CBSE Class–12 Mathematics

NCERT solution

Chapter - 13

Probability - Exercise 13.5

1. A die is thrown 6 times. If 'getting an odd number' is a success, what is the probability of:

(i) 5 successes?

(ii) at least 5 successes?

(iii) at most 5 successes?

Ans. We know that the repeated throws of a die are Bernoulli's triangls., Then

S = {1, 2, 3, 4, 5, 6} $\Rightarrow n(S)$ = 6

A = {1, 3, 5} $\Rightarrow n(A)$ = 3

p = P (a success) = $\frac{n(A)}{n(S)}=\frac{3}{6}=\frac{1}{2}$ and $q=1-p=1-\frac{1}{2}=\frac{1}{2}$

n = 6 and P (X = r) = $C(n,r)p^r q^{n-r}$

(i) $r=5$

P (X = 5) = C (6, 5) $\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)$ = $6\times\frac{1}{64}=\frac{3}{32}$

(ii) $r=5,6$

P (at least 5 success) = P (X = 5) + P (X = 6) = $\frac{6}{64}+\left(\frac{1}{2}\right)^6$ = $\frac{6}{64}+\frac{1}{64}=\frac{7}{64}$

(iii) P (at most 5 success) = 1 – P (X = 6) = $1-\frac{1}{64}=\frac{63}{64}$

2. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of two successes.

Ans. $n(\mathrm{S})$ = 36, A = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)} $\Rightarrow n(\mathrm{A})$ = 6

p = P (a success) = $\frac{n(\mathrm{A})}{n(\mathrm{S})} = \frac{6}{36} = \frac{1}{6}$ and $q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$ [q = P(a failure)]

n = 4 and r = 2

P (X = r) = C$(n, r)\, p^r q^{n-r}$

P (X = 2) = C (4, 2) $\left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$ = $6 \times 25 \times \left(\frac{1}{6}\right)^4 = \frac{25}{216}$

3. There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?

Ans. Let p = Probability of a success and q = Probability of a failure

$p = \frac{5}{100} = \frac{1}{20}$ and $q = 1 - p = 1 - \frac{1}{20} = \frac{19}{20}$

n = 10, r = 0, 1

P (not more than one defective items) = P (X = 0) + P (X = 1)

= C (10, 0) $\left(\frac{19}{20}\right)^{10}$ + C (10, 1) $\left(\frac{19}{20}\right)^9 \left(\frac{1}{20}\right)$

= $\left(\frac{19}{20}\right)^9 \left(\frac{19}{20} + \frac{1}{20}\right) = \frac{29}{20}\left(\frac{19}{20}\right)^9$

4. Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that:

(i) all the five cards are spade?

(ii) only 3 cards are spades?

(iii) none is spade?

Ans. Let p = Probability of a success and q = Probability of a failure

S = 52 cards $\Rightarrow n(\mathrm{S})$ = 52 and A = {13 spades} $\Rightarrow n(\mathrm{A})$ = 13

$$p=\frac{n(\mathrm{A})}{n(\mathrm{S})}=\frac{13}{52}=\frac{1}{4} \text{ and } q=1-p=1-\frac{1}{4}=\frac{3}{4}$$

(i) n = 5, r = 5

P (X = r) = ${}^{n}C_{r}\,p^{r}q^{n-r}$

P (X = 5) = ${}^{5}C_{5}\left(\frac{1}{4}\right)^{5}=\frac{1}{1024}$

(ii) n = 5, r = 0

P (X = 3) = ${}^{5}C_{3}\left(\frac{1}{4}\right)^{3}\left(\frac{3}{4}\right)^{2}=\frac{5\times4\times3}{1\times2\times3}\times9\left(\frac{1}{4}\right)^{5}=\frac{90}{1024}=\frac{45}{512}$

(iii) n = 5, r = 0

P (X = 0) = ${}^{5}C_{0}\left(\frac{3}{4}\right)^{5}=\frac{243}{1024}$

5. The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs.

Ans. Let p = Probability of a success and q = Probability of a failure

p = P (a bulb will fuse after 150 days) = 0.05 and q = 1 – 0.05 = 0.96

n = 5 and P (X = r) = $C(n, r)\, p^r q^{n-r}$

(i) No bulb is fused, r = 0

P (X = 0) = C (5, 0), $(0.05)^0 (0.95)^5 = \left(\frac{19}{20}\right)^5 = (0.95)^5$

(ii) Not more than one fused bulb

P (not more than one fused bulb) = P (X = 0) + P (X = 1)

$= \left(\frac{19}{20}\right)^5 + C(5,1)(0.05)(0.95)^4$

$= \left(\frac{19}{20}\right)^5 + 5(0.05)\left(\frac{19}{20}\right)^4$

$= \left(\frac{19}{20}\right)^4 \left(\frac{19}{20} + \frac{5}{20}\right)$

$= \left(\frac{19}{20}\right)^4 \left(\frac{6}{5}\right) = 1.2(0.95)^4$

(iii) P (more than one fused bulb out of 5) = 1 – [P (X = 0) + P (X = 1)]

$= 1 - 1.2(0.95)^4$ [By (ii) part]

(iv) P (at least one fused bulb) = 1 – P (X = 0)

$= 1 - \left(\frac{19}{20}\right)^5 = 1 - (0.95)^5$

6. A bag consists of 10 balls each marked with one of the digits 0 to 9. If four balls are drawn successively with replacement from the bag, what is the probability that none is marked with the digit 0?

Ans. S = {0, 1, 2 3, 4, 5, 6, 7, 8, 9} $\Rightarrow n(S) = 10$

Let A represents that the ball is marked with the digit 0.

$\therefore$ A = {0} $\Rightarrow n(A) = 1$

$$p = \frac{n(A)}{n(S)} = \frac{1}{10} \text{ and } q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$$

$n = 4, r = 0$ and P (X = r) = ${}^{n}C_{r} p^{r} q^{n-r}$

Now P (one is marked with its digit 0) = ${}^{4}C_{0}\left(\frac{1}{10}\right)^{0}\left(\frac{9}{10}\right)^{4} = \frac{6561}{10000}$

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7. In an examination, 20 questions of true-false are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers 'true', if it falls tails, he answers 'false'. Find the probability that he answers at least 12 questions correctly.

Ans. S = {H, T} $\Rightarrow n(S) = 2$

Let A represents a head.

$\therefore$ A = {H} $\Rightarrow n(A) = 1$

$$p = \frac{n(A)}{n(S)} = \frac{1}{2} \text{ and } q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$$

$n = 20, r = 12, 13, \ldots\ldots\ldots 20$ and P (X = r) = ${}^{n}C_{r} p^{r} q^{n-r}$

P (at least 12 success) = P (X = 12) + P (X = 13) + ……… P (X = 20)

$$= {}^{20}C_{12}\left(\frac{1}{2}\right)^{12}\left(\frac{1}{2}\right)^{8} + {}^{20}C_{13}\left(\frac{1}{2}\right)^{13}\left(\frac{1}{2}\right)^{7} + \ldots\ldots\ldots + {}^{20}C_{20}\left(\frac{1}{2}\right)^{20}\left(\frac{1}{2}\right)^{0}$$

= $\left(\frac{1}{2}\right)^{20}\left[{}^{20}C_{12}+{}^{20}C_{13}+{}^{20}C_{14}+{}^{20}C_{15}+{}^{20}C_{16}+{}^{20}C_{17}+{}^{20}C_{18}+{}^{20}C_{19}+{}^{20}C_{20}\right]$

8. Suppose X has a binomial distribution B $\left(6,\frac{1}{2}\right)$. Show that X = 3 is the most likely outcome.

Ans. B $\left(6,\frac{1}{2}\right) \Rightarrow n=6, p=\frac{1}{2}, q=\frac{1}{2}$

x_i = 0, 1, 2, 3, 4, 5, 6

P (X = r) = C$(n,r)\,p^r q^{n-r}$

Since $p=q$

Therefore, P (X = r) = C$(n,r)\,p^n$

Now, out of C (6, 0), C (6, 1), C (6, 2), C (6, 3), C (6, 4), C (6, 5), C (6, 6), here C (6, 3) is maximum.

$\therefore$ P (X = 3) = C (6, 3) $\left(\frac{1}{2}\right)^6$ = $\frac{6\times5\times4}{1\times2\times3}\left(\frac{1}{2}\right)^6$ = $20\times\frac{1}{64}=\frac{5}{16}$

Therefore, P (X = 3) is maximum, i.e., $\frac{5}{16}$

9. On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

Ans. $p=\frac{1}{3}$ and $q=1-p=1-\frac{1}{3}=\frac{2}{3}$

$n=5, r=4,5$ and P (X = r) = ${}^nC_r\,p^r q^{n-r}$

P (Four or more success) = P (X = 4) + P (X = 5)

= ${}^{5}C_{4}\left(\frac{1}{3}\right)^{4}\left(\frac{2}{3}\right)^{1}+{}^{5}C_{5}\left(\frac{1}{3}\right)^{5}$ = $5 \times 2 \times \frac{1}{3}+\left(\frac{1}{3}\right)^{4}$ = $\frac{11}{243}$

10. A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize:

(a) at least once

(b) exactly once

(c) at least twice?

Ans. $p=\frac{1}{100}$ and $q=1-p=1-\frac{1}{100}=\frac{99}{100}$

$n=50$

(a) P (at least one prize) = 1 – P (X = 0) = $1-\left(\frac{99}{100}\right)^{50}$

(b) P (exactly one prize) = P (X = 1) = ${}^{50}C_{1}\left(\frac{1}{100}\right)\left(\frac{99}{100}\right)^{49}=50\times\frac{1}{100}\left(\frac{99}{100}\right)^{49}=\frac{1}{2}\left(\frac{99}{100}\right)^{49}$

(c) P (at least twice) = 1 – [P (X = 0) + P (X = 1)] = $1-\left[\left(\frac{99}{100}\right)^{50}+\frac{1}{2}\left(\frac{99}{100}\right)^{49}\right]$

= $1-\left(\frac{99}{100}\right)^{49}\left(\frac{99}{100}+\frac{1}{2}\right)=1-\left(\frac{99}{100}\right)^{49}\left(\frac{149}{100}\right)$

11. Find the probability of getting 5 exactly twice in 7 throws of a die.

Ans. S = {1, 2, 3, 4, 5, 6} $\Rightarrow n(S)$ = 6

A = {5} $\Rightarrow$ $n(\mathrm{A})$ = 1

$p = \frac{n(\mathrm{A})}{n(\mathrm{S})} = \frac{1}{6}$ and $q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$

$n = 7, r = 2$ and P (X = r) = ${}^{n}C_{r} p^{r} q^{n-r}$

P (X = 2) = ${}^{7}C_{2}\left(\frac{1}{6}\right)^{2}\left(\frac{5}{6}\right)^{5} = \frac{7\times 6}{1\times 2}\left(\frac{1}{6}\right)^{7}(5)^{5} = \frac{7}{12}\left(\frac{5}{6}\right)^{5}$

12. Find the probability of throwing at most 2 sixes in 6 throws of a single die.

Ans. S = {1, 2, 3, 4, 5, 6} $\Rightarrow$ $n(\mathrm{S})$ = 6

Let A represents the favourable event i.e., 6

A = {6} $\Rightarrow$ $n(\mathrm{A})$ = 1

$p = \frac{n(\mathrm{A})}{n(\mathrm{S})} = \frac{1}{6}$ and $q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$

$n = 6, r = 0, 1, 2$ and P (X = r) = ${}^{n}C_{r} p^{r} q^{n-r}$

P (X = 0) = $\left(\frac{5}{6}\right)^{6}$

P (X = 1) = ${}^{6}C_{1}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{5} = \left(\frac{5}{6}\right)^{5}$

P (X = 2) = ${}^{6}C_{2}\left(\frac{1}{6}\right)^{2}\left(\frac{5}{6}\right)^{4} = 15\left(\frac{1}{6}\right)^{2}\left(\frac{5}{6}\right)^{4}$

P (at most 2 success) = P (X = 0) + P (X = 1) + P (X = 2)

$= \left(\frac{5}{6}\right)^6 + \left(\frac{5}{6}\right)^5 + 15\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^4$

$= \left(\frac{5}{6}\right)^4\left(\frac{25}{36}+\frac{5}{6}+\frac{15}{36}\right) = \left(\frac{5}{6}\right)^4\left(\frac{70}{36}\right) = \left(\frac{5}{6}\right)^4\left(\frac{35}{18}\right)$

13. It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample of 12 such articles 9 are defective?

Ans. $p = \frac{10}{100} = \frac{1}{10}$ and $q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$

$n = 12, r = 9$ and P (X = r) = ${}^nC_r p^r q^{n-r}$

P (X = 9) = ${}^{12}C_9\left(\frac{1}{10}\right)^9\left(\frac{9}{10}\right)^3 = \frac{12\times11\times10}{1\times2\times3}\left(\frac{729}{10^{12}}\right)$ = $\frac{220\times729}{(10)^{12}}$

In each of the following, choose the correct answer:

14. Binomial distribution is given this name because:

(A) This distribution was evolved by James binomial.

(B) Each trial has only two outcomes. Namely success and failure.

(C) Its probability function is obtained by general of binomial expansion.

(D) It is obtained by combining two distributions.

Ans. $p = \frac{10}{100} = \frac{1}{10}$ and $q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$

$n = 5, r = 0$

P (X = 0) = $\left(\frac{9}{10}\right)^5$

Therefore, option (C) is correct.

15. In a box containing 100 bulbs, 10 are defective. The probability that out of a sample of 5 bulbs, none is defective is:

(A) 10^{-1}

(B) $\left(\frac{1}{2}\right)^5$

(C) $\left(\frac{9}{10}\right)^5$

(D) $\frac{9}{10}$

Ans. $p=\frac{4}{5}$ and $q=1-p=1-\frac{4}{5}=\frac{1}{5}$

$n=5, r=4$

P (X = 4) = ${}^5C_4\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)$

Therefore, option (A) is correct.