

CBSE Class-12 Mathematics

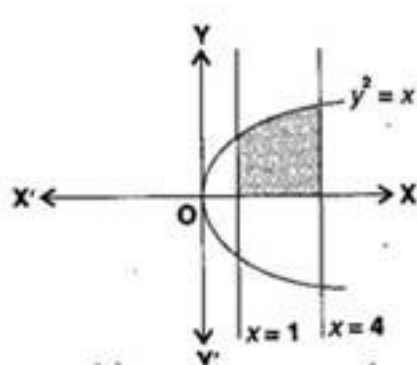
NCERT solution

Chapter - 8

Applications of Integrals - Exercise 8.1

1. Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$, $x = 4$ and the x -axis.

Ans. Equation of the curve (rightward parabola) is $y^2 = x$.



$$\therefore y = \sqrt{x} \dots\dots\dots(i)$$

\therefore Required area (shaded region)

$$= \left| \int_1^4 y \, dx \right| = \left| \int_1^4 \sqrt{x} \, dx \right| \text{ [From eq. (i)]}$$

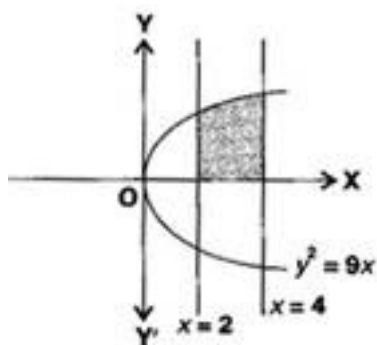
$$= \left| \int_1^4 x^{\frac{1}{2}} \, dx \right| = \left| \frac{\left(x^{\frac{3}{2}} \right)_1^4}{\frac{3}{2}} \right| = \left| \frac{2}{3} \left(4^{\frac{3}{2}} - 1^{\frac{3}{2}} \right) \right|$$

$$= \left| \frac{2}{3} \left(4^{\frac{1}{2} \times 3} - 1^{\frac{1}{2} \times 3} \right) \right| = \left| \frac{2}{3} (8 - 1) \right| = \frac{2}{3} \times 7 = \frac{14}{3} \text{ sq. units}$$

2. Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the x -axis in the first quadrant.

Ans. Equation of the curve (rightward parabola) is $y^2 = 9x$.

$$\therefore y = 3\sqrt{x} \dots\dots\dots(i)$$



\therefore Required area (shaded region) bounded by curve $y^2 = 9x$, (vertical lines $x = 2, x = 4$) and x -axis in first quadrant.

$$= \left| \int_2^4 y dx \right| = \left| \int_2^4 3\sqrt{x} dx \right| \quad [\text{From eq. (i)}]$$

$$= \left| 3 \int_2^4 x^{\frac{1}{2}} dx \right| = \left| 3 \frac{\left(x^{\frac{3}{2}} \right)_2^4}{\frac{3}{2}} \right|$$

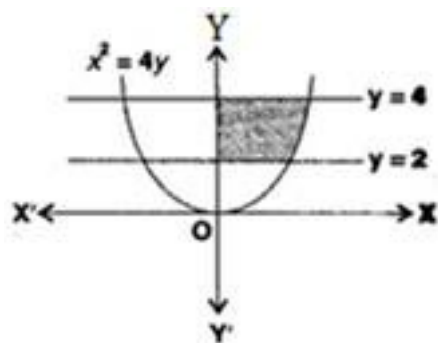
$$= \left| 3 \cdot \frac{2}{3} \left(4^{\frac{3}{2}} - 2^{\frac{3}{2}} \right) \right| = \left| 3 \cdot \frac{2}{3} \left(4^{\frac{1}{2} \times 3} - 2^{\frac{1}{2} \times 3} \right) \right|$$

$$= \left| 2(8 - 2\sqrt{2}) \right| = (16 - 4\sqrt{2}) \text{ sq. units}$$

3. Find the area of the region bounded by $x^2 = 4y$, $y = 2$, $y = 4$ and the y -axis in the first quadrant.

Ans. Equation of curve (parabola) is $x^2 = 4y$. $\therefore x = 2\sqrt{y}$ (i)

\therefore Required (shaded) area bounded by curve $x^2 = 4y$, (Horizontal lines $y = 2$, $y = 4$) and y -axis in first quadrant.

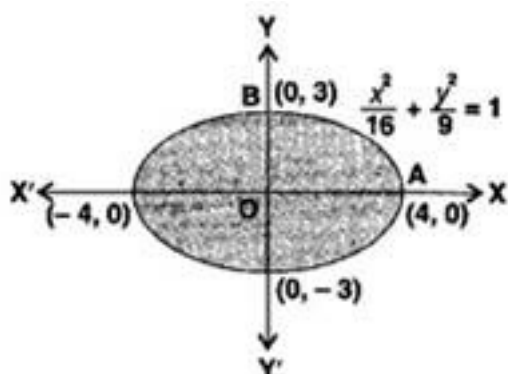


$$= \left| \int_2^4 x \, dy \right| = \left| \int_2^4 2\sqrt{y} \, dy \right| = \left| 2 \int_2^4 y^{\frac{1}{2}} \, dy \right|$$

$$= \left| 2 \frac{\left(y^{\frac{3}{2}} \right)^4}{\frac{3}{2}} \right| = \frac{4}{3} \left(4^{\frac{3}{2}} - 2^{\frac{3}{2}} \right) = \left(\frac{32 - 8\sqrt{2}}{3} \right) \text{ sq. units}$$

4. Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

Ans. Equation of ellipse is $\frac{x^2}{16} + \frac{y^2}{9} = 1$ (i)



Here $a^2 (=16) > b^2 (=9)$

From eq. (i), $\frac{y^2}{9} = 1 - \frac{x^2}{16} = \frac{16-x^2}{16}$

$$\Rightarrow y^2 = \frac{9}{16}(16-x^2)$$

$$\Rightarrow y = \pm \frac{3}{4} \sqrt{16-x^2} \dots \dots \dots (ii)$$

For arc of ellipse in first quadrant.

Ellipse (i) is symmetrical about x -axis,

(\because On changing $y \rightarrow -y$ in eq. (i), it remains unchanged)

Ellipse (i) is symmetrical about y -axis,

(\because On changing $x \rightarrow -x$ in eq. (i), it remains unchanged)

Intersections of ellipse (i) with x -axis ($y=0$)

Putting $y=0$ in eq. (i), $\frac{x^2}{16} = 1 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$

Therefore, Intersections of ellipse (i) with x -axis are $(0, 4)$ and $(0, -4)$.

Intersections of ellipse (i) with y -axis ($x=0$)

Putting $x=0$ in eq. (i), $\frac{y^2}{9} = 1 \Rightarrow y^2 = 9 \Rightarrow y = \pm 3$

Therefore, Intersections of ellipse (i) with y -axis are $(0, 3)$ and $(0, -3)$.

Now Area of region bounded by ellipse (i) = Total shaded area

= 4 x Area OAB of ellipse in first quadrant

$$= 4 \left| \int_0^4 y \, dx \right| \left[\because \text{At end B of arc AB of ellipse; } x=0 \text{ and at end A of arc AB; } x=4 \right]$$

$$= 4 \left| \int_0^4 \frac{3}{4} \sqrt{16-x^2} \, dx \right| = 4 \left| \int_0^4 \frac{3}{4} \sqrt{4^2-x^2} \, dx \right|$$

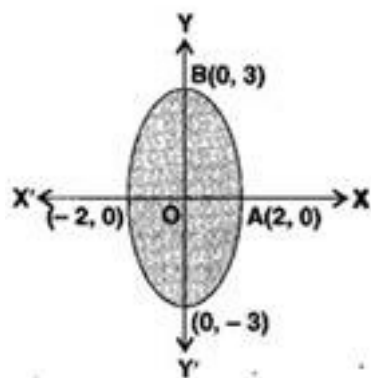
$$= 3 \left[\frac{x}{2} \sqrt{4^2-x^2} + \frac{4^2}{2} \sin^{-1} \frac{x}{4} \right]_0^4 \left[\because \int \sqrt{a^2-x^2} \, dx = \frac{x}{2} \sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]$$

$$= 3 \left[\frac{4}{2} \sqrt{16-16} + 8 \sin^{-1} 1 - (0 + 8 \sin^{-1} 0) \right] = 3 \left[0 + \frac{8\pi}{2} \right]$$

$$= 3(4\pi) = 12\pi \text{ sq. units}$$

5. Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

Ans. Equation of ellipse is $\frac{x^2}{4} + \frac{y^2}{9} = 1$



Here $a^2 (=4) < b^2 (=9)$

$$\text{From eq. (i), } \frac{y^2}{9} = 1 - \frac{x^2}{4} = \frac{4-x^2}{4}$$

$$\Rightarrow y^2 = \frac{9}{4}(4-x^2)$$

$$\Rightarrow y = \pm \frac{3}{2} \sqrt{4 - x^2} \dots\dots\dots (ii)$$

for arc of ellipse in first quadrant.

Ellipse (i) is symmetrical about x -axis,

(\because On changing $y \rightarrow -y$ in eq. (i), it remains unchanged)

Ellipse (i) is symmetrical about y -axis,

(\because On changing $x \rightarrow -x$ in eq. (i), it remains unchanged)

Intersections of ellipse (i) with x -axis ($y = 0$)

Putting $y = 0$ in eq. (i), $\frac{x^2}{4} = 1$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

Therefore, Intersections of ellipse (i) with x -axis are $(0, 2)$ and $(0, -2)$.

Intersections of ellipse (i) with y -axis ($x = 0$)

Putting $x = 0$ in eq. (i), $\frac{y^2}{9} = 1$

$$\Rightarrow y^2 = 9 \Rightarrow y = \pm 3$$

Therefore, Intersections of ellipse (i) with y -axis are $(0, 3)$ and $(0, -3)$.

Now Area of region bounded by ellipse (i) = Total shaded area

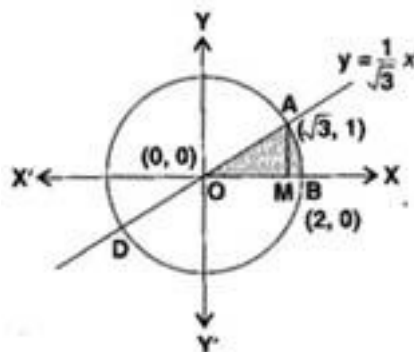
= 4 x Area OAB of ellipse in first quadrant

$$= 4 \left| \int_0^2 y \, dx \right| \left[\because \text{At end B of arc AB of ellipse; } x = 0 \text{ and at end A of arc AB; } x = 2 \right]$$

$$\begin{aligned}
 &= 4 \left| \int_0^2 \frac{3}{2} \sqrt{4-x^2} \, dx \right| = 4 \left| \int_0^4 \frac{3}{2} \sqrt{2^2-x^2} \, dx \right| \\
 &= 6 \left[\frac{x}{2} \sqrt{2^2-x^2} + \frac{2^2}{2} \sin^{-1} \frac{x}{2} \right]_0^4 \left[\because \int \sqrt{a^2-x^2} \, dx = \frac{x}{2} \sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right] \\
 &= 6 \left[\frac{2}{2} \sqrt{4-4} + 2 \sin^{-1} 1 - (0 + 2 \sin^{-1} 0) \right] \\
 &= 6 \left[0 + 2 \cdot \frac{\pi}{2} - 0 \right] = 6\pi \text{ sq. units}
 \end{aligned}$$

6. Find the area of the region in the first quadrant enclosed by x -axis, line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.

Ans. Step I. To draw the graphs and shade the region whose area we are to find.



Equation of the circle is $x^2 + y^2 = 2^2$ (i)

We know that eq. (i) represents a circle whose centre is (0, 0) and radius is 2.

Equation of the given line is $x = \sqrt{3}y$

$$\Rightarrow y = \frac{1}{\sqrt{3}}x \text{(ii)}$$

We know that eq. (ii) being of the form $y = mx$ where $m = \frac{1}{\sqrt{3}} = \tan 30^\circ = \tan \theta \Rightarrow$

$\theta = 30^\circ$ represents a straight line passing through the origin and making angle of 30° with x -axis.

Step II. To find values of x and y . Putting $y = \frac{x}{\sqrt{3}}$ from eq. (ii) in eq. (i),

$$x^2 + \frac{x^2}{3} = 4 \Rightarrow 3x^2 + x^2 = 12 \Rightarrow 4x^2 = 12$$

$$\Rightarrow x^2 = 3 \Rightarrow x = \pm 3$$

Putting $x = \pm 3$ in $y = \frac{x}{\sqrt{3}}$, $y = 1$ and $y = -1$

Therefore, the two points of intersections of circle (i) and line (ii) are $A(\sqrt{3}, 1)$ and $D(-\sqrt{3}, -1)$.

Step III. Now shaded area OAM between segment OA of line (ii) and x -axis

$$\begin{aligned} &= \left| \int_0^{\sqrt{3}} y \, dx \right| \left[\because \text{At O, } x=0 \text{ and at A, } x=\sqrt{3} \right] \\ &= \left| \int_0^{\sqrt{3}} \frac{1}{\sqrt{3}} x \, dx \right| = \frac{1}{\sqrt{3}} \left(\frac{x^2}{2} \right)_0^{\sqrt{3}} = \frac{1}{\sqrt{3}} \left(\frac{3}{2} - 0 \right) = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2} \text{ sq. units.....(iii)} \end{aligned}$$

Step IV. Now shaded area AMB between arc AB of circle and x -axis.

$$\begin{aligned} &= \left| \int_{\sqrt{3}}^2 y \, dx \right| \left[\because \text{At O, } x=\sqrt{3} \text{ and at A, } x=2 \right] \\ &= \left| \int_{\sqrt{3}}^2 \sqrt{2^2 - x^2} \, dx \right| \text{ From eq. (ii),} \end{aligned}$$

$$= \left(\frac{x}{2} \sqrt{2^2 - x^2} + \frac{2^2}{2} \sin^{-1} \frac{x}{2} \right)_{\sqrt{3}}^2 = \left[\frac{2}{2} \sqrt{4-4} + 2 \sin^{-1} 1 - \left(\frac{\sqrt{3}}{2} \sqrt{4-3} + 2 \sin^{-1} \frac{\sqrt{3}}{2} \right) \right]$$

$$= 0 + 2 \cdot \frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2 \cdot \frac{\pi}{3} = \pi - \frac{\sqrt{3}}{2} - \frac{2\pi}{3} = \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right) \text{ sq. units.....(iv)}$$

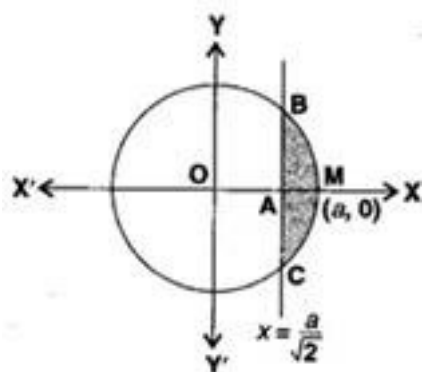
Step V. Required shaded area OAB = Area of OAM + Area of AMB

$$= \frac{\sqrt{3}}{2} + \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right) = \frac{\pi}{3} \text{ sq. units}$$

7. Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line

$$x = \frac{a}{\sqrt{2}}.$$

Ans. Equation of the circle is $x^2 + y^2 = a^2$ (i)



$$\therefore y^2 = a^2 - x^2$$

$$\Rightarrow y = \sqrt{a^2 - x^2} \text{(ii)}$$

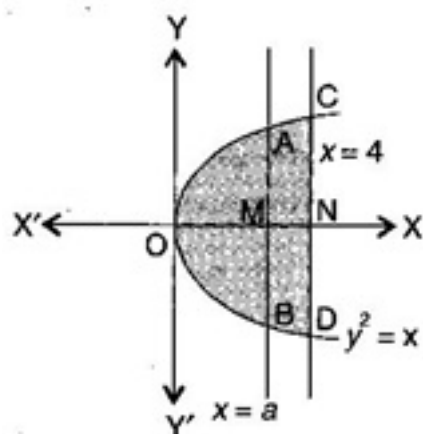
Here Area of smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$ = Area of

ABMC = 2 x Area of ABM

$$\begin{aligned}
 &= 2 \left| \int_{\frac{a}{\sqrt{2}}}^a y \, dx \right| = 2 \left| \int_{\frac{a}{\sqrt{2}}}^a \sqrt{a^2 - x^2} \, dx \right| \text{ [From eq. (ii)]} \\
 &= 2 \left[\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_{\frac{a}{\sqrt{2}}}^a \\
 &= 2 \left[\frac{a}{2} \sqrt{a^2 - a^2} + \frac{a^2}{2} \sin^{-1} 1 - \left(\frac{\frac{a^2}{\sqrt{2}}}{2} \sqrt{a^2 - \frac{a^2}{2}} \sin^{-1} \frac{\frac{a^2}{\sqrt{2}}}{2} \right) \right] \\
 &= 2 \left[0 + \frac{a^2}{2} \cdot \frac{\pi}{2} - \frac{a}{\sqrt{2}} \times \frac{a}{\sqrt{2}} - \frac{a^2}{2} \sin^{-1} \frac{1}{\sqrt{2}} \right] \\
 &= 2 \left[\frac{\pi a^2}{4} - \frac{a}{\sqrt{2}} \times \frac{a}{\sqrt{2}} - \frac{a^2}{2} \cdot \frac{\pi}{4} \right] \\
 &= 2 \left[\frac{\pi a^2}{4} - \frac{a^2}{2} - \frac{\pi a^2}{8} \right] \\
 &= 2a^2 \left[\frac{\pi}{4} - \frac{1}{2} - \frac{\pi}{8} \right] \\
 &= 2a^2 \left[\frac{2\pi - 2 - \pi}{8} \right] \\
 &= \frac{a^2}{4} (\pi - 2) = \frac{\pi a^2}{4} - \frac{a^2}{2} \\
 &\Rightarrow \frac{a^2}{2} \left(\frac{\pi}{2} - 1 \right) \text{ sq. units}
 \end{aligned}$$

8. The area between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, find the value of a .

Ans. Equation of the curve (parabola) is $x = y^2$ (i)



$$\Rightarrow y = \sqrt{x}$$

Now area bounded by parabola (i) and vertical line $x = 4$ is divided into two equal parts by the vertical line $x = a$.

$$\therefore \text{Area OAMB} = \text{Area AMBDNC}$$

$$\Rightarrow 2 \left| \int_0^a y \, dx \right| = 2 \left| \int_a^4 y \, dx \right|$$

$$\Rightarrow 2 \left| \int_0^a x^{\frac{1}{2}} \, dx \right| = 2 \left| \int_a^4 x^{\frac{1}{2}} \, dx \right|$$

$$\Rightarrow \frac{\left(x^{\frac{3}{2}} \right)_0^a}{\frac{3}{2}} = \frac{\left(x^{\frac{3}{2}} \right)_a^4}{\frac{3}{2}}$$

$$\Rightarrow \frac{2}{3} \left[a^{\frac{3}{2}} - 0 \right] = \frac{2}{3} \left[4^{\frac{3}{2}} - a^{\frac{3}{2}} \right]$$

$$\Rightarrow a^{\frac{3}{2}} = 8 - a^{\frac{3}{2}}$$

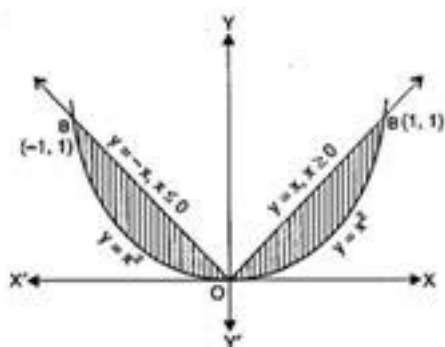
$$\Rightarrow 2a^{\frac{3}{2}} = 8 \Rightarrow a^{\frac{3}{2}} = 4$$

$$\Rightarrow a = 4^{\frac{2}{3}} \text{ units}$$

9. Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$.

Ans. The required area is the area included between the parabola $y = x^2$ and the modulus

$$\text{function } y = |x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x \leq 0 \end{cases}$$



To find: Area between the parabola $y = x^2$ and the ray $y = x$ for $x \geq 0$

Here, Limits of integration $\Rightarrow y = x$

$$\Rightarrow x^2 = x \Rightarrow x^2 - x = 0$$

$$\Rightarrow x(x-1) = 0 \Rightarrow x = 0, x = 1$$

Now, for $y = |x|$, table of values,

$$y = x \text{ if } x \geq 0 \quad y = -x \text{ if } x \leq 0$$

x	0	1	2
y	0	1	2

x	0	-1	-2
y	0	1	2

Now, Area between parabola $y = x^2$ and x -axis between limits $x = 0$ and $x = 1$

$$= \int_0^1 y \, dx = \int_0^1 x^2 \, dx = \left(\frac{x^3}{3} \right)_0^1 = \frac{1}{3} \dots\dots\dots(i)$$

And Area of ray $y = x$ and x -axis,

$$= \int_0^1 y \, dx = \int_0^1 x \, dx = \left(\frac{x^2}{2} \right)_0^1 = \frac{1}{2} \dots\dots\dots(ii)$$

∴ Required shaded area in first quadrant = Area between ray $y = x$ for $x \geq 0$ and x -axis
Area between parabola $y = x^2$ and x -axis in first quadrant

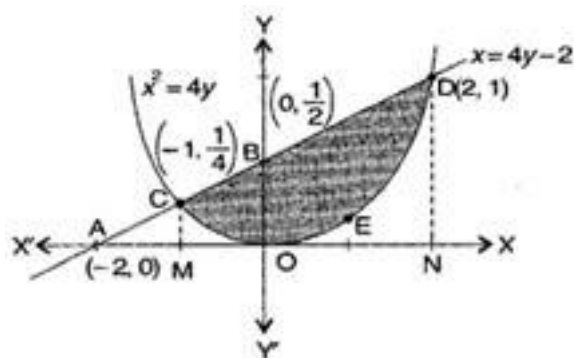
$$= \text{Area given by eq. (ii)} - \text{Area given by eq. (i)} = \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \text{ sq. units}$$

Similarly, shaded area in second quadrant = $\frac{1}{6}$ sq. units

Therefore, total area of shaded region in the above figure = $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$ sq. units

10. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

Ans. Step I. Graphs and region of Integration



Equation of the given curve is

$$x^2 = 4y \dots\dots\dots(i)$$

Equation of the given line is

$$x = 4y - 2 \dots\dots\dots(ii)$$

$$\Rightarrow y = \frac{x+2}{4}$$

x	0	1	-2
y	0	$\frac{1}{2}$	0

Step II. Putting $y = \frac{x^2}{4}$ from eq. (i) in eq. (ii),

$$x = 4 \cdot \frac{x^2}{4} - 2 \Rightarrow x = x^2 - 2 \Rightarrow -x^2 + x + 2 = 0$$

$$\Rightarrow x^2 - x - 2 = 0 \Rightarrow x^2 - 2x + x - 2 = 0 \Rightarrow x(x - 2) + 1(x - 2) = 0$$

$$\Rightarrow (x - 2)(x + 1) = 0 \Rightarrow x = 2 \text{ or } x = -1$$

For $x = 2$, from eq. (i), $y = \frac{x^2}{4} = \frac{4}{4} = 1 \therefore (2, 1)$

For $x = -1$, from eq. (i), $y = \frac{x^2}{4} = \frac{1}{4} \therefore \left(-1, \frac{1}{4}\right)$

Therefore, the two points of intersection of parabola (i) and line (ii) are C $\left(-1, \frac{1}{4}\right)$ and D (2, 1).

Step III. Area CMOEDN between parabola (i) and x -axis = $\left| \int_{-1}^2 y \, dx \right| = \left| \int_{-1}^2 \frac{x^2}{4} \, dx \right|$

$$= \left| \frac{(x^3)^2}{12} \right|_{-1}^2 = \left| \frac{1}{12} \{2^3 - (-1)^3\} \right| = \frac{1}{12} (8 + 1) = \frac{9}{12} = \frac{3}{4} \text{ sq. units.....(iii)}$$

Step IV. Area of trapezium CMND between line (ii) and x -axis = $\left| \int_{-1}^2 y \, dx \right| = \left| \int_{-1}^2 \frac{x+2}{4} \, dx \right|$

$$= \left| \frac{1}{4} \int_{-1}^2 (x+2) dx \right| = \frac{1}{4} \left| \left(\frac{x^2}{2} + 2x \right)_{-1}^2 \right| = \frac{1}{4} \left| \left(\frac{4}{2} + 4 \right) - \left(\frac{1}{2} - 2 \right) \right| = \frac{1}{4} \left| \left(2 + 4 - \frac{1}{2} + 2 \right) \right|$$

$$= \frac{1}{4} \left| 8 - \frac{1}{2} \right| = \frac{1}{4} \times \frac{15}{2} = \frac{15}{8} \text{ sq. units.....(iv)}$$

∴ Required shaded area = Area given by eq. (iv) – Area given by eq. (iii)

$$= \frac{15}{8} - \frac{3}{4} = \frac{15-6}{8} = \frac{9}{8} \text{ sq. units}$$

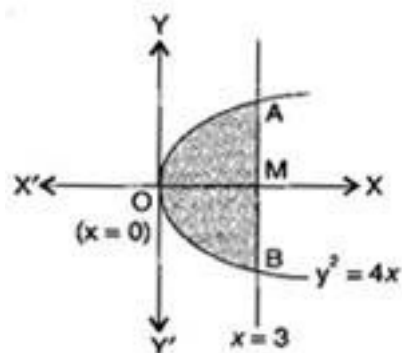
11. Find the area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$.

Ans. Equation of the (parabola) curve is

$$y^2 = 4x \text{(i)}$$

$$\Rightarrow y = 2\sqrt{x} \text{(ii)}$$

Here required shaded area OAMB = 2 x Area OAM



$$= 2 \left| \int_0^3 y dx \right| = 2 \left| \int_0^3 2x^{\frac{1}{2}} dx \right| = 4 \left| \left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right)_0^3 \right|$$

$$= 4 \cdot \frac{2}{3} \left[3^{\frac{3}{2}} - 0 \right] = \frac{8}{3} \cdot 3\sqrt{3} = 8\sqrt{3} \text{ sq. units}$$

12. Choose the correct answer:

Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x=0$ and $x=2$ is

(A) π

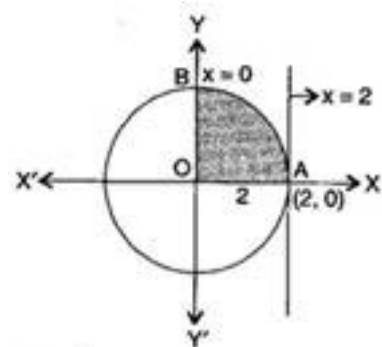
(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{4}$

Ans. Equation of the circle is $x^2 + y^2 = 2^2$ (i)

$\Rightarrow y = \sqrt{2^2 - x^2}$ (ii)



$$\text{Required area} = \left| \int_0^2 y \, dx \right| = \left| \int_0^2 \sqrt{2^2 - x^2} \, dx \right|$$

$$= \left| \left(\frac{x}{2} \sqrt{2^2 - x^2} + \frac{2^2}{2} \sin^{-1} \frac{x}{2} \right) \right|_0^2$$

$$= \frac{2}{2} \sqrt{4 - 4} + 2 \sin^{-1} 1 - (0 + 2 \sin^{-1} 0)$$

$$= 0 + 2 \cdot \frac{\pi}{2} - 0 - 0 = \pi \text{ sq. units}$$

Therefore, option (A) is correct.

13. Choose the correct answer:

Area of the region bounded by the curve $y^2 = 4x$, y – axis and the line $y = 3$ is:

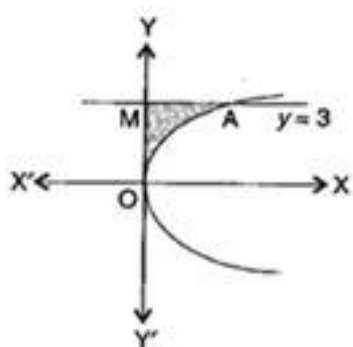
(A) 2

(B) $\frac{9}{4}$

(C) $\frac{9}{3}$

(D) $\frac{9}{2}$

Ans. Equation of the curve (parabola) is $y^2 = 4x$



$$\text{Required area} = \text{Area OAM} = \left| \int_0^3 x \, dy \right| = \left| \int_0^3 \frac{y^2}{4} \, dy \right|$$

$$= \frac{1}{4} \left| \left(\frac{y^3}{3} \right)_0^3 \right| = \frac{1}{4} \left| \frac{27}{3} - 0 \right| = \frac{9}{4} \text{ sq. units}$$

Therefore, option (B) is correct.