CBSE Class–12 Mathematics

NCERT solution

Chapter - 13

Probability - Exercise 13.2

1. If P (A) = $\frac{3}{5}$ and P (B) = $\frac{1}{5}$, find P (A $\cap$ B) if A and B are independent events.

Ans. As A and B are independent events.

Therefore, $P(A \cap B)$ = P (A). P (B)

= $\frac{3}{5} \times \frac{1}{5} = \frac{3}{25}$

2. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

Ans. S = 52 cards $\Rightarrow$ $n(S)$ = 52

Two cards are drawn without replacement.

A = {26 black cards} $\Rightarrow$ $n(A)$ = 26

P (A) = $\frac{26}{52}$

And P (B) i.e., probability that second card is black known that first card is black = $\frac{25}{51}$

P (A and B) = P (A). P (B) = $\frac{26}{52} \times \frac{25}{51} = \frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$

3. A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale otherwise it is rejected. Find the probability that a box containing 15 oranges out of

which 12 are good and 3 are bad ones will be approved for sale.

Ans. S = {12 good oranges, 3 bad oranges}

$\Rightarrow n(S)$ = 15

Probability that first orange drawn is good = $\frac{12}{15}$

Probability that second orange is drawn is good = $\frac{11}{14}$

Probability that third orange is drawn is good when both the first and second are good = $\frac{10}{13}$

P (a box is approved) = $\frac{C(12,3)}{C(15,3)} = \frac{12\times11\times10}{15\times14\times13} = \frac{44}{91}$

4. A fair coin and an unbiased die are tossed. Let A be the event ‘head appears on the coin’ and B be the event ‘3 on the die’. Check whether A and B are independent events or not.

Ans. S = {(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)}

$\Rightarrow n(S)$ = 12

Head appears on the coin = A = {(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6)} $\Rightarrow n(A)$ = 6

P (A) = $\frac{n(A)}{n(S)} = \frac{6}{12} = \frac{1}{2}$

3 appears on the die = B = {(H, 3), (T, 3)} $\Rightarrow n(B)$ = 2

P (B) = $\frac{n(B)}{n(S)} = \frac{2}{12} = \frac{1}{6}$

Now (A $\cap$ B) = {(H, 3)} $\Rightarrow n(A\cap B)$ = 1

$\therefore\ P(A\cap B) = \frac{n(A\cap B)}{n(S)} = \frac{1}{12}$

Again P (A). P (B) = $\frac{1}{2}\times\frac{1}{6}=\frac{1}{12}$

Therefore, $P(A\cap B)=$ P (A). P (B), i.e., events A and B are independent.

5. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event 'number is even' and B be the event 'number is red'. Are A and B independent?

Ans. S = $\left\{\frac{1,2,3,}{\text{red}}\frac{4,5,6}{\text{green}}\right\}$ $\Rightarrow n(S)$ = 6

The number is even = A = {2, 4, 6} $\Rightarrow n(A)$ = 3

P (A) = $\frac{n(A)}{n(S)}=\frac{3}{6}=\frac{1}{2}$

The number is red = B = $\left\{\frac{1,2,3}{\text{red}}\right\}$ $\Rightarrow n(B)$ = 3

P (B) = $\frac{n(B)}{n(S)}=\frac{3}{6}=\frac{1}{2}$

(A $\cap$ B) = {2} $\Rightarrow n(A\cap B)$ = 1

$\therefore\ P(A\cap B)=\frac{n(A\cap B)}{n(S)}=\frac{1}{6}$

Now P (A). P (B) = $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

Therefore, $P(A\cap B)\neq$ P (A). P (B), i.e., A and B are not independent.

6. Let E and F be events with P (E) = $\frac{3}{5}$, P (F) = $\frac{3}{10}$ and P (E $\cap$ F) = $\frac{1}{5}$. Are E and F independent?

Ans. P (E) = $\frac{3}{5}$, P (F) = $\frac{3}{10}$

$\therefore\ P(E\cap F)=\frac{1}{5}$

Now P (E). P (F) = $\frac{3}{5}\times\frac{3}{10}=\frac{9}{50}$

$\Rightarrow\ P(E\cap F)\neq$ P (E). P (F)

Therefore, E and F are not independent events.

7. Given that the events A and B are such that P (A) = $\frac{1}{2}$, P (A $\cup$ B) = $\frac{3}{5}$ and P (B) = p. Find p if they are (i) mutually exclusive, (ii) independent.

Ans. P (A) = $\frac{1}{2}$, $P(A\cup B)$ = $\frac{3}{5}$, P (B) = p

(i) A and B are mutually exclusive events, then $A\cap B=\phi \quad\Rightarrow\quad P(A\cap B)=0$

$\therefore\ P(A\cup B)$ = P (A) + P (B) $-P(A\cap B)\Rightarrow \frac{3}{5}=\frac{1}{2}+p$

$\Rightarrow\ p=\frac{3}{5}-\frac{1}{2}=\frac{6-5}{10}=\frac{1}{10}$

(ii) A and B are independent events.

$\therefore\ P(A\cup B)$ = P (A) + P (B) – P (A). P (B)

$\Rightarrow\ \frac{3}{5}=\frac{1}{2}+p-\frac{1}{2}\times p\ \Rightarrow\ \frac{3}{5}-\frac{1}{2}=\frac{1}{2}p$

$\Rightarrow\ \frac{1}{2}p=\frac{1}{10}\ \Rightarrow\ p=\frac{1}{5}$

8. Let A and B independent events, P (A) = 0.3 and P (B) = 0.4. Find:

(A) P (A $\cap$ B)

(B) P (A $\cup$ B)

(C) P $\left(A|B\right)$

(D) P $\left(B|A\right)$

Ans. P (A) = 0.3, P (B) = 0.4

A and B are independent events.

(i) $P\left(A\cap B\right) =$ P (A). P (B) = 0.3 x 0.4 = 0.12

(ii) $P\left(A\cup B\right)$ = P (A) + P (B) – P (A). P (B) = 0.03 + 0.4 – 0.3 x 0.4 = 0.7 – 0.12 = 0.58

(iii) P$\left(A|B\right)$ = $\frac{P\left(A\cap B\right)}{P\left(B\right)}$ = $\frac{P\left(A\right)P\left(B\right)}{P\left(B\right)}$ = P (A) = 0.3

(iv) P$\left(B|A\right)$ = $\frac{P\left(A\cap B\right)}{P\left(A\right)}$ = $\frac{P\left(A\right)P\left(B\right)}{P\left(A\right)}$ = P (B) = 0.4

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9. If A and B are two events such that P (A) = $\frac{1}{4}$, **P (B) =** $\frac{1}{2}$ **and P (A $\cap$ B) =** $\frac{1}{8}$, **find P (not A and not B).**

Ans. P (A) = $\frac{1}{4}$, P (B) = $\frac{1}{2}$, $P\left(A\cap B\right)=\frac{1}{8}$

P (not A) = 1 – P (A) = $1-\frac{1}{4}=\frac{3}{4}$

P (not B) = 1 – P (B) = $1-\frac{1}{2}=\frac{1}{2}$

Now P (A). P (B) = $\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$

$\therefore$ $P(A\cap B)$ = P (A). P (B)

Thus, A and B are independent events.

Therefore, ‘not A’ and ‘not B’ are independent events.

Hence, P (not A and not B) = P (not A). P (not B)

= $\frac{3}{4}\times\frac{1}{2}=\frac{3}{8}$

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10. Events A and B are such that P (A) = $\frac{1}{2}$, P (B) = $\frac{7}{12}$ and P (not A or not B) = $\frac{1}{4}$. State whether A and B are independent.

Ans. $P(\overline{A}\cup\overline{B})=1-P(A\cap B)$ $\Rightarrow$ $\frac{1}{4}=1-P(A\cap B)$

$\Rightarrow$ $P(A\cap B)=1-\frac{1}{4}=\frac{3}{4}$

P (A) = $\frac{1}{2}$ and P (B) = $\frac{7}{12}$

P (A). P (B) = $\frac{1}{2}\times\frac{7}{12}=\frac{7}{24}$

Therefore, $P(A\cap B)\neq$ P (A). P (B), i.e., A and B are not independent.

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11. Given two independent events A and B such that P (A) = 0.3, P (B) = 0.6. Find:

(A) P (A and B)

(B) P (A and not B)

(C) P (A or B)

(D) P (neither A nor B)

Ans. P (A) = 0.3, P (B) = 0.6

A and B are independent events.

(i) P (A and B) = P (A). P (B) = 0.3 x 0.6 = 0.18

(ii) P (A and not B) = $P\left(A \cap \overline{B}\right)$ = P (A) – $P\left(A \cap B\right)$ = 0.3 – 0.18 = 0.12

(iii) P (A or B) = P (A) + P (B) – P (A and B) = 0.3 + 0.6 – 0.18 = 0.9 – 0.18 = 0.72

(iv) P (neither A nor B) = P [not (A $\cup$ B)] = 1 – P (A $\cup$ B) = 1 – 0.72 = 0.28

12. A die is tossed thrice. Find the probability of getting an odd number at least once.

Ans. S = {1, 2, 3, 4, 5, 6} $\Rightarrow$ $n\left(S\right)$ = 6

Let A represents an odd number.

$\therefore$ A = {1, 3, 5} $\Rightarrow$ $n\left(A\right)$ = 3

P (A) = $\dfrac{n\left(A\right)}{n\left(S\right)} = \dfrac{3}{6} = \dfrac{1}{2}$

$P\left(\overline{A}\right)$ = 1 – P (A) = $1 - \dfrac{1}{2} = \dfrac{1}{2}$

n = 3

Now P (at least one success) = 1 – P (an odd number on none of the three dice)

= $1 - \left(\dfrac{1}{2}\right)^3 = 1 - \dfrac{1}{8} = \dfrac{7}{8}$

13. Two balls are drawn at random with replacement from a box containing 10 black

and 8 red balls. Find the probability that:

(i) both balls are red.

(ii) first ball is black and second is red.

(iii) one of them is black and other is red.

Ans. S = (10 black balls, 8 red balls) $\Rightarrow$ $n(\mathrm{S})$ = = 18

Let drawing of a red ball be a success.

$\therefore$ A = {8 red balls} $\Rightarrow$ $n(\mathrm{A})$ = 8

P (A) = $\frac{n(\mathrm{A})}{n(\mathrm{S})} = \frac{8}{18} = \frac{4}{9}$

And $\mathrm{P}\left(\overline{\mathrm{A}}\right)$ = 1 – P (A) = $1 - \frac{4}{9} = \frac{5}{9}$

(i) P (both are red ball) = P (A). P (B) = $\frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$

(ii) P (first is black ball and second is red) = $\mathrm{P}\left(\overline{\mathrm{A}}\right)$. P (A) = $\frac{5}{9} \times \frac{4}{9} = \frac{20}{81}$

(iii) P (one of them is black and other is red) = $\mathrm{P}\left(\overline{\mathrm{A}}\right)$. P (A) + P (A). $\mathrm{P}\left(\overline{\mathrm{A}}\right)$

= $\frac{5}{9} \times \frac{4}{9} + \frac{4}{9} \times \frac{5}{9} = \frac{40}{81}$

14. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that:

(i) the problem is solved.

(ii) exactly one of them solves the problem.

Ans. P (A) = $\frac{1}{2}$, P (B) = $\frac{1}{3}$, $P\left(\overline{A}\right) = 1-\frac{1}{2}=\frac{1}{2}$ and $P\left(\overline{B}\right)=1-\frac{1}{3}=\frac{2}{3}$

(i) P (the problem is solved) = 1 – P (the problem is not solved)

= $1-P\left(\overline{A}\text{ and }\overline{B}\right)$

= $1-P\left(\overline{A}\right)P\left(\overline{B}\right)$

= $1-\frac{1}{2}\times\frac{2}{3}=1-\frac{2}{6}=\frac{4}{6}=\frac{2}{3}$

(ii) P (exactly one of them solves the problem) = P (A). $P\left(\overline{B}\right)$ + $P\left(\overline{A}\right)$. P (B)

= $\frac{1}{2}\times\frac{2}{3}+\frac{1}{2}\times\frac{1}{3}=\frac{1}{3}+\frac{1}{6}=\frac{3}{6}=\frac{1}{2}$

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15. One card is drawn at random from a well shuffled deck of 52 cards. In which of the following cases are the events E and F independent?

(i) E : 'the card drawn is a spade'

F : 'the card drawn is an ace'

(ii) E : 'the card drawn is black'

F : 'the card drawn is a king'

(iii) E : 'the card drawn is a king or queen'

F : 'the card drawn is a queen or jack'

Ans. S = {All the 52 cards} $\Rightarrow$ $n\left(S\right)$ = 52

(i) E = {13 spades} $\Rightarrow$ $n\left(E\right)$ = 13

$\therefore$ P (E) = $\frac{n(\mathrm{E})}{n(\mathrm{S})}=\frac{13}{52}=\frac{1}{4}$

F = {4 aces} $\Rightarrow$ $n(\mathrm{F})$ = 4

$\therefore$ P (F) = $\frac{n(\mathrm{F})}{n(\mathrm{S})}=\frac{4}{52}=\frac{1}{13}$

Now E $\cap$ F = {An ace of spade} $\Rightarrow$ $n(\mathrm{E} \cap \mathrm{F})$ = 1

$\therefore \mathrm{P}(\mathrm{E} \cap \mathrm{F})=\frac{n(\mathrm{E} \cap \mathrm{F})}{n(\mathrm{S})}$ = $\frac{1}{52}$

Also, P(E). P(F) = $\frac{1}{4} \times \frac{1}{13}=\frac{1}{52}$

Therefore, $\mathrm{P}(\mathrm{E} \cap \mathrm{F})$ = P(E). P(F)

Hence, E and F are independent events.

(ii) E = {26 black cards} $\Rightarrow$ $n(\mathrm{E})$ = 26

$\therefore$ P (E) = $\frac{n(\mathrm{E})}{n(\mathrm{S})}=\frac{26}{52}=\frac{1}{2}$

F = {4 kings} $\Rightarrow$ $n(\mathrm{F})$ = 4

$\therefore$ P (F) = $\frac{n(\mathrm{F})}{n(\mathrm{S})}=\frac{4}{52}=\frac{1}{13}$

Now E $\cap$ F = {2 black kings} $\Rightarrow$ $n(\mathrm{E} \cap \mathrm{F})$ = 2

$\therefore \mathrm{P}(\mathrm{E} \cap \mathrm{F})=\frac{n(\mathrm{E} \cap \mathrm{F})}{n(\mathrm{S})}$ = $\frac{2}{52}=\frac{1}{26}$

Also, P(E). P(F) = $\frac{1}{2}\times\frac{1}{13}=\frac{1}{26}$

Therefore, $P(E\cap F)$ = P(E). P(F)

Hence, E and F are independent events.

(iii) E = {4 kings, 4 queens}$\Rightarrow$ $n(E)$ = 8

$\therefore$ P (E) = $\frac{n(E)}{n(S)}=\frac{8}{52}=\frac{2}{13}$

F = {4 queens, 4 jacks}$\Rightarrow$ $n(F)$ = 8

$\therefore$ P (F) = $\frac{n(F)}{n(S)}=\frac{8}{52}=\frac{2}{13}$

Now E $\cap$ F = {4 queens }$\Rightarrow$ $n(E\cap F)$ = 4

$\therefore$ $P(E\cap F)=\frac{n(E\cap F)}{n(S)}$ = $\frac{4}{52}=\frac{1}{13}$

Also, P(E). P(F) = $\frac{2}{13}\times\frac{2}{13}=\frac{4}{169}$

Therefore, $P(E\cap F)$ $\neq$ P(E). P(F)

Hence, E and F are not independent events.

16. In a hostel 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.

(a) Find the probability that she reads neither Hindi nor English newspapers.

(b) If she reads Hindi newspaper, find the probability that she reads English newspaper.

(c) If she reads English newspapers, find the probability she reads Hindi newspaper.

Ans. Let A represents the student reading Hindi newspaper and B represents the students reading English newspaper.

P (A) = $\frac{60}{100}$, P (B) = $\frac{40}{100}$ and P (A and B) = $\frac{20}{100}$

(a) P (neither Hindi nor English newspaper is read) = 1 – P (A or B)

= 1 – [P (A) + P (B) – P (A and B)]

$= 1-\left[\frac{60}{100}+\frac{40}{100}-\frac{20}{100}\right]$

$= 1-\frac{80}{100}=\frac{20}{100}=\frac{1}{5}$

(b) $P(A\cap B)=\frac{20}{100}$

$P(B|A)=\frac{P(A\cap B)}{P(A)}=\frac{20/100}{60/100}=\frac{20}{60}=\frac{1}{3}$

(c) $P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{20/100}{40/100}=\frac{20}{40}=\frac{1}{2}$

Choose the correct answer in the following:

17. The probability of obtaining an even prime number on each die when a pair of dice is rolled is:

(A) 0 (B) $\frac{1}{3}$ **(C)** $\frac{1}{12}$ **(D)** $\frac{1}{36}$

Ans. $n(S)$ = 36

Let A represents an even prime number on each dice.

$\therefore$ A = {2, 2} $\Rightarrow$ $n(\mathrm{A})$ = 1

P (A) = $\frac{n(\mathrm{A})}{n(\mathrm{S})} = \frac{1}{36}$

Hence option (D) is correct.

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18. Two events A and B are said to be independent, if:

(A) A and B are mutually exclusive.

(B) P (A'B') = [1 – P (A)] [1 – P (B)]

(C) P (A) = P (B)

(D) P (A) + P (B) = 1

Ans. P (A' and B')

= [1 – P (A)] . [1 – P (B)]

= P (A'). P (B')

Hence, option (B) is correct