

CBSE Class-12 Mathematics

NCERT solution

Chapter - 13

Probability - Exercise 13.4

1. State which of the following are not the probability distributions of a random variable. Give reasons for your answer.

(i)

X	0	1	2
P (X)	0.4	0.4	0.2

(ii)

X	0	1	2	3	4
P (X)	0.1	0.5	0.2	- 0.1	0.3

(iii)

Y	- 1	0	1
P (Y)	0.6	0.1	0.2

(iv)

Z	3	2	1	0	- 1
P (Z)	0.3	0.2	0.4	0.1	0.05

Ans. (i) $P(0) + P(1) + P(2) = 0.4 + 0.4 + 0.2 = 1$

Therefore, it is a probability distribution of a random variable.

(ii) $P(3) = -0.1$ which is not possible.

Therefore, it is not a probability distribution.

(iii) $P(-1) + P(0) + P(1) = 0.6 + 0.1 + 0.2 = 0.9 \neq 1$

Therefore, it is not a probability distribution.

$$(iv) P(3) + P(2) + P(1) + P(0) + P(-1) = 0.3 + 0.2 + 0.4 + 0.1 + 0.05 = 1.05 \neq 1$$

Therefore, it is not a probability distribution.

2. An urn contains 5 red and 2 black balls. Two balls are randomly selected. Let X represents the number of black balls. What are the possible values of X ? Is X a random variable?

Ans. There two balls may be selected as BR, RB, BR, BB, where R represents red ball and B represents black ball.

Variable X has the value 0, 1, 2, i.e., there may be no black ball, may be one black ball or both the balls are black.

Since, X is a number whose values are defined on the outcomes of a random experiment, therefore, X is a random variable.

3. Let X represents the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X ?

Ans. Let h denotes the number of heads and t denotes the number of tails, when a coin is tossed 6 times. Then,

$$X = \text{difference between } h \text{ and } t = |h - t|$$

Now, $h : 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

Therefore, $t : 6 \quad 5 \quad 4 \quad 3 \quad 2 \quad 1 \quad 0$

And, hence $X : 6 \quad 4 \quad 2 \quad 0 \quad 2 \quad 4 \quad 6$

Therefore, the possible values of X are 6, 4, 2, 0.

4. Find the probability distribution of:

(i) Number of heads in two tosses of a coin.

(ii) Number of tails in the simultaneous tosses of three coins.

(iii) Number of heads in four tosses of a coin.

Ans. (i) The sample space of the random experiment 'a coin is tossed twice' is $S = \{HH, HT, TH, TT\} \Rightarrow n(S) = 4$

Let X denotes the random variable 'number of heads', then A can take the values 0, 1 or 2.

$$P(X = 0) = P(\bar{A})P(\bar{A}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$P(X = 1) = 2 P(A) P(\bar{A}) = 2 \times \frac{1}{2} \times \frac{1}{2} = \frac{2}{4} = \frac{1}{2}$$

$$P(X = 2) = P(A). P(A) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Probability distribution

X_i	0	1	2
P_i	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

(ii) Three coins tossed once = one coin tossed three times

$\therefore S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\} \Rightarrow n(S) = 8$

Let X denotes the random variable 'number of heads', then A can take the values 0, 1, 2 or 3.

$$P(X = 0) = P(\bar{A})P(\bar{A})P(\bar{A}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$P(X = 1) = 3 P(A) P(\bar{A})P(\bar{A}) = 3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$$

$$P(X = 2) = 3 P(A). P(A). P(\bar{A}) = 3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$$

$$P(X = 3) = P(A) \cdot P(A) \cdot P(A) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

Probability distribution

X_i	0	1	2	3
P_i	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(iii) A coin is tossed four times = $S = \{HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, HTTH, THTT, THTH, TTHH, HTTT, THTT, TTHT, TTTH, TTTT\} \Rightarrow n(S) = 16$

Let X denotes the random variable 'number of heads', then X can take the values 0, 1, 2, 3 or 4.

$$P(X = 0) = P(\bar{A})P(\bar{A})P(\bar{A})P(\bar{A}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$$

$$P(X = 1) = 4 P(A) P(\bar{A})P(\bar{A})P(\bar{A}) = 4 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{4}{16} = \frac{1}{4}$$

$$P(X = 2) = 6 P(A) \cdot P(A) \cdot P(\bar{A})P(\bar{A}) = 6 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{6}{16} = \frac{3}{8}$$

$$P(X = 3) = 4 P(A) \cdot P(A) \cdot P(A) \cdot P(\bar{A}) = 4 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{4}{16} = \frac{1}{4}$$

$$P(X = 4) = P(A) \cdot P(A) \cdot P(A) \cdot P(A) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$$

Probability distribution

X_i	0	1	2	3	4
P_i	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

5. Find the probability distribution of the number of success in two tosses of a die where a success is defined as:

(i) number greater than 4.

(ii) six appears on at least one die.

Ans. $S = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6$

(i) Let A be the set of favourable events. $\Rightarrow n(A) = 1$

$$\therefore P(A) = \frac{n(S)}{n(A)} = \frac{2}{6} = \frac{1}{3} \text{ and } P(\bar{A}) = 1 - \frac{1}{3} = \frac{2}{3}$$

$$n = 2, r = 0, 1, 2$$

$$P(X = 0) = P(\bar{A})P(\bar{A}) = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$$

$$P(X = 1) = 2 P(A) P(\bar{A}) = 2 \times \frac{1}{3} \times \frac{2}{3} = \frac{4}{9}$$

$$P(X = 2) = P(A). P(A) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

Probability distribution

X	0	1	2
P(X)	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

(ii) Let A represents that 6 appears on one die $\Rightarrow A = \{6\} \Rightarrow n(A) = 1$

$$\therefore P(A) = \frac{n(S)}{n(A)} = \frac{1}{6} \text{ and } P(\bar{A}) = 1 - \frac{1}{6} = \frac{5}{6}$$

Let s denotes success and f denotes the failure. Then

$P(s) = P(6 \text{ appears on at least one die})$

$= P(6 \text{ appears on one die or } 6 \text{ appears on both dice})$

= P (6 appears on first dice and does not appear on second dice) + P (6 does not appear on first dice and 6 appears on second dice) + P (6 appears on both dice)

$$= \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} = \frac{11}{36}$$

Let X denotes the numebr of success in two tosses of a dice, then X can take value 0 or 1.

$$P(X = 0) = P(\text{no success}) = P(f) = P(\bar{A})P(\bar{A}) = \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$$

$$P(X = 1) = P(\text{one success}) = P(s) = \frac{11}{36}$$

Probability distribution

X	0	1
P(X)	$\frac{25}{36}$	$\frac{11}{36}$

6. From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

Ans. $n(S) = 30$, $A = \{6 \text{ defective bulbs}\} \Rightarrow n(A) = 6$

$$p = \frac{n(A)}{n(S)} = \frac{6}{30} = \frac{1}{5} \text{ and } q = 1 - \frac{1}{5} = \frac{4}{5}$$

$n = 4$ (4 bulbs are drawn with replacement), $r = 0, 1, 2, 3, 4$

$$P(X = 0) = (q)^4 = \left(\frac{4}{5}\right)^4 = \frac{256}{625}$$

$$P(X = 1) = {}^4C_1 (p) q^3 = 4 \times \frac{1}{5} \times \left(\frac{4}{5}\right)^3 = \frac{256}{625}$$

$$P(X = 2) = {}^4C_2 p^2 q^2 = 6 \times \left(\frac{1}{5}\right)^2 \times \left(\frac{4}{5}\right)^2 = \frac{96}{625}$$

$$P(X = 3) = {}^4C_3 p^3 q = 4 \times \left(\frac{1}{5}\right)^3 \times \left(\frac{4}{5}\right) = \frac{16}{625}$$

$$P(X = 4) = {}^4C_4 p^4 = \left(\frac{1}{5}\right)^4 = \frac{1}{625}$$

Probability distribution:

X_i	0	1	2	3	4
$P(X_i)$	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

7. A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.

Ans. Let p represents the appearance of tail and q represents the appearance of head.

Now $q = 3p$

Since $p + q = 1 \Rightarrow p + 3p = 1 \Rightarrow p = \frac{1}{4}$ and $q = 1 - \frac{1}{4} = \frac{3}{4}$

$$P(X = 0) = {}^2C_0 (q)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

$$P(X = 1) = {}^2C_1 q.p = 2 \times \frac{3}{4} \times \frac{1}{4} = \frac{6}{16}$$

$$P(X = 2) = {}^2C_2 p^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

Probability distribution:

x_i	0	1	2
p_i	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

8. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P (X)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$2k^2 + k$

Determine:

(i) k

(ii) $P(X < 3)$

(iii) $P(X > 6)$

(iv) $P(0 < X < 3)$

Ans. (i) Since, the sum of all the probabilities of a distribution is 1.

$$\therefore P(X = 0) + P(X = 1) + \dots + P(X = 7) = 1$$

$$\Rightarrow 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$

$$\Rightarrow 10k^2 + 9k - 1 = 0$$

$$\Rightarrow (10k - 1)(k + 1) = 0$$

$$\Rightarrow 10k - 1 = 0 \text{ or } k + 1 = 0$$

$$\Rightarrow k = \frac{1}{10} \text{ or } k = -1$$

Since, $k \geq 0$, therefore $k = -1$ is not possible.

$$\therefore k = \frac{1}{10}$$

(ii) $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$$= 0 + k + 2k$$

$$= 3k = 3 \times \frac{1}{10} = \frac{3}{10}$$

(iii) $P(X > 6) = P(X = 7)$

$$= 7k^2 + k = 7\left(\frac{1}{10}\right)^2 + \frac{1}{10} = \frac{17}{100}$$

(iv) $P(0 < X < 3)$

$$= P(X = 1) + P(X = 2)$$

$$= k + 2k = 3k = \frac{3}{10}$$

9. The random variable X has a probability distribution $P(x)$ of the following form, where k is some number:

$$P(x) = \begin{cases} k & \text{if } x = 0 \\ 2k & \text{if } x = 1 \\ 3k & \text{if } x = 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine the value of k .

(b) Find $P(x < 2)$, $P(x \leq 2)$, $P(x \geq 2)$

Ans. Probability distribution:

x_i	0	1	2
$P(x_i)$	k	$2k$	$3k$

(a) $P(X = 0) + P(X = 1) + P(X = 2) = 1$

$$\Rightarrow k + 2k + 3k = 1 \Rightarrow 6k = 1$$

$$\Rightarrow k = \frac{1}{6}$$

(b) $P(X < 2) = P(X = 0) + P(X = 1)$

$$= k + 2k = 3k = 3 \times \frac{1}{6} = \frac{1}{2}$$

$$P(X \geq 2) = P(X = 2) = 3k = 3 \times \frac{1}{6} = \frac{1}{2}$$

10. Find the mean number of heads in three tosses of fair coin.

Ans. $n(S) = 8$

Let A denotes the appearance of head on a toss.

$$A = \{h\} \Rightarrow n(A) = 1$$

$$\therefore p = \frac{n(A)}{n(S)} = \frac{1}{2} \text{ and } q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$$

$$n = 3, r = 0, 1, 2, 3$$

$$P(X = 0) = q^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$$P(X = 1) = 3q^2p = 3\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) = \frac{3}{8}$$

$$P(X = 2) = 3qp^2 = 3\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^2 = \frac{3}{8}$$

$$P(X = 3) = p^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

Probability distribution:

X_i	0	1	2	3
P_i	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$$\text{Mean} = \sum p_i x_i = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8} = \frac{12}{8} = \frac{3}{2}$$

11. Two dice are thrown simultaneously. If X denotes the number of sixes, find expectation of X.

Ans. Two dice thrown simultaneously is the same the die thrown 2 times.

$$\text{Let } S = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6 \times 6 = 36$$

$$\text{Let A denotes the number 6} \Rightarrow A = \{6\} \Rightarrow n(A) = 1$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{6} \text{ and } P(\bar{A}) = 1 - \frac{1}{6} = \frac{5}{6}$$

$$X = 0, 1, 2$$

$$P(X = 0) = P(\bar{A}) \cdot P(\bar{A}) = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$$

$$P(X = 1) = 2 \cdot P(A) \cdot P(\bar{A}) = 2 \times \frac{1}{6} \times \frac{5}{6} = \frac{10}{36}$$

$$P(X = 2) = P(A) \cdot P(A) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

$$E(X) = \sum_{i=1}^2 x_i p(x_i) = 0 \times \frac{25}{36} + 1 \times \frac{10}{36} + 2 \times \frac{1}{36} = \frac{12}{36} = \frac{1}{3}$$

12. Two numbers are selected at random (without replacement), from the first six positive integers. Let X denotes the larger of two numbers obtained. Find E (X).

Ans. $S = \{(1, 2), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1),$

$(1, 3), (2, 3), (3, 2), (4, 2), (5, 2), (6, 2),$

$(1, 4), (2, 4), (3, 4), (4, 3), (5, 3), (6, 3),$

$(1, 5), (2, 5), (3, 5), (4, 5), (5, 4), (6, 4),$

$(1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 6)\}$

$$n(S) = 30$$

Let X denotes the larger of the two numbers obtained.

x_i	f_i	p_i	$p_i x_i$
2	2	$\frac{2}{30}$	$\frac{4}{30}$
3	4	$\frac{4}{30}$	$\frac{12}{30}$
4	6	$\frac{6}{30}$	$\frac{24}{30}$
5	8	$\frac{8}{30}$	$\frac{40}{30}$
6	10	$\frac{10}{30}$	$\frac{60}{30}$
	30		$\sum p_i x_i = \frac{140}{30}$

$$E(X) = \sum p_i x_i = \frac{140}{30} = \frac{14}{3} = 4\frac{2}{3}$$

13. Let X denotes the sum of the numbers obtained when two fair dice are rolled. Find the variance and standard deviation of X.

Ans. $S = \{(1, 1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1),$

$(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2),$

$(1, 3), (2, 3), (3, 3), (4, 3), (5, 3), (6, 3),$

$(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4),$

$(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5),$

$(1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 6)\}$

$$n(S) = 36$$

Let A denotes the sum of the numbers = 2, B denotes the sum of the numbers = 3

C denotes the sum of the numbers = 4, D denotes the sum of the numbers = 5

E denotes the sum of the numbers = 6, F denotes the sum of the numbers = 7

G denotes the sum of the numbers = 8, H denotes the sum of the numbers = 9

I denotes the sum of the numbers = 10, J denotes the sum of the numbers = 11

K denotes the sum of the numbers = 12

$$A = \{1, 1\}, n(A) = 1, P(A) = \frac{1}{36}$$

$$B = \{(1, 2), (2, 1)\}, n(B) = 2, P(A) = \frac{2}{36}$$

$$C = \{(1, 3), (2, 2), (3, 1)\}, n(C) = 3, P(A) = \frac{3}{36}$$

$$D = \{(1, 4), (2, 3), (3, 2), (4, 1)\}, n(D) = 4, P(A) = \frac{4}{36}$$

$$E = \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\},$$

$$n(E) = 5, P(A) = \frac{5}{36}$$

$$F = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\},$$

$$n(F) = 6, P(A) = \frac{6}{36}$$

$$G = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\},$$

$$n(G) = 5, P(A) = \frac{5}{36}$$

$$H = \{(3, 6), (4, 5), (5, 4), (6, 3)\},$$

$$n(H) = 4, P(A) = \frac{4}{36}$$

$$I = \{(4, 6), (5, 5), (6, 4)\},$$

$$n(I) = 3, P(A) = \frac{3}{36}$$

$$J = \{(5, 6), (6, 5)\},$$

$$n(J) = 2, P(A) = \frac{2}{36}$$

$$K = \{6, 6\}, n(K) = 1, P(A) = \frac{1}{36}$$

x_i	$P(x_i)$	x_i	$P(x_i)$	x_i	$P(x_i)$
2	$\frac{1}{36}$	6	$\frac{5}{36}$	10	$\frac{3}{36}$
3	$\frac{2}{36}$	7	$\frac{6}{36}$	11	$\frac{2}{36}$
4	$\frac{3}{36}$	8	$\frac{5}{36}$	12	$\frac{1}{36}$
5	$\frac{4}{36}$	9	$\frac{4}{36}$		

$$\text{Mean} = \mu = \sum p_i x_i$$

$$= \frac{1}{36} \times 1 + \frac{2}{36} \times 3 + \frac{3}{36} \times 4 + \frac{4}{36} \times 5 + \frac{5}{36} \times 6 + \frac{6}{36} \times 7 + \frac{5}{36} \times 8 + \frac{4}{36} \times 9 + \frac{3}{36} \times 10 + \frac{2}{36} \times 11 + \frac{1}{36} \times 12$$

$$= \frac{1}{36} (2 + 6 + 12 + 20 + 30 + 42 + 40 + 36 + 30 + 22 + 12)$$

$$= \frac{252}{36} = 7$$

$$\text{Now} \quad \sum p_i x_i^2 =$$

$$\frac{1}{36} (2)^2 + \frac{2}{36} (3)^2 + \frac{3}{36} (4)^2 + \frac{4}{36} (5)^2 + \frac{5}{36} (6)^2 + \frac{6}{36} (7)^2 + \frac{5}{36} (8)^2 + \frac{4}{36} (9)^2$$

$$\begin{aligned}
 & + \frac{3}{36}(10)^2 + \frac{2}{36}(11)^2 + \frac{1}{36}(12)^2 \\
 & = \frac{1}{36}(4+18+48+100+180+294+320+324+300+242+144) \\
 & = \frac{1}{36} \times 1974 = \frac{329}{6}
 \end{aligned}$$

$$\text{Variance} = \sum p_i x_i^2 - \left(\sum p_i x_i \right)^2 = \frac{329}{6} - (7)^2 = 54.83 - 49 = 5.83$$

$$\text{Standard deviation} = \sqrt{5.83} = 2.4 \text{ (nearly)}$$

14. A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find mean, variance and standard deviation of X.

$$\text{Ans. } n(S) = 15, P(A) = \frac{1}{15}$$

x_i	f_i	p_i	$p_i x_i$	$p_i x_i^2$
14	2	$\frac{2}{15}$	$\frac{28}{15}$	$\frac{392}{15}$
15	1	$\frac{1}{15}$	$\frac{15}{15}$	$\frac{225}{15}$
16	2	$\frac{2}{15}$	$\frac{32}{15}$	$\frac{512}{15}$
17	3	$\frac{3}{15}$	$\frac{51}{15}$	$\frac{867}{15}$
18	1	$\frac{1}{15}$	$\frac{18}{15}$	$\frac{324}{15}$

19	2	$\frac{2}{15}$	$\frac{38}{15}$	$\frac{722}{15}$
20	3	$\frac{3}{15}$	$\frac{60}{15}$	$\frac{1200}{15}$
21	1	$\frac{1}{15}$	$\frac{21}{15}$	$\frac{441}{15}$
	15		$\sum p_i x_i = \frac{263}{15}$	$\sum p_i x_i^2 = \frac{4683}{15}$

$$\text{Mean} = \sum p_i x_i = \frac{263}{15} = 17.53$$

$$\text{Variance} = \sum p_i x_i^2 - \left(\sum p_i x_i \right)^2 = \frac{4683}{15} - \left(\frac{263}{15} \right)^2 = 312.20 - (17.53)^2$$

$$= 312.20 - 307.42 = 4.78$$

$$\text{Standard deviation} = \sqrt{4.78} = 2.19$$

15. In a meeting 70% of the members favour a certain proposal, 30% being opposed. A member is selected at random and we let $X = 0$ if the opposed and $X = 1$ if he is in favour. Find $E(X)$ and $\text{Var}(X)$.

Ans.

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	$\frac{30}{100}$	0	0
1	$\frac{70}{100}$	$\frac{70}{100}$	$\frac{70}{100}$
		$\sum p_i x_i = \frac{70}{100}$	$\sum p_i x_i^2 = \frac{70}{100}$

$$E(X) = \text{Mean} = \sum p_i x_i = \frac{70}{100} = 0.7$$

$$\text{Variance}(X) = \sum p_i x_i^2 - \left(\sum p_i x_i \right)^2 = \frac{70}{100} - \left(\frac{70}{100} \right)^2 = \frac{7}{10} - \frac{49}{100} = \frac{21}{100} = 0.21$$

Choose the correct answer in each of the following:

16. The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is:

(A) 1

(B) 2

(C) 5

(D) $\frac{8}{3}$

Ans.

x_i	p_i	$p_i x_i$
1	$\frac{3}{6}$	$\frac{3}{6}$
2	$\frac{2}{6}$	$\frac{4}{6}$
5	$\frac{1}{6}$	$\frac{5}{6}$
		$\sum p_i x_i = \frac{12}{6} = 2$

Therefore, option (B) is correct.

17. Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. What is the value of E(X)?

(A) $\frac{37}{221}$

(B) $\frac{5}{13}$

(C) $\frac{1}{13}$

(D) $\frac{2}{13}$

Ans. $n(S) = 52, n(A) = 4$

$$P(X = 0) = \frac{{}^{48}C_2}{{}^{52}C_2} = \frac{48 \times 47}{52 \times 51} = \frac{188}{221}$$

x_i	p_i	$p_i x_i$
0	$\frac{188}{221}$	0
1	$\frac{32}{221}$	$\frac{32}{221}$
2	$\frac{1}{221}$	$\frac{2}{221}$
		$\sum p_i x_i = \frac{34}{221} = \frac{2}{13}$

$$P(X = 1) = \frac{{}^{48}C_2 \times {}^4C_1}{{}^{52}C_2} = \frac{2 \times 48 \times 4}{52 \times 51} = \frac{32}{221}$$

$$P(X = 2) = \frac{{}^4C_2}{{}^{52}C_2} = \frac{4 \times 3}{52 \times 51} = \frac{1}{221}$$

Now $E(X) = \frac{2}{13}$

Therefore, option (D) is correct.