CBSE Class–12 Mathematics

NCERT solution

Chapter - 10

Vector Algebra - Exercise 10.2

1. Compute the magnitude of the following vectors:

$\vec{a}=\hat{i}+\hat{j}+\hat{k},\quad \vec{b}=2\hat{i}-7\hat{j}-3\hat{k},\quad \vec{c}=\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}-\frac{1}{\sqrt{3}}\hat{k}$

Ans. Given: $\vec{a}=\hat{i}+\hat{j}+\hat{k}$

$\therefore\ \left|\vec{a}\right|=\sqrt{x^2+y^2+z^2}=\sqrt{(1)^2+(1)^2+(1)^2}=\sqrt{1+1+1}=\sqrt{3}$

And $\vec{b}=2\hat{i}-7\hat{j}-3\hat{k}$

$\therefore\ \left|\vec{b}\right|=\sqrt{x^2+y^2+z^2}=\sqrt{(2)^2+(-7)^2+(-3)^2}=\sqrt{4+49+9}=\sqrt{62}$

Also $\vec{c}=\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}-\frac{1}{\sqrt{3}}\hat{k}$

$\therefore\ \left|\vec{c}\right|=\sqrt{x^2+y^2+z^2}=\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2+\left(\frac{1}{\sqrt{3}}\right)^2+\left(\frac{-1}{\sqrt{3}}\right)^2}=\sqrt{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}=1$

2. Write two different vectors having same magnitude.

Ans. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}-\hat{k}$

Clearly, $\vec{a}\neq\vec{b}$, [$\because$ Coefficients of $\hat{i}$ and $\hat{j}$ are same in vectors $\vec{a}$ and $\vec{b}$ coefficients of $\hat{k}$ in $\vec{a}$ and $\vec{b}$ are unequal as $1\neq-1$.]

But $\left|\vec{a}\right|=\sqrt{x^2+y^2+z^2}=\sqrt{1+1+1}=\sqrt{3}$

And $\left|\vec{b}\right|=\sqrt{x^2+y^2+z^2}=\sqrt{1+1+1}=\sqrt{3}$

Such possible answers are infinite.

3. Write two different vectors having same direction.

Ans. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ = $2\vec{a}$

$\therefore$ $\vec{b} = m\vec{a}$ where m = 2 > 0

$\therefore$ Vectors $\vec{a}$ and $\vec{b}$ have the same direction.

But $\vec{b} \neq \vec{a}\left[\because\ \vec{b} = 2\vec{a} \Rightarrow \left|\vec{b}\right| = |2|\left|\vec{a}\right| = \left|\vec{a}\right|\right]$

Such possible vectors are infinite.

4. Find the values of x and y so that the vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ are equal.

Ans. Given: $2\hat{i} + 3\hat{j} = x\hat{i} + y\hat{j}$

If vectors are equal, then their respective components are equal.

Comparing coefficients of $\hat{i}$ and $\hat{j}$ on both sides, we have,

$x = 2$ and $y = 3$.

5. Find the scalar and vector components of the vector with initial point (2, 1) and terminal point $(-5, 7)$.

Ans. Let $\overrightarrow{AB}$ be the vector with initial point A (2, 1) and terminal point B$(-5, 7)$.

$\Rightarrow$ Position vector of point A is (2, 1) = $2\hat{i} + \hat{j}$ and position vector of point B is $(-5, 7)$ = $-5\hat{i} + 7\hat{j}$.

$\therefore$ $\overrightarrow{AB}$ = Position vector of point B – Position vector of point A

= $\left(-5\hat{i}+7\hat{j}\right)-\left(2\hat{i}+\hat{j}\right)$

= $-5\hat{i}+7\hat{j}-2\hat{i}-\hat{j}$

$\Rightarrow \overrightarrow{AB}$ = $-7\hat{i}+6\hat{j}$

$\therefore$ Scalar components of the vectors $\overrightarrow{AB}$ are coefficients of $\hat{i}$ and $\hat{j}$ in $\overrightarrow{AB}$ i.e., -7 and 6 and vector components of the vector $\overrightarrow{AB}$ are $-7\hat{i}$ and $6\hat{j}$.

6. Find the sum of the vectors: $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$, $\vec{b}=-2\hat{i}+4\hat{j}+5\hat{k}$ and $\vec{c}=\hat{i}-6\hat{j}-7\hat{k}$

Ans. Given: $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$, $\vec{b}=-2\hat{i}+4\hat{j}+5\hat{k}$ and $\vec{c}=\hat{i}-6\hat{j}-7\hat{k}$

Adding, $\vec{a}+\vec{b}+\vec{c}=\hat{i}-2\hat{j}+\hat{k}-2\hat{i}+4\hat{j}+5\hat{k}+\hat{i}-6\hat{j}-7\hat{k}$

= $0\hat{i}-4\hat{j}-\hat{k}=-4\hat{j}-\hat{k}$

7. Find the unit vector in the direction of the vector $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$.

Ans. We know that a unit vector in the direction of the vector $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$ is

$$\hat{a}=\frac{\vec{a}}{\left|\vec{a}\right|}=\frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{(1)^2+(1)^2+(2)^2}}=\frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{1+1+4}}=\frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{6}}$$

$$\Rightarrow \hat{a}=\frac{1}{\sqrt{6}}\hat{i}+\frac{1}{\sqrt{6}}\hat{j}+\frac{2}{\sqrt{6}}\hat{k}$$

8. Find the unit vector in the direction of the vector $\overrightarrow{PQ}$ where P and Q are the points (1, 2, 3) and (4, 5, 6) respectively.

Ans. Given: Points P (1, 2, 3) and Q (4, 5, 6)

$\therefore$ Position vector of point P = $\overrightarrow{OP}$ = $\hat{i}+2\hat{j}+3\hat{k}$ and position vector of Q = $\overrightarrow{OQ}$ = $4\hat{i}+5\hat{j}+6\hat{k}$, where O is the origin.

$\therefore$ $\overrightarrow{PQ}$ = Position vector of Q – Position vector of P = $\overrightarrow{OQ} - \overrightarrow{OP}$

= $4\hat{i}+5\hat{j}+6\hat{k} - \hat{i}-2\hat{j}-3\hat{k}$ = $3\hat{i}+3\hat{j}+3\hat{k}$

Therefore, the unit vector in the direction of vector $\overrightarrow{PQ} = \dfrac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|}$

$$= \frac{3\hat{i}+3\hat{j}+3\hat{k}}{\sqrt{(3)^2+(3)^2+(3)^2}} = \frac{3\hat{i}+3\hat{j}+3\hat{k}}{\sqrt{9+9+9}} = \frac{3\hat{i}+3\hat{j}+3\hat{k}}{\sqrt{27}} = \frac{3\hat{i}+3\hat{j}+3\hat{k}}{3\sqrt{3}}$$

$$= \frac{3\left(\hat{i}+\hat{j}+\hat{k}\right)}{3\sqrt{3}} = \frac{\left(\hat{i}+\hat{j}+\hat{k}\right)}{\sqrt{3}} = \frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}$$

9. For given vectors $\vec{a} = 2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b} = -\hat{i}+\hat{j}-\hat{k}$, find the unit vector in the direction of $\vec{a}+\vec{b}$.

Ans. Given: Vectors $\vec{a} = 2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b} = -\hat{i}+\hat{j}-\hat{k}$

$\therefore$ $\vec{a}+\vec{b} = 2\hat{i}-\hat{j}+2\hat{k}-\hat{i}+\hat{j}-\hat{k}$ = $\hat{i}+0\hat{j}+\hat{k}$

Therefore, $\left|\vec{a}+\vec{b}\right| = \dfrac{\hat{i}+0\hat{j}+\hat{k}}{\sqrt{(1)^2+(0)^2+(1)^2}}$

$$\Rightarrow \left|\vec{a}+\vec{b}\right| = \frac{\hat{i}+0\hat{j}+\hat{k}}{\sqrt{1+0+1}} = \frac{\hat{i}+\hat{k}}{\sqrt{2}} = \frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{k}$$

10. Find the vector in the direction of vector $5\hat{i}-\hat{j}+2\hat{k}$ which has magnitude 8 units.

Ans. Let $\vec{a} = 5\hat{i}-\hat{j}+2\hat{k}$

A vector in the direction of vector $\vec{a}$ which has magnitude 8 units = $8\hat{a}$

$$8.\frac{\vec{a}}{|\vec{a}|} = \frac{8\left(5\hat{i}-\hat{j}+2\hat{k}\right)}{\sqrt{(5)^2+(-1)^2+(2)^2}}$$

$\Rightarrow \quad 8\frac{\vec{a}}{|\vec{a}|} = \frac{8\left(5\hat{i}-\hat{j}+2\hat{k}\right)}{\sqrt{25+1+4}} = \frac{8}{\sqrt{30}}\left(5\hat{i}-\hat{j}+2\hat{k}\right)$

$= \frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$

11. Show that the vectors $2\hat{i}-3\hat{j}+4\hat{k}$ and $-4\hat{i}+6\hat{j}-8\hat{k}$ are collinear.

Ans. Let $\vec{a} = 2\hat{i}-3\hat{j}+4\hat{k}$ and $\vec{b} = -4\hat{i}+6\hat{j}-8\hat{k} = -2\left(2\hat{i}-3\hat{j}+4\hat{k}\right) = -2\vec{a}$

$\Rightarrow \vec{b} = -2\vec{a} = m\vec{a}$ where $m = -2 < 0$

Therefore, vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.

12. Find the direction cosines of the vector $\hat{i}+2\hat{j}+3\hat{k}$.

Ans. The given vector is $\left(\vec{a}\right) = \hat{i}+2\hat{j}+3\hat{k}$

$\therefore \hat{a} = \frac{\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{(1)^2+(2)^2+(3)^2}} = \frac{\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{1+4+9}} = \frac{\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{14}} = \frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}$

We know that the direction cosines of a vector $\vec{a}$ are coefficients of $\hat{i}, \hat{j}, \hat{k}$ in $\hat{a}$ i.e., $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$.

13. Find the direction cosines of the vector joining the points A$(1, 2, -3)$ and B $(-1, -2, 1)$ directed from A to B.

Ans. Given: Points A$(1, 2, -3)$ and B$(-1, -2, 1)$

$\therefore$ Position vector of point A = $\overrightarrow{OA} = \hat{i}+2\hat{j}-3\hat{k}$

And Position vector of point B = $\overrightarrow{OB}$ = $-\hat{i}-2\hat{j}+\hat{k}$

$\therefore$ Vector $\overrightarrow{AB}$ = $\overrightarrow{OB}$ − $\overrightarrow{OA}$ = $-\hat{i}-2\hat{j}+\hat{k}$ − $\hat{i}-2\hat{j}+3\hat{k}$ = $-2\hat{i}-4\hat{j}+4\hat{k}$

Now $\left|\overrightarrow{AB}\right|=\sqrt{(-2)^2+(-4)^2+(4)^2}$ = $\sqrt{4+16+16}=\sqrt{36}=6$

$\therefore$ A unit vector along $\overrightarrow{AB}=\dfrac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|}=\dfrac{-2\hat{i}-4\hat{j}+4\hat{k}}{6}$

= $\dfrac{-2}{6}\hat{i}+\dfrac{-4}{6}\hat{j}+\dfrac{4}{6}\hat{k}=\dfrac{-1}{3}\hat{i}+\dfrac{-2}{3}+\dfrac{2}{3}\hat{k}$

Therefore, the direction cosines of vector $\overrightarrow{AB}$ = $\dfrac{-1}{3},\dfrac{-2}{3},\dfrac{2}{3}$

14. Show that the vector $\hat{i}+\hat{j}+\hat{k}$ is equally inclined to the axes OX, OY and OZ.

Ans. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, then $\left|\vec{a}\right|=\sqrt{(1)^2+(1)^2+(1)^2}=\sqrt{3}$

Let us find angle θ_1 (say) between vector $\vec{a}$ and OX $\left(\Rightarrow \hat{i}\right)$

$$\Rightarrow \cos\theta_1=\frac{\vec{a}.\hat{i}}{\left|\vec{a}\right|.\left|\hat{i}\right|} = \frac{\left(\hat{i}+\hat{j}+\hat{k}\right).\left(\hat{i}+0\hat{j}+0\hat{k}\right)}{\left|\hat{i}+\hat{j}+\hat{k}\right|\left|\hat{i}+0\hat{j}+0\hat{k}\right|} = \frac{1(1)+1(0)+1(0)}{\sqrt{1+1+1}.\sqrt{1+0+0}}=\frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta_1=\cos^{-1}\frac{1}{\sqrt{3}}$$

Similarly angle θ_2 (say) between vector $\vec{a}$ and OY $\left(\Rightarrow \hat{j}\right)$ is $\cos^{-1}\dfrac{1}{\sqrt{3}}$

And angle θ_3 (say) between vector $\vec{a}$ and OZ $\left(\Rightarrow \hat{k}\right)$ is $\cos^{-1}\dfrac{1}{\sqrt{3}}$

$\therefore\ \theta_1 = \theta_2 = \theta_3$

$\therefore$ Vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ is equally inclined to OX, OY and OZ.

15. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively, in the ratio 2 : 1 (i) internally (ii) externally.

Ans. Position vector of P is $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$ and Position vector of Q is $\vec{b} = -\hat{i} + \hat{j} + \hat{k}$

(i) Position vector of point R dividing PQ internally (i.e., R lies within the segment PQ) in the ratio 2 : 1 = $m : n$ = PR : QR is $\dfrac{m\vec{b} + n\vec{a}}{m+n}$

= $\dfrac{2\left(-\hat{i} + \hat{j} + \hat{k}\right) + \hat{i} + 2\hat{j} - \hat{k}}{2+1} = \dfrac{-2\hat{i} + 2\hat{j} + 2\hat{k} + \hat{i} + 2\hat{j} - \hat{k}}{3}$

= $\dfrac{-\hat{i} + 4\hat{j} + \hat{k}}{3} = \dfrac{-1}{3}\hat{i} + \dfrac{4}{3}\hat{j} + \dfrac{1}{3}\hat{k}$

(ii) Position vector of point R dividing PQ externally (i.e., R lies outside the segment PQ and to the right of point Q because ratio 2 : 1 > 1 i.e., PR : QR = 2 : 1) is $\dfrac{m\vec{b} - n\vec{a}}{m-n}$

= $\dfrac{2\left(-\hat{i} + \hat{j} + \hat{k}\right) - \left(\hat{i} + 2\hat{j} - \hat{k}\right)}{2-1} = \dfrac{-2\hat{i} + 2\hat{j} + 2\hat{k} - \hat{i} - 2\hat{j} + \hat{k}}{1}$

= $-3\hat{i} + 3\hat{k}$

16. Find the position vector of the mid-point of the vector joining the points P (2, 3, 4) and Q $(4, 1, -2)$.

Ans. Given: Point P (2, 3, 4) and Q $(4, 1, -2)$

$\therefore$ Position vector of point P is $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

And Position vector of point Q is $\vec{b} = 4\hat{i} + \hat{j} - 2\hat{k}$

And Position vector of mid-point R of PQ is $\frac{\vec{a}+\vec{b}}{2} = \frac{2\hat{i}+3\hat{j}+4\hat{k}+4\hat{i}+\hat{j}-2\hat{k}}{2}$

$= \frac{6\hat{i}+4\hat{j}+2\hat{k}}{2} = 3\hat{i}+2\hat{j}+\hat{k}$

17. Show that the points A, B and C with position vectors $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k},$ respectively form the vertices of a right angled triangle.

Ans. Given: Position vector of point A is $\vec{a} = \overrightarrow{OA} = 3\hat{i} - 4\hat{j} - 4\hat{k}$

Position vector of point B is $\vec{b} = \overrightarrow{OB} = 2\hat{i} - \hat{j} + \hat{k}$ and

Position vector of point C is $\vec{c} = \overrightarrow{OC} = \hat{i} - 3\hat{j} - 5\hat{k}$ where O is the origin.

Now $\overline{AB}$ = Position vector of point B – Position vector of point A

$= 2\hat{i} - \hat{j} + \hat{k} - \left(3\hat{i} - 4\hat{j} - 4\hat{k}\right) = 2\hat{i} - \hat{j} + \hat{k} - 3\hat{i} + 4\hat{j} + 4\hat{k}$

$= -\hat{i} + 3\hat{j} + 5\hat{k}$ ....(i)

$\overline{BC}$ = Position vector of point C – Position vector of point B

$= \hat{i} - 3\hat{j} - 5\hat{k} - \left(2\hat{i} - \hat{j} + \hat{k}\right) = \hat{i} - 3\hat{j} - 5\hat{k} - 2\hat{i} + \hat{j} - \hat{k}$

$= -\hat{i} - 2\hat{j} - 6\hat{k}$ ....(ii)

$\overline{AC}$ = Position vector of point C – Position vector of point A

$= \hat{i} - 3\hat{j} - 5\hat{k} - \left(3\hat{i} - 4\hat{j} - 4\hat{k}\right) = \hat{i} - 3\hat{j} - 5\hat{k} - 3\hat{i} + 4\hat{j} + 4\hat{k}$

= $-2\hat{i}+\hat{j}-\hat{k}$ ....(iii)

Adding eq. (i) and (ii),

$\overrightarrow{AB}$ + $\overrightarrow{BC}$ = $-\hat{i}+3\hat{j}+5\hat{k}$ $-\hat{i}-2\hat{j}-6\hat{k}$ = $-2\hat{i}+\hat{j}-\hat{k}$ = $\overrightarrow{AC}$ [By eq. (iii)]

Now From eq. (i),AB = $\left|\overrightarrow{AB}\right|=\sqrt{1+9+25}=\sqrt{35}$

From eq. (ii),BC = $\left|\overrightarrow{BC}\right|=\sqrt{1+4+36}=\sqrt{41}$

From eq. (iii),AC = $\left|\overrightarrow{AC}\right|=\sqrt{4+1+1}=\sqrt{6}$

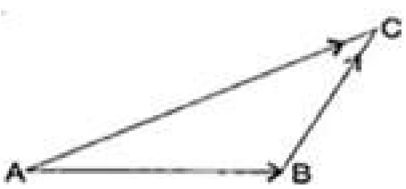
Here, we can observe that (Longest side BC)^2 = $\left(\sqrt{41}\right)^2=41=35+6=$ AB^2 + AC^2

Also, $\overrightarrow{AB}.\overrightarrow{CA}=(+1)(2)+(-3)(-1)+(-5)(1)$ = 2 + 3 - 5 = 0

Thus, $\overrightarrow{AB}\perp\overrightarrow{CA}\Rightarrow\angle A=90^\circ$

Therefore, A B, C are the vertices of a right angled triangle.

18. In triangle ABC (Fig. below), which of the following is not true:



(A) $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}$

(B) $\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{AC}=\vec{0}$

(C) $\overrightarrow{AB}-\overrightarrow{CB}+\overrightarrow{CA}=\vec{0}$

Ans. We know by Triangle law of Addition of vectors that $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$ $\Rightarrow$

$\overrightarrow{AB} + \overrightarrow{BC} = -\overrightarrow{CA}$

$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

Therefore option (C) is not true because in option (C),

$\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \overrightarrow{AC} + \overrightarrow{AC} = 2\overrightarrow{AC} \neq 0$

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19. If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following are incorrect:

(A) $\vec{b} = \lambda\vec{a}$, **for some scalar** λ.

(B) $\vec{a} = \pm\vec{b}$

(C) The respective components of $\vec{a}$ and $\vec{b}$ are proportional.

(D) Both the vectors $\vec{a}$ and $\vec{b}$ have same direction, but different magnitudes.

Ans. Option (D) is not true because two collinear vectors can have different directions and also different magnitudes.

The option (A) and option (C) are true by definition of collinear vectors.

Option (B) is a particular case of option (A) taking $\lambda = \pm 1$.