

CBSE Class-12 Mathematics

NCERT solution

Chapter - 11

Three Dimensional Geometry - Miscellaneous Exercise

1. Show that the line joining the origin to the point (2, 1, 1) is perpendicular to the line determined by the points (3, 5, -1), (4, 3, -1).

Ans. We know that direction ratios of the line joining the origin (0, 0, 0) to the point are

$$x_2 - x_1, y_2 - y_1, z_2 - z_1 = 2 - 0, 1 - 0, 1 - 0 = 2, 1, 1 = a_1, b_1, c_1$$

Similarly, direction ratios of the line joining the points (3, 5, -1) and (4, 3, -1) are

$$x_2 - x_1, y_2 - y_1, z_2 - z_1 = 4 - 3, 3 - 5, -1 - (-1) = 1, -2, 0 = a_2, b_2, c_2$$

For these two lines, $a_1a_2 + b_1b_2 + c_1c_2$

$$= 2(1) + 1(-2) + 1(0) = 2 - 2 + 0 = 0$$

Therefore, the two given lines are perpendicular to each other.

2. If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both of these are $m_1n_2 - m_2n_1, n_1l_2 - n_2l_1, l_1m_2 - l_2m_1$.

Ans. l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of two mutually perpendicular of two given lines L_1 and L_2 . (say)

Let \hat{n}_1 and \hat{n}_2 be the unit vectors along these lines L_1 and L_2 .

$$\therefore \vec{n}_1 = l_1\hat{i} + m_1\hat{j} + n_1\hat{k} \text{ and } \vec{n}_2 = l_2\hat{i} + m_2\hat{j} + n_2\hat{k}$$

Let L be the line perpendicular to both the lines L_1 and L_2 and let \hat{n} be a unit vector along

line L perpendicular both lines L_1 and L_2 .

$$\therefore \text{Cross-product of two vectors} = \hat{n}_1 \times \hat{n}_2 = |\hat{n}_1| |\hat{n}_2| \sin 90^\circ \hat{n}$$

[$\because L_1 \perp L_2$ (given, \therefore angle between them is 90°)]

$$\Rightarrow \hat{n}_1 \times \hat{n}_2 = \hat{n}$$

$$\Rightarrow \hat{n} = \hat{n}_1 \times \hat{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}$$

$$\Rightarrow \hat{n} = (m_1 n_2 - m_2 n_1) \hat{i} - (l_1 n_2 - l_2 n_1) \hat{j} + (l_1 m_2 - l_2 m_1) \hat{k}$$

Since, \hat{n} is a unit vector, therefore its components are its direction cosines.

Thus, direction cosines of \hat{n} are $m_1 n_2 - m_2 n_1, l_1 n_2 - l_2 n_1, l_1 m_2 - l_2 m_1$

$$\Rightarrow \text{direction cosines of line L are } m_1 n_2 - m_2 n_1, l_1 n_2 - l_2 n_1, l_1 m_2 - l_2 m_1$$

3. Find the angle between the lines whose direction ratios are a, b, c and $b - c, c - a, a - b$.

Ans. Direction ratios of one line are a, b, c

$$\Rightarrow \text{A vector along this line is } \vec{b}_1 = a\hat{i} + b\hat{j} + c\hat{k}$$

Direction ratios of second line are $b - c, c - a, a - b$

$$\Rightarrow \text{A vector along second line is } \vec{b}_2 = (b - c)\hat{i} + (c - a)\hat{j} + (a - b)\hat{k}$$

Let θ be the angle between the two lines, then

$$\begin{aligned}\cos \theta &= \frac{|\vec{b_1} \cdot \vec{b_2}|}{|\vec{b_1}| |\vec{b_2}|} = \frac{a(b-c) + b(c-a) + c(a-b)}{\sqrt{a^2 + b^2 + c^2} \sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}} \\ &= \frac{ab - ac + bc - ab + ac - bc}{\sqrt{a^2 + b^2 + c^2} \sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}} = 0 = \cos 90^\circ \\ \Rightarrow \theta &= 90^\circ\end{aligned}$$

4. Find the equation of the line parallel to x -axis and passing through the origin.

Ans. We know that a unit vector along x -axis is $\hat{i} = \hat{i} + 0\hat{j} + 0\hat{k}$

\therefore Direction cosines of x -axis are coefficients of $\hat{i}, \hat{j}, \hat{k}$ in the unit vector

i.e., $1, 0, 0 = l, m, n$

\therefore Equation of the required line passing through the origin $(0, 0, 0)$ and parallel to x -axis is

$$\frac{x-0}{1} = \frac{y-0}{0} = \frac{z-0}{0} \Rightarrow \frac{x}{1} = \frac{y}{0} = \frac{z}{0}$$

Vector equation of the required line is $\vec{r} = \vec{a} + \lambda \vec{b}$

$$\Rightarrow \vec{r} = \vec{0} + \lambda \hat{i} \quad [\vec{a} = \vec{0} \text{ and } \vec{b} = \hat{i}]$$

$$\Rightarrow \vec{r} = \lambda \hat{i}$$

5. If the coordinates of the points A, B, C, D be $(1, 2, 3)$, $(4, 5, 7)$, $(-4, 3, -6)$ and $(2, 9, 2)$ respectively, then find the angle between the lines AB and CD.

Ans. Given: Points A $(1, 2, 3)$, B $(4, 5, 7)$, C $(-4, 3, -6)$ and D $(2, 9, 2)$.

\therefore Direction ratios of line AB are $x_2 - x_1, y_2 - y_1, z_2 - z_1$

$$\Rightarrow 4 - 1, 5 - 2, 7 - 3 = 3, 3, 4 = a_1, b_1, c_1$$

∴ A vector along the line AB is $\vec{b}_1 = 3\hat{i} + 3\hat{j} + 4\hat{k}$

Similarly, direction ratios of line CD are $x_2 - x_1, y_2 - y_1, z_2 - z_1$

$$\Rightarrow 2 - (-4), 9 - 3, 2 - (-6) = 6, 6, 8 = a_2, b_2, c_2$$

∴ A vector along the line AB is $\vec{b}_2 = 6\hat{i} + 6\hat{j} + 8\hat{k}$

Let θ be the angle between the two lines, then

$$\cos \theta = \frac{|\vec{b}_1 \cdot \vec{b}_2|}{|\vec{b}_1| |\vec{b}_2|} = \frac{|3(6) + 3(6) + 4(8)|}{\sqrt{9+9+16} \sqrt{36+36+64}} = \frac{|18+18+32|}{\sqrt{34} \sqrt{136}} = \frac{68}{\sqrt{34 \times 34 \times 4}} = \frac{68}{34 \times 2} = 1$$

$$= \cos 0^\circ$$

$$\Rightarrow \theta = 0^\circ$$

Therefore, lines AB and CD are parallel.

6.If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular, find the value of k .

Ans. Given: Equation of one line is $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$

Direction ratios of this line are its denominators, i.e., $-3, 2k, 2 = a_1, b_1, c_1$

∴ A vector along this line is $\vec{b}_1 = -3\hat{i} + 2k\hat{j} + 2\hat{k}$

Again, equation of second line is $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$

Direction ratios of this line are its denominators, i.e., $3k, 1, -5 = a_2, b_2, c_2$

∴ A vector along this line is $\vec{b}_2 = 3\hat{i} + \hat{j} - 5\hat{k}$

Since these given lines are perpendicular.

$$\therefore \vec{b}_1 \cdot \vec{b}_2 = a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\Rightarrow (-3)(3k) + (2k)(1) + 2(-5) = 0 \Rightarrow -9k + 2k - 10 = 0$$

$$\Rightarrow -7k = 10 \Rightarrow k = \frac{-10}{7}$$

7. Find the vector equation of the line passing through (1, 2, 3) and perpendicular to the plane $\vec{r} \cdot (\hat{i} + 2\hat{j} - 5\hat{k}) + 9 = 0$.

Ans. The required line passes through the point P (1, 2, 3).

∴ Position vector \vec{a} (say) of point P is (1, 2, 3)

$$\Rightarrow \vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$$

Equation of the given plane is $\vec{r} \cdot (\hat{i} + 2\hat{j} - 5\hat{k}) + 9 = 0$

$$\Rightarrow \vec{r} \cdot (\hat{i} + 2\hat{j} - 5\hat{k}) = -9$$

Comparing with $\vec{r} \cdot \vec{n} = \vec{d}$, $\vec{n} = \hat{i} + 2\hat{j} - 5\hat{k}$

Since, the required line is perpendicular to the given plane, therefore, vector \vec{b} along the required line is $\vec{b} = \vec{n} = \hat{i} + 2\hat{j} - 5\hat{k}$

∴ Equation of the required line is $\vec{r} = \vec{a} + \lambda \vec{b}$

$$\therefore \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(\hat{i} + 2\hat{j} - 5\hat{k})$$

8. Find the equation of the plane passing through (a, b, c) and parallel to the plane

$$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 2.$$

Ans. Equation of any plane parallel to the plane $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$ is $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = \lambda$ (i)

Plane (i) passes through (a, b, c)

\therefore Putting $\vec{r} = (a, b, c) = a\hat{i} + b\hat{j} + c\hat{k}$ in eq. (i), we get

$$(a\hat{i} + b\hat{j} + c\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = \lambda$$

$$\Rightarrow a(1) + b(1) + c(1) = \lambda \Rightarrow \lambda = a + b + c$$

Putting the value of λ in eq. (i), to get the required plane is

$$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = a + b + c$$

9. Find the shortest distance between lines $\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$ and

$$\vec{r} = 4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k}).$$

Ans. Given: Vector equation of one line is $\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$

Comparing with $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$, we get

$$\vec{a_1} = 6\hat{i} + 2\hat{j} + 2\hat{k} \text{ and } \vec{b_1} = \hat{i} - 2\hat{j} + 2\hat{k}$$

Again given: Vector equation of another line is $\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$

Comparing with $\vec{r} = \vec{a_2} + \mu\vec{b_2}$, we get

$$\vec{a_2} = -4\hat{i} - \hat{k} \text{ and } \vec{b_2} = 3\hat{i} - 2\hat{j} - 2\hat{k}$$

We know that length of shortest distance between two (skew) lines is $\frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$..

(i)

$$\text{Now } \vec{a}_2 - \vec{a}_1 = -4\hat{i} - \hat{k} - (6\hat{i} + 2\hat{j} + 2\hat{k}) = -4\hat{i} - \hat{k} - 6\hat{i} - 2\hat{j} - 2\hat{k} = -10\hat{i} - 2\hat{j} - 3\hat{k}$$

$$\text{Again } \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}$$

Expanding along first row,

$$= \hat{i}(4 + 4) - \hat{j}(-2 - 6) + \hat{k}(-2 + 6) = 8\hat{i} + 8\hat{j} + 4\hat{k}$$

$$\begin{aligned} \therefore (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) &= (-10)(8) + (-2)(8) + (-3)4 \\ &= -80 - 16 - 12 = -108 \end{aligned}$$

$$\text{And } |\vec{b}_1 \times \vec{b}_2| = \sqrt{(8)^2 + (8)^2 + (4)^2} = \sqrt{64 + 64 + 16} = \sqrt{144} = 12$$

$$\text{Putting these values in eq. (i), length of shortest distance} = \frac{|-108|}{12} = \frac{108}{12} = 9$$

10. Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the YZ-plane.

Ans. Given: A line through the points A (5, 1, 6) and B (3, 4, 1)

\therefore Direction ratios of this line AB are $x_2 - x_1, y_2 - y_1, z_2 - z_1$

$$\Rightarrow 3 - 5, 4 - 1, 1 - 6 \Rightarrow -2, 3, -5 = a, b, c$$

$$\therefore \text{Equation of the line AB is } \frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$$\Rightarrow \frac{x-5}{-2} = \frac{y-1}{3} = \frac{z-6}{-5} \dots\dots\dots(i)$$

Now we have to find the coordinates of the point where this line AB crosses the YZ-plane

i.e., $x = 0 \dots\dots\dots(ii)$

Putting $x = 0$ in eq. (i), we get

$$\frac{-5}{-2} = \frac{y-1}{3} = \frac{z-6}{-5} \Rightarrow \frac{y-1}{3} = \frac{5}{2} \text{ and } \frac{z-6}{-5} = \frac{5}{2}$$

$$\Rightarrow 2y - 2 = 15 \text{ and } 2z - 12 = -25 \Rightarrow 2y = 17 \text{ and } 2z = -13$$

$$\Rightarrow y = \frac{17}{2} \text{ and } z = \frac{-13}{2}$$

Thus, required point is $P\left(0, \frac{17}{2}, \frac{-13}{2}\right)$

11. Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the ZX-plane.

Ans. Given: A line through the points A (5, 1, 6) and B (3, 4, 1)

\therefore Direction ratios of this line AB are $x_2 - x_1, y_2 - y_1, z_2 - z_1$

$$\Rightarrow 3 - 5, 4 - 1, 1 - 6 \Rightarrow -2, 3, -5 = a, b, c$$

\therefore Equation of the line AB is $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

$$\Rightarrow \frac{x-5}{-2} = \frac{y-1}{3} = \frac{z-6}{-5} \dots\dots\dots(i)$$

Now we have to find the coordinates of the point where this line AB crosses the ZX-plane

i.e., $y = 0 \dots\dots\dots(ii)$

Putting $y = 0$ in eq. (i), we get

$$\frac{x-5}{-2} = \frac{-1}{3} = \frac{z-6}{-5} \Rightarrow \frac{x-5}{-2} = \frac{-1}{3} \text{ and } \frac{z-6}{-5} = \frac{-1}{3}$$

$$\Rightarrow 3x-15=2 \text{ and } 3z-18=5 \Rightarrow 3x=17 \text{ and } 3z=23$$

$$\Rightarrow x = \frac{17}{3} \text{ and } z = \frac{23}{3}$$

Thus, required point is $P\left(\frac{17}{3}, 0, \frac{23}{3}\right)$

12. Find the coordinates of the point where the line through $(3, -4, -5)$ and $(2, -3, 1)$ crosses the plane $2x + y + z = 7$.

Ans. Direction ratios of the line joining the points A $(3, -4, -5)$ and B $(2, -3, 1)$ are

$$2-3, -3-(-4), 1-(-5) \Rightarrow -1, 1, 6$$

$$\therefore \text{Equation of the line AB are } \frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} \dots\dots\dots(i)$$

$$\text{Equation of the plane is } 2x + y + z = 7 \dots\dots\dots(ii)$$

Now to find the point where line (i) crosses plane (ii),

$$\text{From eq. (i) } \frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = \lambda \text{ (say)}$$

$$\Rightarrow x-3 = -\lambda, \quad y+4 = \lambda, \quad z+5 = 6\lambda$$

$$\Rightarrow x = 3 - \lambda, \quad y = -4 + \lambda, \quad z = -5 + 6\lambda \dots\dots\dots(iii)$$

Putting the values of x, y, z in eq. (ii), we get

$$2(3 - \lambda) + (-4 + \lambda) + (-5 + 6\lambda) = 7$$

$$\Rightarrow 6 - 2\lambda - 4 + \lambda - 5 + 6\lambda = 7 \Rightarrow 5\lambda = 10 \Rightarrow \lambda = 2$$

Putting $\lambda = 2$ in eq. (iii), point of intersection of line (i) and plane (ii) is

$$x = 3 - 2 = 1, \quad y = -4 + 2 = -2, \quad z = -5 + 12 = 7$$

Thus, required point of intersection is $(1, -2, 7)$.

13. Find the equation of the plane passing through the point $(-1, 3, 2)$ and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

Ans. Since equation of any plane through the point $(-1, 3, 2)$ is

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

$$\therefore a(x+1) + b(y-3) + c(z-2) = 0 \dots\dots\dots(i)$$

$$\Rightarrow ax + a + by - 3b + cz - 2c = 0 \Rightarrow ax + by + cz = -a + 3b + 2c$$

This required plane is perpendicular to the plane $x + 2y + 3z = 5$ ($a_1a_2 + b_1b_2 + c_1c_2 = 0$)

$$\therefore \text{Product of coefficients} \Rightarrow a(1) + b(2) + c(3) = 0 \dots\dots\dots(ii)$$

Again the required plane is perpendicular to the plane $3x + 3y + z = 0$

$$\therefore \text{Product of coefficients} \Rightarrow a(3) + b(3) + c(1) = 0 \dots\dots\dots(iii)$$

Solving eq. (ii) and (iii), we get

$$\frac{a}{2-9} = \frac{b}{9-1} = \frac{c}{3-6}$$

$$\Rightarrow \frac{a}{-7} = \frac{b}{8} = \frac{c}{-3}$$

Putting these values of a, b, c in eq. (i), we get

$$-7(x+1) + 8(y-3) - 3(z-2) = 0$$

$$\Rightarrow -7x - 7 + 8y - 24 - 3z + 6 = 0$$

$$\Rightarrow -7x + 8y - 3z - 25 = 0$$

$$\Rightarrow 7x - 8y + 3z + 25 = 0$$

14. If the points $(1, 1, p)$ and $(-3, 0, 1)$ be equidistant from the plane

$$\vec{r} \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0, \text{ then find the value of } p.$$

Ans. Equation of the given plane is $\vec{r} \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0$$

[$\because \vec{r}$ = Position vector of any point (x, y, z) on the plane $x\hat{i} + y\hat{j} + z\hat{k}$]

$$\Rightarrow 3x + 4y - 12z + 13 = 0 \dots\dots\dots(i)$$

Also, the point $(1, 1, p)$ and $(-3, 0, 1)$ are equidistant from plane (i)

\Rightarrow (Perpendicular) distance of point $(1, 1, p)$ from plane (i)

= Distance of point $(-3, 0, 1)$ from plane (i)

$$\Rightarrow \frac{|3(1) + 4(1) - 12(p) + 13|}{\sqrt{9 + 16 + 144}} = \frac{|3(-3) + 4(0) - 12(1) + 13|}{\sqrt{9 + 16 + 144}}$$

$$\Rightarrow \frac{|3 + 4 - 12p + 13|}{13} = \frac{|-9 - 12 + 13|}{13}$$

$$\Rightarrow |20 - 12p| = |-8|$$

$$\Rightarrow 20 - 12p = \pm 8 \quad [\because \text{If } |x| = a, a \geq 0, \text{ then } x = \pm a]$$

Taking positive sign, $20 - 12p = 8$

$$\Rightarrow -12p = -12$$

$$\Rightarrow p = 1$$

Taking negative sign, $20 - 12p = -8$

$$\Rightarrow -12p = -28$$

$$\Rightarrow p = \frac{-28}{-12} = \frac{7}{3}$$

Hence, the values of p are 1 or $\frac{7}{3}$.

15. Find the equation of the plane passing through the line of intersection of the planes

$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$ and parallel to x -axis.

Ans. Equation of one plane is $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$

$$\Rightarrow \vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1 = 0 \dots\dots\dots(i)$$

Equation of the second plane is $\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0 \dots\dots\dots(ii)$

Since, equation of any plane passing through the line intersection of these two planes is

L.H.S. of I + λ (L.H.S. of II) = 0

$$\Rightarrow \vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1 + \lambda [\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4] = 0$$

$$\Rightarrow \vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1 + \lambda \vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4\lambda = 0$$

$$\Rightarrow \vec{r} \cdot [\hat{i} + \hat{j} + \hat{k} + \lambda (2\hat{i} + 3\hat{j} - \hat{k})] - 1 + 4\lambda = 0$$

$$\Rightarrow \vec{r} \cdot [\hat{i} + \hat{j} + \hat{k} + 2\lambda\hat{i} + 3\lambda\hat{j} - \lambda\hat{k}] = 1 - 4\lambda$$

$$\Rightarrow \vec{r} \cdot [(1+2\lambda)\hat{i} + (1+3\lambda)\hat{j} + (1-\lambda)\hat{k}] = 1-4\lambda \dots\dots\dots(i)$$

Comparing $\vec{r} \cdot \vec{n} = d$, we have

$$\vec{n} = (1+2\lambda)\hat{i} + (1+3\lambda)\hat{j} + (1-\lambda)\hat{k}$$

Now required plane (i) is parallel to x -axis (\Rightarrow a vector \vec{b} along x -axis is $\vec{b} = \hat{i} = \hat{i} + 0\hat{j} + 0\hat{k}$) $\vec{b} \cdot \vec{n} = 0$

$$(1+2\lambda)(1) + (1+3\lambda)(0) + (1-\lambda)(0) = 0$$

$$\Rightarrow 1+2\lambda = 0 \Rightarrow 2\lambda = -1 \Rightarrow \lambda = \frac{-1}{2}$$

Putting $\lambda = \frac{-1}{2}$ in eq. (i), the equation of required plane,

$$\vec{r} \cdot \left[\left(1+2 \cdot \frac{-1}{2}\right)\hat{i} + \left(1+3 \cdot \frac{-1}{2}\right)\hat{j} + \left(1-\frac{-1}{2}\right)\hat{k} \right] = 1-4 \cdot \frac{-1}{2}$$

$$\Rightarrow \vec{r} \cdot \left[(1-1)\hat{i} + \left(1-\frac{3}{2}\right)\hat{j} + \left(1+\frac{1}{2}\right)\hat{k} \right] = 1+2$$

$$\Rightarrow \vec{r} \cdot \left[\frac{-1}{2}\hat{j} + \frac{3}{2}\hat{k} \right] = 3$$

$$\Rightarrow \vec{r} \cdot (-\hat{j} + 3\hat{k}) = 6$$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (0\hat{i} - \hat{j} + 3\hat{k}) = 6$$

$$\Rightarrow -y + 3z = 6$$

$$\Rightarrow -y + 3z - 6 = 0$$

$$\Rightarrow y - 3z + 6 = 0$$

16.If O be the origin and the coordinates of P be $(1, 2, -3)$, then find the equation of the plane passing through P and perpendicular to OP.

Ans. Given: Origin O $(0, 0, 0)$ and point P $(1, 2, -3)$

To find: Equation of the plane passing through P $(1, 2, -3) = (x_1, y_1, z_1)$

∴ Direction ratios of normal OP to the plane are $1-0, 2-0, -3-0$

$$\Rightarrow 1, 2, -3 = (a, b, c)$$

∴ Equation of the required plane is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

$$\Rightarrow 1(x-1) + 2(y-2) - 3(z+3) = 0$$

$$\Rightarrow x-1+2y-4-3z-9=0$$

$$\Rightarrow x+2y-3z-14=0$$

17.Find the equation of the plane which contains the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0$, $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ and which is perpendicular to the plane $\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$.

Ans. Equation of any plane passing through (or containing) the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0$ and $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ is L.H.S. of I + λ (L.H.S. of II) = 0

$$\Rightarrow \vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 + \lambda [\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5] = 0$$

$$\Rightarrow \vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 + \lambda \vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5\lambda = 0$$

$$\Rightarrow \vec{r} \cdot [\hat{i} + 2\hat{j} + 3\hat{k} + \lambda(2\hat{i} + \hat{j} - \hat{k})] - 4 + 5\lambda = 0$$

$$\Rightarrow \vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k} + 2\lambda\hat{i} + \lambda\hat{j} - \lambda\hat{k}) = 4 - 5\lambda$$

$$\Rightarrow \vec{r} \cdot [(1+2\lambda)\hat{i} + (2+\lambda)\hat{j} + (3-\lambda)\hat{k}] = 4 - 5\lambda \dots\dots\dots(i)$$

Comparing with $\vec{r} \cdot \vec{n_1} = d_1$ we have, $\vec{n_1} = (1+2\lambda)\hat{i} + (2+\lambda)\hat{j} + (3-\lambda)\hat{k}$

Now plane (i) is perpendicular to the given plane $\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$

$$\Rightarrow \vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) = -8$$

Comparing with $\vec{r} \cdot \vec{n_2} = d_2$ we have, $\vec{n_2} = 5\hat{i} + 3\hat{j} - 6\hat{k}$

For perpendicular planes $\vec{n_1} \cdot \vec{n_2} = 0$

$$\Rightarrow (1+2\lambda)5 + (2+\lambda)3 + (3-\lambda)(-6) = 0$$

$$\Rightarrow 5 + 10\lambda + 6 + 3\lambda - 18 + 6\lambda = 0$$

$$\Rightarrow 19\lambda - 7 = 0$$

$$\Rightarrow 19\lambda = 7$$

$$\Rightarrow \lambda = \frac{7}{19}$$

Putting $\lambda = \frac{7}{19}$ in eq. (i), equation of required plane is

$$\vec{r} \cdot \left[\left(1 + 2 \cdot \frac{7}{19}\right)\hat{i} + \left(2 + \frac{7}{19}\right)\hat{j} + \left(3 - \frac{7}{19}\right)\hat{k} \right] = 4 - 5 \cdot \frac{7}{19}$$

$$\Rightarrow \vec{r} \cdot \left[\left(1 + \frac{14}{19}\right)\hat{i} + \left(2 + \frac{7}{19}\right)\hat{j} + \left(3 - \frac{7}{19}\right)\hat{k} \right] = 4 - \frac{35}{19}$$

$$\Rightarrow \vec{r} \cdot (33\hat{i} + 45\hat{j} + 50\hat{k}) = 41$$

$$\Rightarrow (\hat{x}\hat{i} + \hat{y}\hat{j} + \hat{z}) \cdot (33\hat{i} + 45\hat{j} + 50\hat{k}) = 41$$

$$\Rightarrow 33x + 45y + 50z = 41$$

18. Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$.

Ans. Given: A point P (say) $(-1, -5, -10)$

and equation of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ (i)

equation of the plane is $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$

Putting the value of \vec{r} from eq. (i) in eq. (ii),

$$\left[(2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k}) \right] \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$$

$$\Rightarrow (2\hat{i} - \hat{j} + 2\hat{k}) \cdot (\hat{i} - \hat{j} + \hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k}) \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$$

$$\Rightarrow 2 + 1 + 2 + \lambda(3 - 4 + 2) = 5$$

$$\Rightarrow 5 + \lambda = 5$$

$$\Rightarrow \lambda = 0$$

Putting $\lambda = 0$ in eq. (i), $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + 0(3\hat{i} + 4\hat{j} + 2\hat{k})$

$$\Rightarrow \vec{r} = 2\hat{i} - \hat{j} + 2\hat{k}$$

Therefore, Point of intersection is $(x, y, z) = (-2, 1, 2)$

\therefore Distance of the given point P $(-1, -5, -10)$ from the point of intersection is

$$\sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2}$$

$$= \sqrt{9+16+144}$$

$$= \sqrt{169} = 13 \text{ units}$$

19. Find the vector equation of the line passing through $(1, 2, 3)$ and parallel to the plane $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$.

Ans. The required line passes through the point A $(1, 2, 3) = \vec{a}$

$$\therefore \vec{a} = \text{Position vector of point A} = \hat{i} + 2\hat{j} + 3\hat{k}$$

Let \vec{b} be any vector along the required line.

$$\therefore \text{Vector equation of required line is } \vec{r} = \vec{a} + \lambda \vec{b}$$

$$\Rightarrow \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda \vec{b} \dots\dots\dots(i)$$

Since required line is parallel to the plane $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$

$$\therefore \vec{b} \cdot \vec{n_1} = 0 \text{ and } \vec{b} \cdot \vec{n_2} = 0$$

Comparing with $\vec{r} \cdot \vec{n_1} = d_1$ we have, $\vec{n_1} = \hat{i} - \hat{j} + 2\hat{k}$

And Comparing with $\vec{r} \cdot \vec{n_2} = d_2$ we have, $\vec{n_2} = 3\hat{i} + \hat{j} + \hat{k}$

Since \vec{b} is perpendicular to both $\vec{n_1}$ and $\vec{n_2}$

$$\therefore \vec{b} = \vec{n_1} \times \vec{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{vmatrix}$$

Expanding along first row,

$$\vec{b} = \hat{i}(-1-2) - \hat{j}(1-6) + \hat{k}(1+3) = -3\hat{i} + 5\hat{j} + 4\hat{k}$$

Putting this value of \vec{b} in eq. (i), vector equation of required line,

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k})$$

20. Find the vector equation of the line passing through the point $(1, 2, -4)$ and

perpendicular to the two lines: $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

Ans. Given: A point on the required line is A $(1, 2, -4)$

\therefore Position vector of point A is $\vec{a} = (1, 2, -4) = \hat{i} + 2\hat{j} - 4\hat{k}$

Also given equations of two lines

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

\therefore Direction ratios of given two lines are $\vec{b}_1 = 3\hat{i} - 16\hat{j} + 7\hat{k}$ and $\vec{b}_2 = 3\hat{i} + 8\hat{j} - 5\hat{k}$

$$\text{Now } \vec{b} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$$

Expanding along first row,

$$= \hat{i}(80 - 56) - \hat{j}(-15 - 21) + \hat{k}(24 + 48) = 24\hat{i} + 36\hat{j} + 72\hat{k}$$

$$\Rightarrow \vec{b} = 12(2\hat{i} + 3\hat{j} + 6\hat{k})$$

\therefore Equation of the required line is $\vec{r} = \vec{a} + \lambda\vec{b}$

$$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda (12)(2\hat{i} + 3\hat{j} + 6\hat{k})$$

Again replacing 12λ by λ ,

$$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda (2\hat{i} + 3\hat{j} + 6\hat{k})$$

21. Prove that if a plane has the intercepts a, b, c and is at a distance of p units from the origin, then $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$.

Ans. We know that equation of plane making intercepts a, b, c (on the axes) is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} - 1 = 0$$

Given: Perpendicular distance of the origin $(0, 0, 0)$ from plane = p

$$\therefore \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}} = \frac{\left| \frac{0}{a} + \frac{0}{b} + \frac{0}{c} - 1 \right|}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} = p$$

$$\Rightarrow \frac{|-1|}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} = p$$

Squaring both sides, $\frac{1}{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}} = p^2$

$$\Rightarrow p^2 \left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} \right) = 1$$

$$\Rightarrow \left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} \right) = \frac{1}{p^2}$$

Choose the correct answer in Exercise Q. 22 and 23.

22.Distance between the two planes: $2x + 3y + 4z = 4$ and $4x + 6y + 8z = 12$ is

(A) 2 units(B) 4 units(C) 8 units(D) $\frac{2}{\sqrt{29}}$ units

Ans. Equation of one plane is $2x + 3y + 4z = 4 \Rightarrow 2x + 3y + 4z - 4 = 0$

Equation of second plane is $4x + 6y + 8z = 12 \Rightarrow 4x + 6y + 8z - 12 = 0$

Here $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{4}{8} = \frac{1}{2}$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ therefore, the given two lines are parallel.

We know that the distance of the parallel lines = $\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$

$$\Rightarrow \frac{|-4 - (-6)|}{\sqrt{(2)^2 + (3)^2 + (4)^2}}$$

$$= \frac{|-4 + 6|}{\sqrt{4 + 9 + 16}}$$

$$= \frac{2}{\sqrt{29}}$$

Therefore, option (D) is correct.

23. The planes: $2x - y + 4z = 5$ and $5x - 2.5y + 10z = 6$ are

(A) Perpendicular(B) Parallel

(C) intersect y -axis (D) passes through $\left(0, 0, \frac{5}{4}\right)$

Ans. Equations of the given planes are $2x - y + 4z = 5$ ($a_1x + b_1y + c_1z + d = 0$)

and $5x - 2.5y + 10z = 6$ ($a_2x + b_2y + c_2z + d = 0$)

For perpendicular $a_1a_2 + b_1b_2 + c_1c_2 = 2(5) + (-1)(-2.5) + 4(10) = 10 + 2.5 + 40 = 52.5$

$$\therefore a_1a_2 + b_1b_2 + c_1c_2 \neq 0$$

\therefore Planes are not perpendicular.

For parallel $\frac{a_1}{a_2} = \frac{2}{5}, \frac{b_1}{b_2} = \frac{-1}{-2.5} = \frac{10}{25} = \frac{2}{5}, \frac{c_1}{c_2} = \frac{4}{10} = \frac{2}{5}$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

\therefore given planes are parallel.

Therefore, option (B) is correct.