ii)

Adding eq. (i) and (ii),

2I = $\int_0^a \left(\frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} + \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}\right) dx$

= $\int_0^a \left(\frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}\right) dx$

= $\int_0^a 1\ dx$ = $(x)_0^a$ = a

$\Rightarrow$ I = $\frac{a}{2}$ Ans.

18. $\int_0^4 |x-1|\ dx$

Ans. Let I = $\int_0^4 |x-1|\ dx$ ....(i)

Here $x-1=0$

$\Rightarrow x=1\in(0,4)$

$\therefore$ From eq. (i),

I = $\int_0^1 |x-1|\ dx+\int_1^4 |x-1|\ dx$

= $-\int_0^1 (x-1)\ dx+\int_1^4 (x-1)\ dx$

$\Rightarrow$ I = $-\left(\frac{x^2}{2}-x\right)_0^1+\left(\frac{x^2}{2}-x\right)_1^4$

= $-\left\{\left(\frac{1}{2}-1\right)-0\right\}+\left\{\frac{16}{2}-4-\left(\frac{1}{2}-1\right)\right\}$

$=\frac{1}{2}+8-4+\frac{1}{2}$

= 9 - 4

= 5 Ans.

19. Show that $\int_0^a f(x)g(x)dx=2\int_0^a f(x)dx$ **if f and g are defined as** $f(x)=f(a-x)$

and $g(x)+g(a-x)=4$

Ans. Here $f(x)=f(a-x)$ ....(i) and $g(x)+g(a-x)=4$ ....(ii)

Let I = $\int_0^a f(x)g(x)\ dx$ ....(iii)

$\therefore$ I = $\int_0^a f(a-x)g(a-x)\ dx$

$\left[\because \int_0^a f(x)\ dx = \int_0^a f(a-x)\ dx = \right]$

= $\int_0^a f(x)g(a-x)\ dx$ [From eq. (i)] ....(iv)

Adding eq. (iii) and (iv)

2I = $\int_0^a \left(f(x)g(x)+f(x)g(a-x)\right)\ dx$

= $\int_0^a f(x)\left(g(x)+g(a-x)\right)\ dx$

$\Rightarrow$ 2I = $\int_0^a f(x)(4)\ dx$ [From eq. (ii)]

$\Rightarrow$ 2I = $4\int_0^a f(x)\ dx$

$\Rightarrow$ I = $2\int_0^a f(x)\ dx$

Hence proved.

Choose the correct answer in Exercises 20 and 21.

20. The value of $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\left(x^3+x\cos x+\tan^5 x+1\right)\,dx$ **is:**

(A) 0

(B) 2

(C) π

(D) 1

Ans. Let I = $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\left(x^3+x\cos x+\tan^5 x+1\right)\,dx$

= $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} x^3\,dx+\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} x\cos x\,dx+\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \tan^5 x\,dx+\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} 1\,dx$

$\Rightarrow$ I = $0+0+0+\left(x\right)_{\frac{-\pi}{2}}^{\frac{\pi}{2}}$

Because x^3 , xcosx and $\tan^5x$ all are odd functions.

= $\frac{\pi}{2}-\left(\frac{-\pi}{2}\right)=\pi$

Therefore, option (C) is correct.

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21. The value of $\int_0^{\frac{\pi}{2}}\log\left(\frac{4+3\sin x}{4+3\cos x}\right)\,dx$ **is:**

(A) 2

(B) $\frac{3}{4}$

(C) 0

(D) -2

Ans. Let I = $\int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\sin x}{4+3\cos x}\right) dx$ ....(i)

$\Rightarrow$ I = $\int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\sin\left(\frac{\pi}{2}-x\right)}{4+3\cos\left(\frac{\pi}{2}-x\right)}\right) dx$

= $\int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\cos x}{4+3\sin x}\right) dx$ ....(ii)

Adding eq. (i) and (ii),

2I = $\int_0^{\frac{\pi}{2}} \left(\log\left(\frac{4+3\sin x}{4+3\cos x}\right) + \log\left(\frac{4+3\cos x}{4+3\sin x}\right)\right) dx$

= $\int_0^{\frac{\pi}{2}} \left(\log\left(\frac{4+3\sin x}{4+3\cos x}\right).\left(\frac{4+3\cos x}{4+3\sin x}\right)\right) dx$

$\Rightarrow$ 2I = $\int_0^{\frac{\pi}{2}} (\log 1)\ dx$

= $\int_0^{\frac{\pi}{2}} 0\ dx$ = 0

$\Rightarrow$ I = 0

Option (C) is correct.