CBSE Class–12 Mathematics
NCERT solution
Chapter - 7
Integrals - Exercise 7.9

Evaluate the definite integrals in Exercises 1 to 11.

1. $\int_{-1}^{1}(x+1)\ dx$

Ans. $\int_{-1}^{1}(x+1)\ dx$

$= \left(\frac{x^2}{2}+x\right)_{-1}^{1}$

$= \left(\frac{1^2}{2}+1\right)-\left(\frac{(-1)^2}{2}-1\right)$

$= \frac{1}{2}+1-\left(\frac{1}{2}-1\right)$

$= \frac{1}{2}+1-\frac{1}{2}+1$

=2 Ans.

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2. $\int_{2}^{3}\frac{1}{x}\ dx$

Ans. $\int_{2}^{3}\frac{1}{x}\ dx$

$= \left(\log|x|\right)_2^3$

$= \log|3| - \log|2|$

$= \log 3 - \log 2$

$= \log\frac{3}{2}$ Ans.

3. $\int_1^2 \left(4x^3 - 5x^2 + 6x + 9\right)\, dx$

Ans. $\int_1^2 \left(4x^3 - 5x^2 + 6x + 9\right)\, dx$

$= \left(4\frac{x^4}{4} - 5\frac{x^3}{3} + 6\frac{x^2}{2} + 9x\right)_1^2$

$= \left(x^4 - \frac{5}{3}x^3 + 3x^2 + 9x\right)_1^2$

$= \left(2^4 - \frac{5}{3}(2)^3 + 3(2)^2 + 9(2)\right) - \left(1 - \frac{5}{3} + 3 + 9\right)$

$= \left(16 - \frac{40}{3} + 12 + 18\right) - \left(13 - \frac{5}{3}\right)$

$= 46 - \frac{40}{3} - \left(13 - \frac{5}{3}\right)$

$= 46 - \frac{40}{3} - 13 + \frac{5}{3}$

$= 33 - \frac{40}{3} + \frac{5}{3}$

= $\dfrac{99-40+5}{3}$

= $\dfrac{64}{3}$ Ans.

4. $\int_0^{\frac{\pi}{4}} \sin 2x\, dx$

Ans. $\int_0^{\frac{\pi}{4}} \sin 2x\, dx$

= $\left(\dfrac{-\cos 2x}{2}\right)_0^{\frac{\pi}{4}}$

= $\dfrac{-\cos\frac{\pi}{2}}{2} - \left(\dfrac{-\cos 0^\circ}{2}\right)$

= $\dfrac{0}{2} - \left(\dfrac{-1}{2}\right)$

= $0 + \dfrac{1}{2}$

= $\dfrac{1}{2}$ Ans.

5. $\int_0^{\frac{\pi}{2}} \cos 2x\, dx$

Ans. $\int_0^{\frac{\pi}{2}} \cos 2x \, dx$

= $\left(\frac{\sin 2x}{2}\right)_0^{\frac{\pi}{2}}$

= $\frac{\sin \pi}{2} - \frac{\sin 0^\circ}{2}$

= $\frac{0}{2} - \frac{0}{2}$

= 0 Ans.

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6. $\int_4^5 e^x \, dx$

Ans. $\int_4^5 e^x \, dx$

= $\left(e^x\right)_4^5$

= $e^5 - e^4$

= $e^4(e-1)$ Ans.

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7. $\int_0^{\frac{\pi}{4}} \tan x \, dx$

Ans. $\int_0^{\frac{\pi}{4}} \tan x \, dx$

= $\left(\log|\sec x|\right)_0^{\frac{\pi}{4}}$

= $\log\left|\sec\frac{\pi}{4}\right| - \log|\sec 0°|$

= $\log\left|\sqrt{2}\right| - \log|1|$

= $\log\sqrt{2} - \log 1$

= $\log 2^{\frac{1}{2}} - 0$

= $\frac{1}{2}\log 2$ Ans.

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8. $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \, dx$

Ans. $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \, dx$

= $\left(\log|\operatorname{cosec} x - \cot x|\right)_{\frac{\pi}{6}}^{\frac{\pi}{4}}$

= $\log\left|\operatorname{cosec}\frac{\pi}{4} - \cot\frac{\pi}{4}\right| - \log\left|\operatorname{cosec}\frac{\pi}{6} - \cot\frac{\pi}{6}\right|$

= $\log\left|\sqrt{2} - 1\right| - \log\left|2 - \sqrt{3}\right|$

= $\log\left(\sqrt{2} - 1\right) - \log\left(2 - \sqrt{3}\right)$

= $\log\left(\frac{\sqrt{2}-1}{2-\sqrt{3}}\right)$ Ans.

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9. $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

Ans. $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

= $\left(\sin^{-1} x\right)_0^1$

= $\sin^{-1} 1 - \sin^{-1} 0$

= $\frac{\pi}{2} - 0$

= $\frac{\pi}{2}$ Ans.

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10. $\int_0^1 \frac{dx}{1+x^2}$

Ans. $\int_0^1 \frac{dx}{1+x^2}$

= $\left(\tan^{-1} x\right)_0^1$

= $\tan^{-1} 1 - \tan^{-1} 0$

= $\frac{\pi}{4} - 0$

= $\frac{\pi}{4}$ Ans.

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11. $\int_2^3 \frac{dx}{x^2-1}$

Ans. $\int_2^3 \frac{dx}{x^2-1} = \int_2^3 \frac{1}{x^2-1^2}\ dx$

$= \left(\frac{1}{2(1)}\log\left|\frac{x-1}{x+1}\right|\right)_2^3$

$= \frac{1}{2}\log\left|\frac{3-1}{3+1}\right| - \frac{1}{2}\log\left|\frac{2-1}{2+1}\right|$

$= \frac{1}{2}\log\left|\frac{1}{2}\right| - \frac{1}{2}\log\left|\frac{1}{3}\right|$

$= \frac{1}{2}\left(\log\frac{1}{2} - \log\frac{1}{3}\right)$

$= \frac{1}{2}\log\frac{1/2}{1/3}$

$= \frac{1}{2}\log\frac{3}{2}$ Ans.

Evaluate the definite integrals in Exercises 12 to 20.

12. $\int_0^{\frac{\pi}{2}} \cos^2 c\ dx$

Ans. $\int_0^{\frac{\pi}{2}} \cos^2 x\ dx$

$= \int_0^{\frac{\pi}{2}} \frac{1+\cos 2x}{2}\ dx$

$= \frac{1}{2}\int_0^{\frac{\pi}{2}} (1+\cos 2x)\ dx$

$= \frac{1}{2}\left(x+\frac{\sin 2x}{2}\right)_0^{\frac{\pi}{2}}$

$= \frac{1}{2}\left[\frac{\pi}{2}+\frac{1}{2}\sin \pi-\left(0+\frac{1}{2}\sin 0^\circ\right)\right]$

$= \frac{1}{2}\left[\frac{\pi}{2}+0-0\right]$

$= \frac{\pi}{4}$ Ans.

13. $\int_2^3 \frac{xdx}{x^2+1}$

Ans. $\int_2^3 \frac{x\ dx}{x^2+1} = \frac{1}{2}\int_2^3 \frac{2x}{x^2+1}\ dx$

$= \frac{1}{2}\left(\log\left|x^2+1\right|\right)_2^3$

$= \frac{1}{2}\left(\log|10|-\log|5|\right)$

$= \frac{1}{2}\left(\log 10-\log 5\right)$

= $\frac{1}{2}\log\frac{10}{5}$

= $\frac{1}{2}\log 2$ Ans.

14. $\int_0^1 \frac{2x+3}{5x^2+1}\,dx$

Ans. $\int_0^1 \frac{2x+3}{5x^2+1}\,dx$

= $\int_0^1 \left(\frac{2x}{5x^2+1}+\frac{3}{5x^2+1}\right)dx$

= $\int_0^1 \frac{2x}{5x^2+1}\,dx+\int_0^1 \frac{3}{5x^2+1}\,dx$

= $\int_0^1 \frac{2x}{5x^2+1}\,dx+3\int_0^1 \frac{1}{\left(\sqrt{5}x\right)^2+1^2}\,dx$

$= \frac{1}{5}\int_0^1 \frac{10x}{5x^2+1}dx + 3\int_0^1 \frac{1}{(\sqrt{5}x)^2+1^2}dx$

= $\frac{1}{5}\left(\log\left|5x^2+1\right|\right)_0^1+3.\frac{1}{1}\frac{\left(\tan^{-1}\frac{\sqrt{5}x}{1}\right)_0^1}{\sqrt{5}\rightarrow \text{Coeff. of } x}$

$= \frac{1}{5}\left(\log|5(1)^2+1| - \log|5(0)^2+1\right) + \frac{3}{\sqrt{5}}\left(\tan^{-1}\sqrt{5} - \tan^{-1}\sqrt{0}\right)$

$= \frac{1}{5}(\log 6 - \log 1) + \frac{3}{\sqrt{5}}\left(\tan^{-1}\sqrt{5} - 0\right)$

$= \frac{1}{5}(\log 6 - 0) + \frac{3}{\sqrt{5}}\left(\tan^{-1}\sqrt{5} - 0\right)$

$= \frac{1}{5}\log 6 + \frac{3}{\sqrt{5}}\tan^{-1}\sqrt{5}$ Ans.

15. $\int_0^1 xe^{x^2}\ dx$

Ans. $\int_0^1 xe^{x^2}\ dx$

First we evaluate $\int xe^{x^2}\ dx = \frac{1}{2}\int e^{x^2}\ (2x)\ dx$ ....(i)

Putting $x^2 = t$

$\Rightarrow 2x = \frac{dt}{dx}$

$\Rightarrow 2x\ dx = dt$

$\therefore$ From eq. (i), $\frac{1}{2}\int e^t\ dt = \frac{1}{2}e^t = \frac{1}{2}e^{x^2}$

$\therefore \int_0^1 xe^{x^2}\ dx = \frac{1}{2}\left(e^{x^2}\right)_0^1$

$= \frac{1}{2}\left(e^1 - e^0\right)$

$= \frac{1}{2}\left(e - 1\right)$ Ans.

16. $\int_1^2 \frac{5x^2}{x^2+4x+3}\ dx$

Ans. $\int_1^2 \frac{5x^2}{x^2+4x+3}\ dx$

= $\int_1^2 \frac{5x^2}{(x+1)(x+3)}\,dx$

= $\int_1^2 \left(5+\frac{-20x-15}{(x+1)(x+3)}\right) dx$ (On dividing)

= $\int_1^2 5\,dx+\int_1^2\left(\frac{-20x-15}{(x+1)(x+3)}\right) dx$

= $5(x)_1^2 + \mathrm{I}$ where I = $\int_1^2\left(\frac{-20x-15}{(x+1)(x+3)}\right) dx$

= $5(2-1)+\ \mathrm{I}=5+\mathrm{I}$ ....(i)

Now, I = $\int_1^2\left(\frac{-20x-15}{(x+1)(x+3)}\right) dx$ ....(ii)

Let $\frac{-20x-15}{(x+1)(x+3)}=\frac{A}{x+1}+\frac{B}{x+3}$ ....(iii)

$\Rightarrow\ -20x-15=A(x+3)+B(x+1)$

$\Rightarrow\ -20x-15=Ax+3A+Bx+B$

Comparing coefficients of x A + B = –20 ....(iv)

Comparing constants 3A + B = –15 ....(v)

On solving eq. (iv) and (v), we get A = $\frac{5}{2}$, B = $\frac{-45}{2}$

Putting these values in eq. (iii),

$$\frac{-20x-15}{(x+1)(x+3)}=\frac{5/2}{x+1}+\frac{-45/2}{x+3}$$

$\Rightarrow$ I = $\int_1^2 \left(\frac{-20x-15}{(x+1)(x+3)} \right) dx$

= $\frac{5}{2}\int_1^2 \frac{1}{x+1}\ dx - \frac{45}{2}\int_1^2 \frac{1}{x+3}\ dx$

= $\frac{5}{2}\left(\log|x+1|\right)_1^2 - \frac{45}{2}\left(\log|x+3|\right)_1^2$

= $\frac{5}{2}\left(\log|3| - \log|2|\right) - \frac{45}{2}\left(\log|5| - \log|4|\right)$

= $\frac{5}{2}\log\frac{3}{2} - \frac{45}{2}\log\frac{5}{4}$

= $\frac{5}{2}\left(\log\frac{3}{2} - 9\log\frac{5}{4} \right)$

Putting this value of I in eq. (i),

$\int_1^2 \frac{5x^2}{x^2+4x+3}\ dx$

= $5 + \frac{5}{2}\left(\log\frac{3}{2} - 9\log\frac{5}{4} \right)$

= $5 - \frac{5}{2}\left(9\log\frac{5}{4} - \log\frac{3}{2} \right)$ Ans.

17. $\int_0^{\frac{\pi}{4}} \left(2\sec^2 x + x^3 + 2\right)\ dx$

Ans. $\int_0^{\frac{\pi}{4}} \left(2\sec^2 x + x^3 + 2\right)\ dx$

= $2\int_0^{\frac{\pi}{4}} \sec^2 x \, dx + \int_0^{\frac{\pi}{4}} x^3 \, dx + 2\int_0^{\frac{\pi}{4}} 1 \, dx$

= $2(\tan x)_0^{\frac{\pi}{4}} + \left(\frac{x^4}{4}\right)_0^{\frac{\pi}{4}} + 2(x)_0^{\frac{\pi}{4}}$

= $2\left(\tan\frac{\pi}{4} - \tan 0°\right) + \frac{\left(\frac{\pi}{4}\right)^4}{4} - 0 + 2\left(\frac{\pi}{4} - 0\right)$

= $2(1-0) + \frac{\left(\frac{\pi^4}{256}\right)}{4} + \frac{2\pi}{4}$

= $2 + \frac{\pi^4}{1024} + \frac{\pi}{2}$

= $\frac{\pi^4}{1024} + \frac{\pi}{2} + 2$ Ans.

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18. $\int_0^{\pi}\left(\sin^2\frac{x}{2} - \cos^2\frac{x}{2}\right) dx$

Ans. $\int_0^{\pi}\left(\sin^2\frac{x}{2} - \cos^2\frac{x}{2}\right) dx$

= $\int_0^{\pi}\left(\frac{1-\cos x}{2} - \frac{1+\cos x}{2}\right) dx$

= $\int_0^{\pi}\left(\frac{1-\cos x - 1 - \cos x}{2}\right) dx$

= $\int_0^{\pi}\left(\frac{-2\cos x}{2}\right) dx$

$= -\int_0^{\pi} \cos x \, dx$

$= -(\sin x)_0^{\pi}$

$= -(\sin \pi - \sin 0°)$

$= -(0-0) = 0$ Ans.

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19. $\int_0^2 \frac{6x+3}{x^2+4} \, dx$

Ans. $\int_0^2 \frac{6x+3}{x^2+4} \, dx$

$= \int_0^2 \left(\frac{6x}{x^2+4} + \frac{3}{x^2+4} \right) dx$

$= \int_0^2 \frac{6x}{x^2+4} \, dx + \int_0^2 \frac{3}{x^2+4} \, dx$

$= 3\int_0^2 \frac{2x}{x^2+4} \, dx + 3\frac{1}{2}\left(\tan^{-1} \frac{x}{2} \right)_0^2$

$= 3\left(\log \left| x^2+4 \right| \right)_0^2 + \frac{3}{2}\left(\tan^{-1} 1 - \tan^{-1} 0 \right)$

$= 3(\log 8 - \log 4) + \frac{3}{2}\left(\frac{\pi}{4} - 0 \right)$

$= 3\log \frac{8}{4} + \frac{3\pi}{8}$

$= 3\log 2 + \frac{3\pi}{8}$ Ans.

20. $\int_0^1 \left(xe^x + \sin \frac{\pi x}{4} \right) dx$

Ans. $\int_0^1 \left(xe^x + \sin \frac{\pi x}{4} \right) dx$

= $\int_0^1 xe^x \, dx + \int_0^1 \sin \frac{\pi x}{4} \, dx$

[Applying Product Rule on first definite integral]

= $\left(xe^x \right)_0^1 - \int_0^1 1.e^x \, dx - \dfrac{\left(\cos \frac{\pi x}{4} \right)_0^1}{\frac{\pi}{4} \to \text{Coeff. of } x \text{ in } \frac{\pi}{4}}$

= $e^1 - 0 - \int_0^1 e^x \, dx - \frac{4}{\pi}\left[\cos \frac{\pi}{4} - \cos 0^\circ \right]$

= $e - \left(e^x \right)_0^1 - \frac{4}{\pi}\left(\frac{1}{\sqrt{2}} - 1 \right)$

= $e - \left(e - e^0 \right) - \frac{4}{\pi\sqrt{2}} + \frac{4}{\pi}$

= $e - e + 1 - \frac{2.2}{\pi\sqrt{2}} + \frac{4}{\pi}$

= $1 + \frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}$ Ans.

Choose the correct answer in Exercises 21 and 22.

21. $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ **equals:**

(A) $\frac{\pi}{3}$

(B) $\frac{2\pi}{3}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{12}$

Ans. $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$

= $\left(\tan^{-1} x\right)_1^{\sqrt{3}}$

= $\tan^{-1}\sqrt{3} - \tan^{-1} 1$

= $\frac{\pi}{3} - \frac{\pi}{4}$

= $\frac{\pi}{12}$

Therefore, option (D) is correct.

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22. $\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$ **equals:**

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{12}$

(C) $\frac{\pi}{24}$

(D) $\frac{\pi}{4}$

Ans. $\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$

$= \int_0^{\frac{2}{3}} \frac{dx}{(3x)^2+2^2}$

$= \left[\frac{1}{2}\frac{\tan^{-1}\frac{3x}{2}}{3 \to \text{Coeff. of } x \text{ in } 3x}\right]$

$= \frac{1}{6}\left[\tan^{-1}\frac{3x}{2}\right]_0^{\frac{2}{3}}$

$= \frac{1}{6}\left[\tan^{-1}\left(\frac{3}{2}\times\frac{2}{3}\right)-\tan^{-1}0\right]$

$= \frac{1}{6}\left(\tan^{-1}1-\tan^{-1}0\right)$

$= \frac{1}{6}\left(\frac{\pi}{4}-0\right)=\frac{\pi}{24}$

Therefore, option (C) is correct.