CBSE Class–12 Mathematics

NCERT solution

Chapter - 9

Differential Equations - Exercise 9.4

For each of the differential equations in Questions 1 to 4, find the general solution:

1. $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

Ans. Given: Differential equation $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

$$\Rightarrow \; dy = \frac{1-\cos x}{1+\cos x}\; dx$$

Integrating both sides, $\int dy = \int \frac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}}\; dx$

$$\Rightarrow \; y = \int \tan^2 \frac{x}{2}\; dx$$

$$\Rightarrow \; \int\left(\sec^2 \frac{x}{2} - 1\right) dx$$

$$\Rightarrow \; \frac{\tan \frac{x}{2}}{\frac{1}{2}} - x + c$$

$$\Rightarrow \; y = 2\tan \frac{x}{2} - x + c$$

2. $\frac{dy}{dx}=\sqrt{4-y^2}\left(-2<y<2\right)$

Ans. Given: Differential equation $\frac{dy}{dx}=\sqrt{4-y^2}$

$\Rightarrow dy=\sqrt{4-y^2}\ dx$

$\Rightarrow \frac{dy}{\sqrt{4-y^2}}=dx$

Integrating both sides,

=> $\int\frac{dy}{\sqrt{4-y^2}}\ dy=\int 1\ dx$

$\Rightarrow \sin^{-1}\frac{y}{2}=x+c\ \left[\because \int\frac{1}{\sqrt{a^2-x^2}}\ dx=\sin^{-1}\frac{x}{a}\right]$

$\Rightarrow \frac{y}{2}=\sin\left(x+c\right)$

$\Rightarrow y=2\sin\left(x+c\right)$

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3. $\frac{dy}{dx}+y=1\left(y\neq 1\right)$

Ans. Given: Differential equation $\frac{dy}{dx}+y=1$

$\Rightarrow \frac{dy}{dx}=1-y$

$\Rightarrow dy=\left(1-y\right)\ dx$

$\Rightarrow dy=-\left(y-1\right)\ dx$

$$\Rightarrow \frac{dy}{y-1} = -\,dx$$

Integrating both sides,

$$=> \int \frac{dy}{y-1}\ dy = -\int 1\ dx$$

$$\Rightarrow \log|y-1| = -x + c$$

$$\Rightarrow |y-1| = e^{-x+c} \quad \left[\because \text{ if } \log x = t, \text{then } x = e^{t}\right]$$

$$\Rightarrow y - 1 = \pm e^{-x+c}$$

$$\Rightarrow y = 1 \pm e^{-x} e^{c}$$

$$\Rightarrow y = 1 \pm e^{c} e^{-x}$$

$$\Rightarrow y = 1 + \mathrm{A}e^{-x}, \text{ where } \mathrm{A} = \pm e^{c}$$

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4. $\sec^2 x \tan y\ dx + \sec^2 y \tan x\ dy = 0$

Ans. Given: Differential equation $\sec^2 x \tan y\ dx + \sec^2 y \tan x\ dy = 0$

Dividing by $\tan x \tan y,$ we have

$$\frac{\sec^2 x}{\tan x} dx + \frac{\sec^2 y}{\tan y} dy = 0$$

$$\Rightarrow \log|\tan x| + \log|\tan y| = \log c \quad \left[\because \int \frac{f'(x)}{f(x)} dx = \log|f(x)|\right]$$

$$\Rightarrow \log|\tan x \tan y| = \log c$$

$$\Rightarrow |\tan x \tan y| = c$$

$\Rightarrow \tan x \tan y = \pm c = C \quad \left[\because |t| = a(a \geq 0) \Rightarrow t = \pm a\right]$

5. $\left(e^x + e^{-x}\right) dy - \left(e^x - e^{-x}\right) dx = 0$

Ans. Given: Differential equation $\left(e^x + e^{-x}\right) dy - \left(e^x - e^{-x}\right) dx = 0$

$\Rightarrow \left(e^x + e^{-x}\right) dy = \left(e^x - e^{-x}\right) dx$

$\Rightarrow dy = \left(\dfrac{e^x - e^{-x}}{e^x + e^{-x}}\right) dx$

Integrating both sides,

=> $\int dy = \int \left(\dfrac{e^x - e^{-x}}{e^x + e^{-x}}\right) dx$

$\Rightarrow y = \log\left|e^x + e^{-x}\right| + c \quad \left[\because \int \dfrac{f'(x)}{f(x)} dx = \log|f(x)|\right]$

6. $\dfrac{dy}{dx} = \left(1 + x^2\right)\left(1 + y^2\right)$

Ans. Given: Differential equation $\dfrac{dy}{dx} = \left(1 + x^2\right)\left(1 + y^2\right)$

$\Rightarrow dy = \left(1 + x^2\right)\left(1 + y^2\right) dx$

$\Rightarrow \dfrac{dy}{1 + y^2} = \left(1 + x^2\right) dx$ [Separating variables]

Integrating both sides,

=> $\int \dfrac{1}{y^2 + 1} dy = \int \left(x^2 + 1\right) dx$

$\Rightarrow \tan^{-1} y = \frac{x^3}{3} + x + c$

7. $y \log y\ dx - x\ dy = 0$

Ans. Given: Differential equation $y \log y\ dx - x\ dy = 0$

$\Rightarrow -x\ dy = -y \log y\ dx$

$\Rightarrow \frac{dy}{y \log y} = \frac{dx}{x}$ [Separating variables]

Integrating both sides,

=> $\int \frac{dy}{y \log y} = \int \frac{dx}{x}$

Putting $\log y = t$ on L.H.S., we get

$\frac{1}{y} = \frac{dt}{dy}$

$\Rightarrow \frac{dy}{y} = dt$

Now eq. (i) becomes

$\int \frac{dt}{t} = \int \frac{dx}{x}$

$\Rightarrow \log |t| = \log |x| + \log |c| = \log |xc|$

[If all the terms in the solution of a differential equation involve log, it is better to use $\log c$ or $\log |c|$ instead of c in the solution.]

$\Rightarrow |t| = |xc|$

$\Rightarrow t = \pm xc$

$\Rightarrow \log y = \pm xc = ax$ where $a = \pm c$

$\Rightarrow y = e^{ax}$

8. $x^5 \frac{dy}{dx} = -y^5$

Ans. Given: Differential equation $x^5 \frac{dy}{dx} = -y^5$

$\Rightarrow x^5\ dy = -y^5\ dx$

$\Rightarrow \frac{dy}{y^5} = -\frac{dx}{x^5}$ [Separating variables]

$\Rightarrow y^{-5}\ dy = -x^{-5}\ dx$

Integrating both sides

=> $\int y^{-5}\ dy = -\int x^{-5}\ dx$

$\frac{y^{-4}}{-4} = -\frac{x^{-4}}{-4} + c$

$\Rightarrow y^{-4} = -x^{-4} - 4c$

$\Rightarrow x^{-4} + y^{-4} = -4c$

$\Rightarrow y^{-4} = x^{-4} + y^{-4} = C$, where $C = -4c$

9. $\frac{dy}{dx} = \sin^{-1} x$

Ans. Given: Differential equation $\frac{dy}{dx} = \sin^{-1} x$

$\Rightarrow dy = \sin^{-1} x\ dx$

Integrating both sides, $\int 1\, dy = \int \sin^{-1} x\, dx$

$\Rightarrow\ y = \int \sin^{-1} x . 1\, dx$

Applying product rule,

$$y = \left(\sin^{-1} x\right) \int 1\, dx - \int \frac{d}{dx}\left(\sin^{-1} x\right) \int 1\, dx\, dx$$

$= x \sin^{-1} x - \int \frac{1}{\sqrt{1-x^2}} x\, dx$ ....(i)

To evaluate $\int \frac{x}{\sqrt{1-x^2}}\, dx = -\frac{1}{2} \int \frac{-2x}{\sqrt{1-x^2}}\, dx$

Putting, $1-x^2 = t,$ differentiate $-2x\, dx = dt$

$$\Rightarrow \int \frac{x}{\sqrt{1-x^2}}\, dx = -\frac{1}{2} \int \frac{dt}{\sqrt{t}} = -\frac{1}{2} \int t^{-1/2}\, dt = -\frac{1}{2} \cdot \frac{t^{1/2}}{1/2} = -\sqrt{t} = -\sqrt{1-x^2}$$

Putting this value in eq. (i), the required general solution is

$$y = x \sin^{-1} x + \sqrt{1-x^2} + c$$

10. $e^x \tan y\, dx + \left(1-e^x\right) \sec^2 y\, dy = 0$

Ans. Given: Differential equation $e^x \tan y\, dx + \left(1-e^x\right) \sec^2 y\, dy = 0$

Dividing each term by $\left(1-e^x\right) \tan y,$ we get

$\frac{e^x}{1-e^x}\, dx + \frac{\sec^2 y}{\tan y}\, dy = 0$ [Separating variables]

Integrating both sides,

$$\int \frac{e^x}{1-e^x}\ dx + \int \frac{\sec^2 y}{\tan y}\ dy = c$$

$$\Rightarrow\ -\int \frac{-e^x}{1-e^x}\ dx + \log|\tan y| = c$$

$$\Rightarrow\ -\log\left|1-e^x\right| + \log|\tan y| = c$$

$$\Rightarrow\ \log \frac{|\tan y|}{\left|1-e^x\right|} = \log c'$$

$$\Rightarrow\ \frac{|\tan y|}{\left|1-e^x\right|} = c'$$

$$\Rightarrow\ \tan y = C\left(1-e^x\right)\ \left[\because |t| = c' \Rightarrow t = \pm c' = C\ (\text{say})\right]$$

For each of the differential equations in Question 11 to 14, find a particular solution satisfying the given condition:

11. $\left(x^3+x^2+x+1\right)\frac{dy}{dx} = 2x^2+x$, $y=1$, **when** $x=0$

Ans. Given: $\left(x^3+x^2+x+1\right)\frac{dy}{dx} = 2x^2+x$

$$\Rightarrow\ \left(x^3+x^2+x+1\right)\ dy = \left(2x^2+x\right)\ dx$$

$$\Rightarrow\ dy = \frac{2x^2+x}{x^3+x^2+x+1}\ dx$$ [Separating variables]

$$\Rightarrow\ dy = \frac{2x^2+x}{x^2(x+1)+(x+1)}\ dx$$

$\Rightarrow \quad dy = \frac{2x^2 + x}{(x+1)(x^2+1)} \; dx$

Integrating both sides,

$\int 1 \; dy = \int \frac{2x^2 + x}{(x+1)(x^2+1)} \; dx$

$\Rightarrow \; y = \int \frac{2x^2 + x}{(x+1)(x^2+1)} \; dx$ ...(i)

Let $\frac{A}{x+1} + \frac{Bx+C}{x^2+1}$ [Partial fraction] ....(ii)

$\Rightarrow 2x^2 + x = A(x^2+1) + (Bx+C)(x+1)$

Comparing the coefficients of x^2 on both sides, A + B = 2 ....(iii)

Comparing the coefficients of x on both sides, B + C = 1 ....(iv)

Comparing constants on both sides, A + C = 0 ....(v)

From eq. (iii) – (iv), we haveA – C = 1 ....(vi)

Adding eq. (v) and (vi), we have 2A = 1

$\Rightarrow$ A = $\frac{1}{2}$

From eq. (v), we have C = – A = $\frac{-1}{2}$

Putting the value of C in eq. (iv), B – $\frac{1}{2}$ = 1

$\Rightarrow$ B = 1 + $\frac{1}{2}$ = $\frac{3}{2}$

Putting the values of A, B, and C in eq. (ii), we have

$$\frac{2x^2+x}{(x+1)(x^2+1)}=\frac{\frac{1}{2}}{x+1}+\frac{\frac{3}{2}x-\frac{1}{2}}{x^2+1}$$

$$=\frac{1}{2}\cdot\frac{1}{x+1}+\frac{3}{2}\cdot\frac{x}{x^2+1}-\frac{1}{2}\cdot\frac{1}{x^2+1}$$

$$=\frac{1}{2}\cdot\frac{1}{x+1}+\frac{3}{4}\cdot\frac{2x}{x^2+1}-\frac{1}{2}\cdot\frac{1}{x^2+1}$$

Putting this value in eq. (i),

$$y=\frac{1}{2}\int\frac{1}{(x+1)}\,dx+\frac{3}{4}\int\frac{2x}{x^2+1}\,dx-\frac{1}{2}\int\frac{1}{x^2+1}\,dx$$

$$\Rightarrow\ y=\frac{1}{2}\log(x+1)+\frac{3}{4}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}x+c \text{ ....(vii)}$$

$$\left[\because\int\frac{2x}{x^2+1}dx=\int\frac{f'(x)}{f(x)}dx=\log f(x)\right]$$

Now, when $x=0, y=1$, putting these values in eq. (vii),

$$1=\frac{1}{2}\log 1+\frac{3}{4}\log 1-\frac{1}{2}\tan^{-1}0+c$$

$$\Rightarrow\ 1=c\quad\left[\log 1=0, \tan^{-1}0=0\right]$$

Putting value of c in eq. (vii), the required general solution is

$$y=\frac{1}{2}\log(x+1)+\frac{3}{4}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}x+1$$

$$\Rightarrow\ y=\frac{1}{4}\left[2\log(x+1)+3\log\left(x^2+1\right)\right]-\frac{1}{2}\tan^{-1}x+1$$

$$\Rightarrow y=\frac{1}{4}\left[\log(x+1)^2+\log\left(x^2+1\right)^3\right]-\frac{1}{2}\tan^{-1}x+1$$

$$\Rightarrow y=\frac{1}{4}\left[\log(x+1)^2\left(x^2+1\right)^3\right]-\frac{1}{2}\tan^{-1}x+1$$

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12. $x\left(x^2-1\right)\frac{dy}{dx}=1;\ y=0$ **when** $x=2$

Ans. Given: Differential equation $x\left(x^2-1\right)\frac{dy}{dx}=1$

$$\Rightarrow dy=\frac{dx}{x\left(x^2-1\right)}$$

Integrating both sides,

=> $\int 1\,dy=\int\frac{1}{x\left(x^2-1\right)}dx$

$$\Rightarrow y=\int\frac{1}{x(x+1)(x-1)}\,dx+c \quad \text{..........(i)}$$

Let $\frac{1}{x(x+1)(x-1)}=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{x-1}$ ....(ii)

$$\Rightarrow 1=A(x+1)(x-1)+Bx(x-1)+Cx(x+1)$$

$$\Rightarrow 1=A\left(x^2-1\right)+B\left(x^2-x\right)+C\left(x^2+x\right)$$

$$\Rightarrow 1=Ax^2-A+Bx^2-Bx+Cx^2+Cx$$

Comparing the coefficients of x^2 on both sides, A + B + C = 0 ....(iii)

Comparing the coefficients of x on both sides, $-$ B + C = 0

$\Rightarrow$ C = B ....(iv)

Comparing constants on both sides, $-$A = 1

$\Rightarrow$ A = $-$1 ....(v)

Putting A = $-$1 and C = B in eq. (iii),

$-$1 + B + B = 0

$\Rightarrow$ 2B = 1

$\Rightarrow$ B = $\frac{1}{2}$

From eq. (iv),C = B = $\frac{1}{2}$

Putting the values of A, B and C in eq. (ii), we get

$$\frac{1}{x(x+1)(x-1)} = \frac{-1}{x} + \frac{\frac{1}{2}}{x+1} + \frac{\frac{1}{2}}{x-1}$$

$$\Rightarrow \int \frac{1}{x(x+1)(x-1)}\,dx = -\int \frac{1}{x}\,dx + \frac{1}{2}\int \frac{1}{x+1}\,dx + \frac{1}{2}\int \frac{1}{x-1}\,dx$$

$$\Rightarrow \int \frac{1}{x(x+1)(x-1)}\,dx = -\log|x| + \frac{1}{2}\log|x+1| + \frac{1}{2}\log|x-1|$$

$$\Rightarrow \int \frac{1}{x(x+1)(x-1)}\,dx = \frac{1}{2}\left[2 - \log|x| + \log|x+1| + \log|x-1|\right]$$

$$\Rightarrow \int \frac{1}{x(x+1)(x-1)}\,dx = \frac{1}{2}\left[-\log|x|^2 + \log|x+1||x-1|\right]$$

$$\Rightarrow \int \frac{1}{x(x+1)(x-1)}\,dx = \frac{1}{2}\left[\log\frac{|x^2-1|}{|x|^2} = \frac{1}{2}\log\left|\frac{x^2-1}{x^2}\right|\right]$$

Putting this value in eq. (i),

$$y=\frac{1}{2}\log\left|\frac{x^2-1}{x^2}\right|+c \text{ ....(v)}$$

Now, putting $y=0,$ when $x=2$ in eq. (v), we get

$$0=\frac{1}{2}\log\frac{3}{4}+c \Rightarrow c=\frac{-1}{2}\log\frac{3}{4}$$

Putting the value of c in eq. (v), the required general solution is

$$y=\frac{1}{2}\log\left|\frac{x^2-1}{x^2}\right|-\frac{1}{2}\log\frac{3}{4}$$

[NOTE: You can also do, to evaluate $\int\frac{1}{x(x^2-1)}\,dx$ = $\int\frac{x}{x^2(x^2-1)}\,dx$ = $\frac{1}{2}\int\frac{2x}{x^2(x^2-1)}\,dx$ Put $x^2=t$]

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13. $\cos\left(\frac{dy}{dx}\right)=a(a\in\mathrm{R});y=1$ **when** $x=0$

Ans. Given: Differential equation $\cos\frac{dy}{dx}=a(a\in\mathrm{R});y=1$ when $x=0$

$$\Rightarrow \frac{dy}{dx}=\cos^{-1}a$$

$\Rightarrow$ dy = (cos^{-1} a)dx

Integrating both sides,

=> $\int 1\,dy=\int\left(\cos^{-1}a\right)\,dx$

$$\Rightarrow y=\left(\cos^{-1}a\right)\int 1\,dx$$

$\Rightarrow\ y=\left(\cos^{-1}a\right)x+c$ ....(i)

Now putting $y=1$ when $x=0$ in eq. (i), we get $c=1$

Putting $c=1$ in eq. (i), $y=\left(\cos^{-1}a\right)x+1$

$\Rightarrow\ \dfrac{y-1}{x}=\cos^{-1}a$

$\Rightarrow\ \cos\left(\dfrac{y-1}{x}\right)=a$

14. $\dfrac{dy}{dx}=y\tan x;\ y=1$ **when** $x=0$

Ans. Given: Differential equation $\dfrac{dy}{dx}=y\tan x$

$\Rightarrow\ dy=y\tan x\,dx$

$\Rightarrow\ \dfrac{dy}{y}=\tan x\,dx$ [Separating variables]

Integrating both sides,

=> $\int\frac{1}{y}\,dy=\int\tan x\,dx$

$\Rightarrow\ \log|y|=\log|\sec x|+\log|c|$

$\Rightarrow\ \log|y|=\log|c\sec x|$

$\Rightarrow\ |y|=|c\sec x|$

$\Rightarrow\ y=\pm c\sec x$

$\Rightarrow\ y=\mathrm{C}\sec x$ where $\pm c=\mathrm{C}$ ....(i)

Now putting $y=1$ and $x=0$ in eq. (i), we get $1=C\sec 0=C$

Putting C = 1 in eq. (i), we get the required general solution

$\Rightarrow\ y=\sec x$

15. Find the equation of the curve passing through the point (0, 0) and whose differential equation is $y'=e^x\sin x$.

Ans. Given: Differential equation $y'=e^x\sin x$

$$\Rightarrow \frac{dy}{dx}=e^x\sin x$$

$$\Rightarrow dy=e^x\sin x\,dx$$

Integrating both sides

=> $\int 1\,dy=\int e^x\sin x\,dx$

$\Rightarrow\ y=\mathrm{I}+C$ ....(i)

where I = $\int e^x\sin x\,dx$ ....(ii)

Applying product rule,

$$\mathrm{I}=e^x(-\cos x)-\int e^x(-\cos x)\,dx$$

$$\Rightarrow \mathrm{I}=-e^x\cos x+\int e^x\cos x\,dx$$

Again applying product rule,

$$\mathrm{I}=-e^x\cos x+e^x\sin x-\int e^x\sin x\,dx$$

$\Rightarrow\ \mathrm{I}=e^x(-\cos x+\sin x)-\mathrm{I}$ [By eq. (ii)]

$\Rightarrow$ 2I = $e^x(\sin x-\cos x)$

$\Rightarrow$ I = $\dfrac{e^x}{2}(\sin x - \cos x)$

Putting this value of I in eq. (i), we get

$y = \dfrac{1}{2}e^x(\sin x - \cos x) + c$ ....(iii)

Now putting $x = 0$ and $y = 0$ in eq. (iii)

$0 = \dfrac{1}{2}(-1) + c$

$\Rightarrow c = \dfrac{1}{2}$

Putting the value of c in eq. (iii), we get the required general solution

$y = \dfrac{1}{2}e^x(\sin x - \cos x) + \dfrac{1}{2}$

$\Rightarrow 2y = e^x(\sin x - \cos x) + 1$

$\Rightarrow 2y - 1 = e^x(\sin x - \cos x)$

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16. For the differential equation $xy\dfrac{dy}{dx} = (x+2)(y+2)$, find the solution curve passing through the point $(1, -1)$.

Ans. Given: Differential equation $xy\dfrac{dy}{dx} = (x+2)(y+2)$

$\Rightarrow xy\ dy = (x+2)(y+2)\ dx$

$\Rightarrow \dfrac{y}{y+2}\ dy = \dfrac{x+2}{x}\ dx$ [Separating both sides]

Integrating both sides

=> $\int \frac{y}{y+2}\, dy = \int \frac{x+2}{x}\, dx$

$=> \int \frac{y+2-2}{y+2} dy = \int (\frac{x}{x} + \frac{2}{x}) dx$

$=> \int \frac{y+2}{y+2} - \frac{2}{y+2} dy = \int (1 + \frac{2}{x}) dx$

$=> \int (1 - \frac{2}{y+2}) dy = \int (1 + \frac{2}{x}) dx$

$\Rightarrow\ y - 2\log|y+2| = x + 2\log|x| + c$

$\Rightarrow\ y - x = \log(y+2)^2 + \log(x)^2 + c$

$\Rightarrow\ y - x = \log(y+2)^2.(x)^2 + c$ ....(i)

Now putting $x = 1, y = -1$ in eq. (i),

$-1-1 = \log(1) + c$

$\Rightarrow\ c = -2$

Putting this value of c in eq. (i) to get the required solution curve

$y - x = \log(y+2)^2.(x)^2 - 2$

$\Rightarrow\ y - x + 2 = \log(y+2)^2.(x)^2$

17. Find the equation of the curve passing through the point $(0,-2)$ given that at any point (x,y) on the curve the product of the slope of its tangent and $y-$coordinate of the point is equal to the $x-$ coordinate of the point.

Ans. Let P(x,y) be any point on the required curve.

According to the question, Slope of the tangent to the curve at P(x,y) $\times y = x$

$\Rightarrow \frac{dy}{dx}.y = x$

$\Rightarrow y\ dy = x\ dx$

Integrating both sides,

=> $\int y\ dy = \int x\ dx$

$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + c$

$\Rightarrow y^2 = x^2 + 2c$

$\Rightarrow y^2 = x^2 + C$ where $C = 2c$

Now it is given that curve $y^2 = x^2 + C$ passes through the point $(0, -2)$.

Therefore, putting $x = 0$ and $y = -2$ in this equation, we get C = 4

Putting the value of C in the equation $y^2 = x^2 + C$,

$y^2 = x^2 + 4$

$\Rightarrow y^2 - x^2 = 4$

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18. At any point (x, y) of a curve the slope of the tangent is twice the slope of the line segment joining the point of contact to the point $(-4, -3)$. Find the equation of the curve given that it passes through $(-2, 1)$.

Ans. According to the question, slope of the tangent at any point P(x, y) of the required curve

= 2. Slope of the line joining the point of contact P(x, y) to the given point A$(-4, -3)$

$\Rightarrow \dfrac{dy}{dx} = 2\left(\dfrac{y-(-3)}{x-(-4)}\right)$

$\Rightarrow \dfrac{dy}{dx} = 2\left(\dfrac{y+3}{x+4}\right)$

$\Rightarrow (x+4)\ dy = 2(y+3)\ dx$

$\Rightarrow \dfrac{dy}{y+3} = \dfrac{2}{x+4}\ dx$ [Separating variables]

Integrating both side,

=> $\int \dfrac{1}{y+3}\ dy = 2\int \dfrac{1}{x+4}\ dx$

$\Rightarrow \log|y+3| = 2\log|x+4| + \log|c|$

$\Rightarrow \log|y+3| = \log|x+4|^2 + \log|c| = \log|c|(x+4)^2$

$\Rightarrow |y+3| = \pm|c|(x+4)^2$

$\Rightarrow y+3 = \pm|c|(x+4)^2$

$\Rightarrow y+3 = \mathrm{C}(x+4)^2$ where $\pm|c| = \mathrm{C}$ ....(i)

Now it is given that curve (i) passes through the point $(-2,1)$.

Therefore, putting $x=-2$ and $y=1$ in eq. (i),

=> 1+3 = C(-2+4)^2

$\Rightarrow$ 4 = 4C

$\Rightarrow$ C = 1

Putting C = 1 in eq. (i), we get the required solution,

$$y+3=(x+4)^2$$

$$\Rightarrow\ (x+4)^2=y+3$$

19. The volume of the spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of balloon after t seconds.

Ans. Let x be the radius of the spherical balloon at time t.

Given: Rate of change of volume of spherical balloon is constant = k (say)

$$\Rightarrow\ \frac{d}{dt}\left(\frac{4\pi}{3}x^3\right)=k$$

$$\Rightarrow\ \frac{4\pi}{3}3x^2.\frac{dx}{dt}=k$$

$$\Rightarrow\ 4\pi x^2\frac{dx}{dt}=k$$

$\Rightarrow\ 4\pi x^2\ dx=k\ dt$ [Separating variables]

Integrating both sides,

=> $4\pi\int x^2\ dx=k\int 1\ dt$

$\Rightarrow\ 4\pi\frac{x^3}{3}=kt+c$ ....(i)

Now it is given that initially radius is 3 units, when $t=0, x=3$.

Therefore, putting $t=0, x=3$ in eq. (i),

=> $\frac{4\pi\times 27}{3}=c$

$\Rightarrow c = 36\pi$ ....(ii)

Again when $t = 3$ sec, then $x = 6$ units

Therefore, putting $t = 3$ and $x = 6$ in eq. (i),

=> $\frac{4\pi}{3}(6)^3 = 3k + c$

$\Rightarrow \frac{4\pi}{3}(216) = 3k + 36\pi$ [From eq. (ii)]

$\Rightarrow 4\pi(72) - 36\pi = 36$

$\Rightarrow 288\pi - 36\pi = 3k \Rightarrow 3k = 252\pi$

$\Rightarrow k = 84\pi$ ....(iii)

Putting the value of c and k in eq. (i), we get

$\frac{4\pi}{3}x^3 = 84\pi t + 36\pi$

$\Rightarrow \frac{x^3}{3} = 21t + 9$

$\Rightarrow x^3 = 63t + 27$

$\Rightarrow x = (63t + 27)^{\frac{1}{3}}$

20. In a bank principal increases continuously at the rate of $r\%$ per year. Find the value of r if Rs 100 double itself in 10 years. $(\log_e 2 = 0.6931)$

Ans. Let P be the principal (amount) at the end of t years.

According to the given condition, rate of increase of principal per year = $r\%$ (of principal)

$\Rightarrow \frac{dP}{dt} = \frac{r}{100} \times P$

$\Rightarrow \frac{dP}{P} = \frac{r}{100}\ dt$ [Separating variables]

Integrating both sides,

$\log\ P = \frac{r}{100}t + c$ ....(i)

[Since P being principal > 0, hence $\log|P| = \log\ P$]

Now initial principal = Rs 100 (given), i.e., when $t = 0$, then P = 100

Therefore, putting $t = 0$, P = 100 in eq. (i), $\log 100 = c$

Putting $\log 100 = c$ in eq. (i), $\log\ P = \frac{r}{100}t + \log 100$

$\Rightarrow \log\ P - \log 100 = \frac{r}{100}t$

$\Rightarrow \log \frac{P}{100} = \frac{r}{100}t$ ....(ii)

Now putting P = double of itself = 2 x 100 = Rs 200, when $t = 10$ years (given)

$\log \frac{200}{100} = \frac{r}{100} \times 10$

$\Rightarrow \log 2 = \frac{r}{10}$

$\Rightarrow r = 10 \log 2$

$\Rightarrow 10 \times 0.6931 = 6.931\%$

21. In a bank principal increases at the rate of 5% per year. An amount of Rs 1000 is

deposited with this bank, how much will it worth after 10 years? $\left(e^{0.5} = 1.648\right)$

Ans. Let P be the principal (amount) at the end of t years.

According to the given condition, rate of increase of principal per year = 5% (of principal)

$$\Rightarrow \frac{dP}{dt} = \frac{5}{100} \times P$$

$$\Rightarrow \frac{dP}{dt} = \frac{P}{20}$$

$$\Rightarrow \frac{dP}{P} = \frac{dt}{20}$$ [Separating variables]

Integrating both sides,

$$\log\ P = \frac{1}{20}t + c$$ ....(i)

[Since P being principal > 0, hence $\log|P| = \log\ P$]

Now initial principal = Rs 1000 (given), i.e., when $t = 0$, then P = 1000

Therefore, putting $t = 0$, P = 1000 in eq. (i), $\log 1000 = c$

Putting $\log 1000 = c$ in eq. (i), $\log\ P = \frac{1}{20}t + \log 1000$

$$\Rightarrow \log\ P - \log 1000 = \frac{1}{20}t$$

$$\Rightarrow \log \frac{P}{1000} = \frac{1}{20}t$$ ....(ii)

Now putting $t = 10$ years (given)

$$\log \frac{P}{100} = \frac{1}{20} \times 10 = \frac{1}{2} = 0.5$$

$\Rightarrow \dfrac{P}{1000} = e^{0.5}$ $\left[\because \text{ if } x = t, \text{ then } x = e^t\right]$

$\Rightarrow$ P = 1000 x 1.648 = Rs 1648

22. In a culture the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number present.

Ans. Let x be the bacteria present in the culture at time t hours.

According to the question,

Rate of growth of bacteria is proportional to the number present

$\Rightarrow \dfrac{dx}{dt}$ is proportional to x

$\Rightarrow \dfrac{dx}{dt} = kx$ where k is the constant of proportionality

$\Rightarrow dx = kx\ dt$

$\Rightarrow \dfrac{dx}{x} = k\ dt$

Integrating both sides,

=> $\int \dfrac{1}{x}\ dx = k\int 1\ dt$

$\Rightarrow \log x = kt + c$ ....(i)

Now it is given that initially the bacteria count is x_0 (say) = 1,00,000

$\Rightarrow$ when $t = 0$, then $x = x_0$

Putting these values in eq. (i)

$\log x_0 = c$

Putting $\log x_0 = c$ in eq. (i), we get

=> $\log x = kt + \log x_0$

$\Rightarrow \log x - \log x_0 = kt$

$\Rightarrow \log \frac{x}{x_0} = kt$ ....(ii)

Now it is given also that the number of bacteria increased by 10% in 2 hours.

Therefore, increase in bacteria in 2 hours = $\frac{10}{100} \times 100000$ = 10,000

$\therefore$ x, the amount of bacteria at $t = 2$ = 1,00,000 + 10,000 = 1,10,000 = x_1 (say)

Putting $x = x_1$ and $t = 2$ in eq. (ii),

$\log \frac{x_1}{x_0} = 2k$

$\Rightarrow k = \frac{1}{2} \log \frac{x_1}{x_0}$

$\Rightarrow k = \frac{1}{2} \log \frac{110000}{100000}$

$\Rightarrow k = \frac{1}{2} \log \frac{11}{10}$

Putting this value of k in eq. (ii), we get,

$\log \frac{x}{x_0} = \frac{1}{2}\left(\log \frac{11}{10}\right)t$

$\Rightarrow \log \frac{200000}{100000} = \frac{1}{2}\left(\log \frac{11}{10}\right)t$ [when $x = 200000$]

$\Rightarrow \log 2 = \frac{1}{2}\left(\log \frac{11}{10}\right)t$

$\Rightarrow 2\log 2 = \left(\log \frac{11}{10}\right)t$

$\Rightarrow t = \frac{2\log 2}{\log \frac{11}{10}}$ hours

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23. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is:

(A) $e^{x} + e^{-y} = c$

(B) $e^{x} + e^{y} = c$

(C) $e^{-x} + e^{y} = c$

(D) $e^{-x} + e^{-y} = c$

Ans. Given: Differential equation $\frac{dy}{dx} = e^{x+y}$

$\Rightarrow \frac{dy}{dx} = e^{x}.e^{y}$

$\Rightarrow dy = e^{x}.e^{y}\ dx$

$\Rightarrow \frac{dy}{e^{y}} = e^{x}\ dx$ [Separating variables]

$\Rightarrow e^{-y}\ dy = e^{x}\ dx$

Integrating both sides,

$$\int e^{-y}\ dy = \int e^{x}\ dx$$

$$\Rightarrow \frac{e^{-y}}{-1} = e^{x} + c$$

$$\Rightarrow -e^{-y} - e^{x} = c$$

$$\Rightarrow e^{-y} + e^{x} = -c$$

$\Rightarrow e^{-y} + e^{x} = C$ where $C = -c$ which is required solution

Therefore, option (A) is correct.