CBSE Class–12 Mathematics

NCERT solution

Chapter - 11

Three Dimensional Geometry - Exercise 11.2

1.Show that the three lines with direction cosines $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}; \frac{4}{13}, \frac{12}{13}, \frac{3}{13}; \frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$ are mutually perpendicular.

Ans. Given: Direction cosines of three lines are

$$\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13} = l_1, m_1, n_1, \quad \frac{4}{13}, \frac{12}{13}, \frac{3}{13} = l_2, m_2, n_2, \quad \frac{3}{13}, \frac{-4}{13}, \frac{12}{13} = l_3, m_3, n_3,$$

For first two lines, $l_1l_2 + m_1m_2 + n_1n_2 = \left(\frac{12}{13}\right)\left(\frac{4}{13}\right) + \left(\frac{-3}{13}\right)\left(\frac{12}{13}\right) + \left(\frac{-4}{13}\right)\left(\frac{3}{13}\right)$

$$= \frac{48}{169} - \frac{36}{169} - \frac{12}{169} = \frac{48-36-12}{169} = \frac{0}{169} = 0$$

Since, it is 0, therefore, the first two lines are perpendicular to each other.

For second and third lines, $l_2l_3 + m_2m_3 + n_2n_3 = \left(\frac{4}{13}\right)\left(\frac{3}{13}\right) + \left(\frac{12}{13}\right)\left(\frac{-4}{13}\right) + \left(\frac{3}{13}\right)\left(\frac{12}{13}\right)$

$$= \frac{12}{169} - \frac{48}{169} + \frac{36}{169} = \frac{12-48+36}{169} = \frac{0}{169} = 0$$

Since, it is 0, therefore, second and third lines are also perpendicular to each other.

For First and third lines, $l_1l_3 + m_1m_3 + n_1n_3 = \left(\frac{12}{13}\right)\left(\frac{3}{13}\right) + \left(\frac{-3}{13}\right)\left(\frac{-4}{13}\right) + \left(\frac{-4}{13}\right)\left(\frac{12}{13}\right)$

$$= \frac{36}{169} + \frac{12}{169} - \frac{48}{169} = \frac{36+12-48}{169} = \frac{0}{169} = 0$$

Since it is 0, therefore, first and third lines are also perpendicular to each other.

Hence, given three lines are mutually perpendicular to each other.

2.Show that the line through the points $(1,-1,2),(3,4,-2)$ is perpendicular to the line through the points (0, 3, 2) and (3, 5, 6).

Ans. We know that direction ratios of the line joining the points A $(1,-1,2)$ and B $(3,4,-2)$ are $x_2-x_1, y_2-y_1, z_2-z_1$

$\Rightarrow 3-1, 4-(-1), -2-2$

$\Rightarrow 2, 5, -4 = a_1, b_1, c_1$

Again, direction ratios of the line joining the points C (0, 3, 2) and D (3, 5, 6) are

$x_2-x_1, y_2-y_1, z_2-z_1$

$\Rightarrow 3-0, 5-3, 6-2$

$\Rightarrow 3, 2, 4 = a_2, b_2, c_2$ (say)

For lines AB and CD, $a_1a_2+b_1b_2+c_1c_2$ = $2\times3+5\times2+(-4)\times4$ = 6 + 10 – 16 = 0

Since, it is 0, therefore, line AB is perpendicular to line CD.

3.Show that the line through points (4, 7, 8), (2, 3, 4) is parallel to the line through the points $(-1,-2,1),(1,2,5)$.

Ans. We know that direction ratios of the line joining the points A (4, 7, 8) and B (2, 3, 4) are

$x_2-x_1, y_2-y_1, z_2-z_1$

$\Rightarrow 2-4, 3-7, 4-8$

$\Rightarrow -2, -4, -4$ = a_1, b_1, c_1 (say)

Again direction ratios of the line joining the points C $(-1,-2,1)$ and D (1, 2, 5) are

$x_2-x_1, y_2-y_1, z_2-z_1$

$\Rightarrow 1-(-1), 2-(-2), 5-1$

$\Rightarrow 2, 4, 4$ = a_2, b_2, c_2 (say)

For the lines AB and CD, $\frac{a_1}{a_2} = \frac{-2}{2}, \frac{b_1}{b_2} = \frac{-4}{4}, \frac{c_1}{c_2} = \frac{-4}{4} = -1$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore, line AB is parallel to line CD.

4.Find the equation of the line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$.

Ans. A point on the required line is A (1, 2, 3) = x_1, y_1, z_1

$\Rightarrow$ Position vector of a point on the required line is $\vec{a} = \overrightarrow{OA} = (1, 2, 3) = \hat{i} + 2\hat{j} + 3\hat{k}$

The required line is parallel to the vector $\vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$

$\therefore$ direction ratios of the required line are coefficient of $\hat{i}, \hat{j}, \hat{k}$ in $\vec{b}$ are

$3, 2, -2 = a, b, c$

$\therefore$ Vector equation of the required line is

$\vec{r} = \vec{a} + \lambda\vec{b} \Rightarrow \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$

Where λ is a real number.

Cartesian equation of this equation is $\frac{x-1}{3} = \frac{y-2}{2} = \frac{z-3}{-2}$

5.Find the equation of the line in vector and in Cartesian form that passes through the

point with position vector $2\hat{i}-\hat{j}+4\hat{k}$ and is in the direction $\hat{i}+2\hat{j}-\hat{k}$.

Ans. Position vector of a point on the required line is $\vec{a}=2\hat{i}-\hat{j}+4\hat{k}$ = $(2,-1,4)=(x_1,y_1,z_1)$

The required line is in the direction of the vector is $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$

$\Rightarrow$ Direction ratios of required line are coefficients of $\hat{i},\hat{j},\hat{k}$ in $\vec{b}$ = $1,2,-1$ $=a,b,c$

$\therefore$ Equation of the required line in vector form is $\vec{r}=\vec{a}+\lambda\vec{b}$

$$\Rightarrow \vec{r}=\left(2\hat{i}-\hat{j}+4\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}-\hat{k}\right)$$

Where λ is a real number.

Cartesian equation of this equation is $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z-4}{-1}$

6.Find the Cartesian equation of the line which passes through the point $(-2,4,-5)$ and parallel to the line given by $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$.

Ans. Given: A point on the line is $(-2,4,-5)=(x_1,y_1,z_1)$

Equation of the given line in Cartesian form is $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$

$\therefore$ Direction ratios of the given line are its denominators 3, 5, 6 $=a,b,c$

$\therefore$ Equation of the required line is $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$

$$\Rightarrow \frac{x-(-2)}{3}=\frac{y-4}{5}=\frac{z-(-5)}{6} = \frac{x+2}{3}=\frac{y-4}{5}=\frac{z+5}{6}$$

7.The Cartesian equation of a line is $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$. Write its vector form.

Ans. Given: The Cartesian equation of the line is $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$ = λ (say)

$\Rightarrow x-5=3\lambda,\ y+4=7\lambda,\ z-6=2\lambda$

$\Rightarrow x=5+3\lambda,\ y=-4+7\lambda,\ z=6+2\lambda$

General equation for the required line is $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

Putting the values of x, y, z in this equation,

$\vec{r}=(5+3\lambda)\hat{i}+(-4+7\lambda)\hat{j}+(6+2\lambda)\hat{k}$ = $5\hat{i}+3\lambda\hat{i}-4\hat{j}+7\lambda\hat{j}+6\hat{k}+2\lambda\hat{k}$

$\Rightarrow \vec{r}=\left(5\hat{i}-4\hat{j}+6\hat{k}\right)+\lambda\left(3\hat{i}+7\hat{j}+2\hat{k}\right)$ $\left[\text{Since } \vec{r}=\vec{a}+\lambda\vec{b}\right]$

8.Find the vector and Cartesian equations of the line that passes through the origin and $(5,-2,3)$.

Ans. $\vec{a}$ = Position vector of a point here O (say) on the line = (0, 0, 0) = $0\hat{i}+0\hat{j}+0\hat{k}=\vec{0}$

$\vec{b}$ = A vector along the line

= $\overrightarrow{OA}$ = Position vector of point A – Position vector of point O

= $(5,-2,3)-(0,0,0)=(5,-2,3)=5\hat{i}-2\hat{j}+3\hat{k}$

$\therefore$ Vector equation of the line is $\left(\vec{r}=\vec{a}+\lambda\vec{b}\right)$

$\Rightarrow \vec{r}=\vec{0}+\lambda\left(5\hat{i}-2\hat{j}+3\hat{k}\right) \Rightarrow \vec{r}=\lambda\left(5\hat{i}-2\hat{j}+3\hat{k}\right)$

NowCartesian equation of the line

Direction ratios of line OA are $5-0,-2-0,3-0=5,-2,3$

And a point on the line is O (0, 0, 0) = (x_1, y_1, z_1)

$\therefore$ Cartesian equation of the line = $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$

= $\frac{x-0}{5}=\frac{y-0}{-2}=\frac{z-0}{3}$ = $\frac{x}{5}=\frac{y}{-2}=\frac{z}{3}$

Remark: In the solution of the above question we can also take:

$\vec{a}$ = Position vector of point A = $(5, -2, 3) = 5\hat{i}-2\hat{j}+3\hat{k}$ for vector form and point A as (x_1, y_1, z_1) = $(5, -2, 3)$ for Cartesian form.

Then the equation of the line in vector form is $\vec{r} = 5\hat{i}-2\hat{j}+3\hat{k}+\lambda\left(5\hat{i}-2\hat{j}+3\hat{k}\right)$ And equation of line in Cartesian form is $\frac{x-5}{5}=\frac{y+2}{-2}=\frac{z-3}{3}$

9.Find vector and Cartesian equations of the line that passes through the points $(3, -2, -5)$ and $(3, -2, 6)$.

Ans. Let $\vec{a}$ and $\vec{b}$ be the position vectors of the points A$(3, -2, -5)$ and B$(3, -2, 6)$ respectively.

$\therefore$ $\vec{a} = 3\hat{i}-2\hat{j}-5\hat{k}$ and $\vec{b} = 3\hat{i}-2\hat{j}+6\hat{k}$

$\therefore$ A vector along the line = $\overrightarrow{AB}$ = Position vector of point B – Position vector of point A

$\Rightarrow$ $\overrightarrow{AB}$ = $\vec{b}-\vec{a}$ = $3\hat{i}-2\hat{j}+6\hat{k}$ − $3\hat{i}+2\hat{j}+5\hat{k}$ = $11\hat{k}$

$\because$ Vector equation of the line is $\left(\vec{r} = \vec{a}+\lambda\vec{b}\right)$

$\therefore$ $\vec{r} = 3\hat{i}-2\hat{j}-5\hat{k}+\lambda\left(11\hat{k}\right)$

And another vector equation for the same line is $\vec{r} = \vec{a}+\lambda\overrightarrow{AB}$ =

$\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k} + \lambda\left(11\hat{k}\right)$

Cartesian equation

Direction ratios of line AB are $3-3, -2+2, 6+5 = 0, 0, 11$

$\therefore$ Equation of the line is $\dfrac{x-x_1}{a} = \dfrac{y-y_1}{b} = \dfrac{z-z_1}{c}$

$\Rightarrow \dfrac{x-3}{0} = \dfrac{y+2}{0} = \dfrac{z+5}{11}$

---

10. Find the angle between the following pairs of lines:

(i) $\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ **and** $\vec{r} = 7\hat{i} - 6\hat{k} + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$

(ii) $\vec{r} = 3\hat{i} + \hat{j} - 2\hat{k} + \lambda\left(\hat{i} - \hat{j} - 2\hat{k}\right)$ **and** $\vec{r} = 2\hat{i} - \hat{j} - 5\hat{k} + \mu\left(3\hat{i} - 5\hat{j} - 4\hat{k}\right)$

Ans. (i) Equation of the first line is $\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$

Comparing with $\left(\vec{r} = \vec{a} + \lambda\vec{b}\right)$,

$\overrightarrow{a_1} = 2\hat{i} - 5\hat{j} + \hat{k}$ and $\overrightarrow{b_1} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

(vector $\vec{a}$ is the position vector of a point on line and $\vec{b}$ is a vector along the line)

Again, equation of the second line is $\vec{r} = 7\hat{i} - 6\hat{k} + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$

Comparing with $\left(\vec{r} = \vec{a} + \mu\vec{b}\right)$,

$\overrightarrow{a_2} = 7\hat{i} - 6\hat{k}$ and $\overrightarrow{b_2} = \hat{i} + 2\hat{j} + 2\hat{k}$

(vector $\vec{a}$ is the position vector of a point on line and $\vec{b}$ is a vector along the line)

Let θ be the angle between these two lines, then

$$\cos\theta = \frac{\vec{b_1}.\vec{b_2}}{\left|\vec{b_1}\right|.\left|\vec{b_2}\right|} = \frac{3(1)+2(2)+6(2)}{\sqrt{9+4+36}\sqrt{1+4+4}} = \frac{3+4+12}{\sqrt{49}\sqrt{9}} = \frac{19}{7\times 3}$$

$$\cos\theta = \frac{19}{21} \Rightarrow \theta = \cos^{-1}\frac{19}{21}$$

(ii)Comparing the first and second equations with $\left(\vec{r} = \vec{a} + \lambda\vec{b}\right)$ and $\left(\vec{r} = \vec{a} + \mu\vec{b}\right)$ resp.

$\vec{b_1} = \hat{i} - \hat{j} - 2\hat{k}$ and $\vec{b_2} = 3\hat{i} - 5\hat{j} - 4\hat{k}$

Let θ be the angle between these two lines, then

$$\cos\theta = \frac{\vec{b_1}.\vec{b_2}}{\left|\vec{b_1}\right|.\left|\vec{b_2}\right|} = \frac{1(3)+(-1)(-5)+(-2)(-4)}{\sqrt{1+1+4}\sqrt{9+25+16}} = \frac{3+5+8}{\sqrt{6}\sqrt{50}} = \frac{16}{\sqrt{300}}$$

$$\cos\theta = \frac{16}{10\sqrt{3}} = \frac{8}{5\sqrt{3}} \Rightarrow \theta = \cos^{-1}\frac{8}{5\sqrt{3}} \Rightarrow \theta = \cos^{-1}\frac{8\sqrt{3}}{15}$$

11.Find the angle between the following pair of lines:

(i) $\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3}$ **and** $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$

(ii) $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ **and** $\frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$

Ans. (i) Given: Equation of first line is $\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3}$

The direction ratios of this line i.e., a vector along the line is

$\vec{b_1} = (2, 5, -3) = 2\hat{i} + 5\hat{j} - 3\hat{k}$

Now, equation of second line is $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$

The direction ratios of this line i.e., a vector along the line is

$\overrightarrow{b_2} = (-1, 8, 4) = -\hat{i} + 8\hat{j} + 4\hat{k}$

Let θ be the angle between these two lines, then

$$\cos\theta = \frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{\left|\overrightarrow{b_1}\right|.\left|\overrightarrow{b_2}\right|} = \frac{2(-1)+(5)(8)+(-3)(4)}{\sqrt{4+25+9}\sqrt{1+64+16}} = \frac{-2+40-12}{\sqrt{38}\sqrt{81}} = \frac{26}{9\sqrt{38}}$$

$$\Rightarrow \theta = \cos^{-1}\frac{26}{9\sqrt{38}}$$

(ii)Given: Equation of first line is $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$

The direction ratios of this line i.e., a vector along the line is

$\overrightarrow{b_1} = (2, 2, 1) = 2\hat{i} + 2\hat{j} + \hat{k}$

Nowequation of second line is $\frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$

The direction ratios of this line i.e., a vector along the line is

$\overrightarrow{b_2} = (4, 1, 8) = 4\hat{i} + \hat{j} + 8\hat{k}$

Let θ be the angle between these two lines, then

$$\cos\theta = \frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{\left|\overrightarrow{b_1}\right|.\left|\overrightarrow{b_2}\right|} = \frac{2(4)+(2)(1)+(1)(8)}{\sqrt{4+4+1}\sqrt{16+1+64}} = \frac{8+2+8}{\sqrt{9}\sqrt{81}} = \frac{18}{3\times 9} = \frac{2}{3}$$

$$\Rightarrow \theta = \cos^{-1}\frac{2}{3}$$

12.Find the values of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and

$\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ **are at right angles.**

Ans. Given: Equation of one line $\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ $\Rightarrow$

$$\frac{-(x-1)}{3}=\frac{7(y-2)}{2p}=\frac{z-3}{2}$$

$$\Rightarrow \frac{-(x-1)}{3}=\frac{y-2}{\frac{2p}{7}}=\frac{z-3}{2}$$

$\therefore$ Direction ratios of this line are $-3, \frac{2p}{7}, 2=a_1, b_1, c_1$ (say)

Again, equation of another line $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ $\Rightarrow$

$$\frac{-7(x-1)}{3p}=\frac{y-5}{1}=\frac{-(z-6)}{5}$$

$$\Rightarrow \frac{x-1}{\frac{-3p}{7}}=\frac{y-5}{1}=\frac{z-6}{-5}$$

$\therefore$ Direction ratios of this line are $\frac{-3p}{7}, 1, -5=a_2, b_2, c_2$ (say)

Since, these two lines are perpendicular.

Therefore, $a_1a_2+b_1b_2+c_1c_2=0$

$$\Rightarrow (-3)\left(\frac{-3p}{7}\right)+\left(\frac{2p}{7}\right)(1)+(2)(-5)=0 \Rightarrow \frac{9p}{7}+\frac{2p}{7}-10=0$$

$$\Rightarrow \frac{11p}{7}=10 \Rightarrow p=\frac{70}{11}$$

13.Show that the lines $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ are perpendicular to each other.

Ans. Equation of one line $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$

$\therefore$ Direction ratios of this line are $7,-5,1$ = a_1, b_1, c_1

$\Rightarrow \overrightarrow{b_1}=7\hat{i}-5\hat{j}+\hat{k}$

Again equation of another line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$

$\therefore$ Direction ratios of this line are 1, 2, 3 = a_2, b_2, c_2

$\Rightarrow \overrightarrow{b}=\hat{i}+2\hat{j}+3\hat{k}$

Now $\overrightarrow{b_1}.\overrightarrow{b_2}=a_1a_2+b_1b_2+c_1c_2$ = $7\times1+(-5)\times2+1\times3$ = $7-10+3=0$

Hence, the given two lines are perpendicular to each other.

14.Find the shortest distance between the lines $\vec{r}=\hat{i}+2\hat{j}+\hat{k}+\lambda(\hat{i}-\hat{j}+\hat{k})$ and $\vec{r}=2\hat{i}-\hat{j}-\hat{k}+\mu(2\hat{i}+\hat{j}+2\hat{k})$.

Ans. Comparing the given equations with $\vec{r}=\overrightarrow{a_1}+\lambda\overrightarrow{b_1}$ and $\vec{r}=\overrightarrow{a_2}+\lambda\overrightarrow{b_2}$, we get

$\overrightarrow{a_1}=\hat{i}+2\hat{j}+\hat{k},\quad \overrightarrow{b_1}=\hat{i}-\hat{j}+\hat{k}$ and $\overrightarrow{a_2}=2\hat{i}-\hat{j}-\hat{k},\quad \overrightarrow{b_2}=2\hat{i}+\hat{j}+2\hat{k}$

Since, the shortest distance between the two skew lines is given by

$$d=\frac{\left|\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right).\left(\overrightarrow{b_1}\times\overrightarrow{b_2}\right)\right|}{\left|\overrightarrow{b_1}\times\overrightarrow{b_2}\right|} \quad ....(i)$$

Here, $\overrightarrow{a_2}-\overrightarrow{a_1}=\left(2\hat{i}-\hat{j}-\hat{k}\right)-\left(\hat{i}+2\hat{j}+\hat{k}\right)=\hat{i}-3\hat{j}-2\hat{k}$

$$\overrightarrow{b_1}\times\overrightarrow{b_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 1\\ 2 & 1 & 2\end{vmatrix}=(-2-1)\hat{i}-(2-2)\hat{j}+(1+2)\hat{k}=-3\hat{i}+3\hat{k}$$

$$\left|\overrightarrow{b_1}\times\overrightarrow{b_2}\right|=\sqrt{(-3)^2+(0)^2+(3)^2}=\sqrt{18}=3\sqrt{2}$$

$$\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right).\left|\overrightarrow{b_1}\times\overrightarrow{b_2}\right|=\left(\hat{i}-3\hat{j}-2\hat{k}\right).\left(-3\hat{i}+3\hat{k}\right)=1\times(-3)+(-3\times 0)+(-2\times 3)=-9$$

Putting these values in eq. (i),

Shortest distance $(d)=\dfrac{|-9|}{3\sqrt{2}}=\dfrac{9}{3\sqrt{2}}=\dfrac{3}{\sqrt{2}}=\dfrac{3\sqrt{2}}{2}$

15.Find the shortest distance between the lines $\dfrac{x+1}{7}=\dfrac{y+1}{-6}=\dfrac{z+1}{1}$ and $\dfrac{x-3}{1}=\dfrac{y-5}{-2}=\dfrac{z-7}{1}$.

Ans. Equation of one line is $\dfrac{x+1}{7}=\dfrac{y+1}{-6}=\dfrac{z+1}{1}$

Comparing this equation with $\dfrac{x-x_1}{a_1}=\dfrac{y-y_1}{b_1}=\dfrac{z-z_1}{c_1}$, we have

$x_1=-1,\ y_1=-1,\ z_1=-1,\ a_1=7,\ b_1=-6,\ c_1=1$

Again equation of another line is $\dfrac{x-3}{1}=\dfrac{y-5}{-2}=\dfrac{z-7}{1}$

Comparing this equation with $\dfrac{x-x_2}{a_2}=\dfrac{y-y_2}{b_2}=\dfrac{z-z_2}{c_2}$, we have

$x_2 = 3,\ \ y_2 = 5,\ \ z_2 = 7,\ \ a_2 = 1,\ \ b_2 = -2,\ \ c_2 = 1$

$$\therefore \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 3+1 & 5+1 & 7+1 \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

Expanding by first row = $4(-6+2) - 6(7-1) + 8(-14+6)$ = $-16 - 36 - 64 = -116$

And $\sqrt{(a_1b_2 - a_2b_1) + (b_1c_2 - b_2c_1) + (c_1a_2 - c_2a_1)}$

= $\sqrt{(-14+6)^2 + (-6+2)^2 + (1-7)^2}$ = $\sqrt{64+16+36}$ = $\sqrt{116}$

$$\therefore \text{ Length of shortest distance } = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(a_1b_2 - a_2b_1) + (b_1c_2 - b_2c_1) + (c_1a_2 - c_2a_1)}}$$

= $\dfrac{-116}{\sqrt{116}} = -\sqrt{116} = \sqrt{116}$ (numerically)

= $\sqrt{4 \times 29} = 2\sqrt{29}$

16.Find the shortest distance between the lines whose vector equations are

$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(\hat{i} - 3\hat{j} + 2\hat{k}\right)$ **and** $\vec{r} = 4\hat{i} + 5\hat{j} + 6\hat{k} + \mu\left(2\hat{i} + 3\hat{j} + \hat{k}\right)$

Ans. Equation of the first line is $\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - 3\hat{j} + 2\hat{k}\right)$

Comparing this equation with $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$,

$\vec{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b_1} = \hat{i} - 3\hat{j} + 2\hat{k}$

Again equation of second line $\vec{r} = \left(4\hat{i} + 5\hat{j} + 6\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + \hat{k}\right)$

Comparing this equation with $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$,

$\overrightarrow{a_2} = 4\hat{i} + 5\hat{j} + 6\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$

Now shortest distance (d) = $\dfrac{\left|\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right).\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)\right|}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|}$ ....(i)

Here $\overrightarrow{a_2} - \overrightarrow{a_1} = \left(4\hat{i} + 5\hat{j} + 6\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 3\hat{i} + 3\hat{j} + 3\hat{k}$

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix} = (-3-6)\hat{i} - (1-4)\hat{j} + (3+6)\hat{k} = -9\hat{i} + 3\hat{j} + 9\hat{k}$$

$$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \sqrt{(-9)^2 + (3)^2 + (9)^2} = \sqrt{171} = 3\sqrt{19}$$

$$\left(\overline{a_2} - \overline{a_1}\right).\left|\overline{b_1} \times \overline{b_2}\right| = 3 \times (-9) + (3 \times 3) + (3 \times 9) = -27 + 9 + 27 = 9$$

Putting these values in eq. (i),

Shortest distance $(d) = \dfrac{|9|}{3\sqrt{19}} = \dfrac{9}{3\sqrt{19}} = \dfrac{3}{\sqrt{19}}$

17.Find the shortest distance between the lines whose vector equations are

$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ **and** $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$.

Ans. Equation of first line is $\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$

= $\hat{i} - t\hat{i} + t\hat{j} - 2\hat{j} + 3\hat{k} - 2t\hat{k}$ = $\left(\hat{i} - 2\hat{j} + 3\hat{k}\right) + t\left(-\hat{i} + \hat{j} - 2\hat{k}\right)$

Comparing this equation with $\overrightarrow{a_1} + t\overrightarrow{b_1}$,

$\overrightarrow{a_1} = \hat{i} - 2\hat{j} + 3\hat{k},\quad \overrightarrow{b_1} = -\hat{i} + \hat{j} - 2\hat{k}$

Equation of second line is $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$

$= s\hat{i} + \hat{i} + 2s\hat{j} - \hat{j} - 2s\hat{k} - \hat{k} = \left(\hat{i} - \hat{j} - \hat{k}\right) + s\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$

Comparing this equation with $\overrightarrow{a_2} + s\overrightarrow{b_2}$,

$\overrightarrow{a_2} = \hat{i} - \hat{j} - \hat{k},\quad \overrightarrow{b_2} = \hat{i} + 2\hat{j} - 2\hat{k}$

Now Shortest distance (d) = $\dfrac{\left|\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right).\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)\right|}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|}$ ....(i)

Here $\overrightarrow{a_2} - \overrightarrow{a_1} = \left(\hat{i} - \hat{j} - \hat{k}\right) - \left(\hat{i} - 2\hat{j} + 3\hat{k}\right) = \hat{j} - 4\hat{k}$

$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i}\hat{j}\hat{k} \\ -11-2 \\ 12-2 \end{vmatrix} (-2+4)\,\hat{i} - (2+2)\,\hat{j} + (-2-1)\,\hat{k}$

$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \sqrt{(2)^2 + (-4)^2 + (-3)^2} = \sqrt{29}$

$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right).\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \left(\hat{j} - 4\hat{k}\right).\left(2\hat{i} - 4\hat{j} - 3\hat{k}\right) = 0 \times 2 + 1 \times (-4) + (-4)(-3) = 8$

Putting these values in eq. (i),

Shortest distance $(d) = \dfrac{|8|}{\sqrt{29}} = \dfrac{8}{\sqrt{29}}$