CBSE Class–12 Mathematics

NCERT solution

Chapter - 8

Applications of Integrals - Miscellaneous Exercise

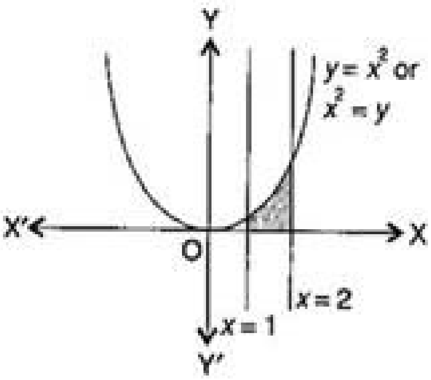
1. Find the area under the given curves and given lines:

(i) $y = x^2$, $x = 1$, $x = 2$ **and** $x-$ **axis.**

(ii) $y = x^4$, $x = 1$, $x = 5$ **and** $x-$ **axis.**

Ans. (i) Equation of the curve (parabola) is

$y = x^2$ ....(i)



Required area bounded by curve (i), vertical line $x = 1$, $x = 2$ and $x-$axis

$= \left|\int_1^2 y \, dx\right|$

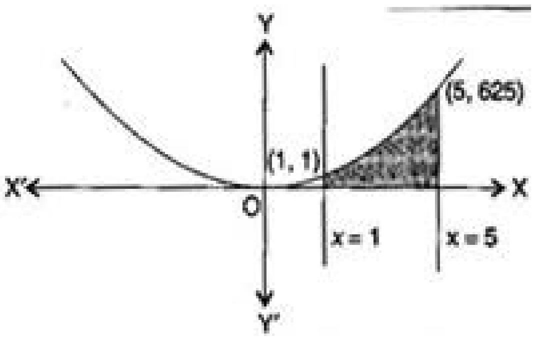
$= \left|\int_1^2 x^2 \, dx\right|$

$= \left(\frac{x^3}{3}\right)_1^2$

= $\frac{8}{3}-\frac{1}{3}=\frac{7}{3}$ sq. units

(ii) Equation of the curve

$y=x^4$ .....(i)



It is clear that curve (i) passes through the origin because $x=0$, from (i) $y=0$.

Table of values for curve $y=x^4$ for $x=1$ and $x=5$ (given)

x	1	2	3	4	5
y	1	$2^4=16$	$3^4=81$	$4^4=256$	$5^4=625$

Required shaded area between the curve $y=x^4$, vertical lines $x=1, x=5$ and $x-$ axis

= $\left|\int_1^5 y\ dx\right|$ = $\left|\int_1^5 x^4\ dx\right|$

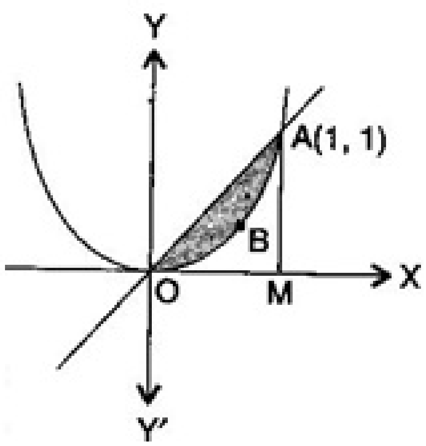
= $\left(\frac{x^5}{5}\right)_1^5$ = $\frac{5^5}{5}-\frac{1^5}{5}$

= $\frac{3125-1}{5}=\frac{3124}{5}$

= 624.8 sq. units

2. Find the area between the curves $y=x$ and $y=x^2$.

Ans. Equation of one curve (straight line) is $y=x$ .....(i)



Equation of second curve (parabola) is $y = x^2$ .....(ii)

Solving eq. (i) and (ii), we get $x = 0$ or $x = 1$ and $y = 0$ or $y = 1$

$\therefore$ Points of intersection of line (i) and parabola (ii) are O (0, 0) and A (1, 1).

Now Area of triangle OAM

= Area bounded by line (i) and $x-$ axis

= $\left|\int_0^1 y\ dx\right|$ = $\left|\int_0^1 x\ dx\right|$ = $\left(\frac{x^2}{2}\right)_0^1$

= $\frac{1}{2} - 0 = \frac{1}{2}$ sq. units

Also Area OBAM = Area bounded by parabola (ii) and $x-$ axis

= $\left|\int_0^1 y\ dx\right|$ = $\left|\int_0^1 x^2\ dx\right|$ = $\left(\frac{x^3}{3}\right)_0^1$

= $\frac{1}{3} - 0 = \frac{1}{3}$ sq. units

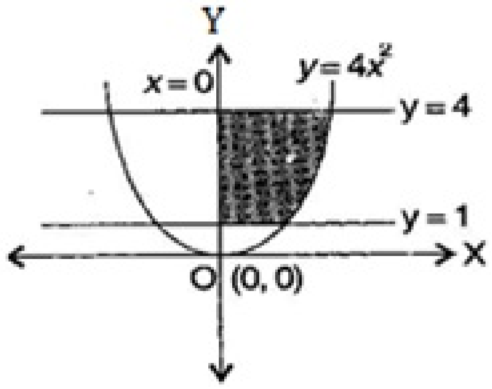
$\therefore$ Required area OBA between line (i) and parabola (ii)

= Area of triangle OAM – Area of OBAM

$= \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$ sq. units

3. Find the area of the region lying in the first quadrant and bounded by $y = 4x^2$, $x = 0$, $y = 1$ and $y = 4$.

Ans. Equation of the curve (parabola) is $y = 4x^2$



$\Rightarrow x^2 = \frac{y}{4}$ ....(i)

$\Rightarrow x = \frac{\sqrt{y}}{2}$ ....(ii)

Here required shaded area of the region lying in first quadrant bounded by parabola (i), $x = 0$

and the horizontal lines $y = 1$ and $y = 4$ is

$\left|\int_1^4 x\, dy\right| = \left|\int_1^4 \frac{\sqrt{y}}{2}\, dy\right| = \frac{1}{2}\left|\int_1^4 y^{\frac{1}{2}}\right|$

$= \frac{1}{2}\left|\frac{\left(y^{\frac{3}{2}}\right)_1^4}{\frac{3}{2}}\right|$

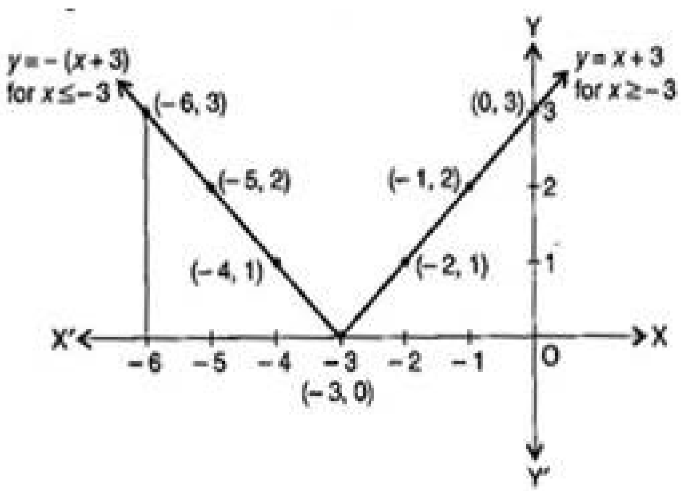
= $\frac{1}{2}.\frac{2}{3}\left(4^{\frac{3}{2}}-1^{\frac{3}{2}}\right)$

= $\frac{1}{3}\left(4\sqrt{4}-1\right)$

= $\frac{1}{3}(8-1)$ = $\frac{7}{3}$ sq. units

4. Sketch the graph of $y=|x+3|$ and evaluate $\int_{-6}^{0}|x+3|\,dx$.

Ans. Equation of the given curve is $y=|x+3|$ ....(i)



$\because$ $y=|x+3|$ $\geq$ 0 for all real x

$\therefore$ Graph of curve is only above the $x-$axis i.e., in first and second quadrant only.

$\therefore$ $y=|x+3|$

= $x+3$

If $x+3\geq$ 0

$\Rightarrow$ $x\geq -3$ ....(ii)

And $y = |x+3|$

= $-(x+3)$

If $x+3 \leq 0$

$\Rightarrow x \leq -3$ ....(iii)

Table of values for $y = x+3$ for $x \geq -3$

x	-3	-2	-1	0
y	0	1	2	3

Table of values for $y = x+3$ for $x \leq -3$

x	-3	-4	-5	-6
y	0	1	2	3

Now, $\int_{-6}^{0} |x+3| \, dx$

= $\int_{-6}^{-3} |x+3| \, dx$ + $\int_{-3}^{0} |x+3| \, dx$

= $\int_{-6}^{-3} -(x+3) \, dx$ + $\int_{-3}^{0} (x+3) \, dx$

= - $\left(\frac{x^2}{2} + 3x\right)_{-6}^{-3} + \left(\frac{x^2}{2} + 3x\right)_{-3}^{0}$

= $-\left[\frac{9}{2} - 9 - (18 - 18)\right] + \left[0 - \left(\frac{9}{2} - 9\right)\right]$

= $-\frac{9}{2} + 9 + 0 + 0 - \frac{9}{2} + 9$

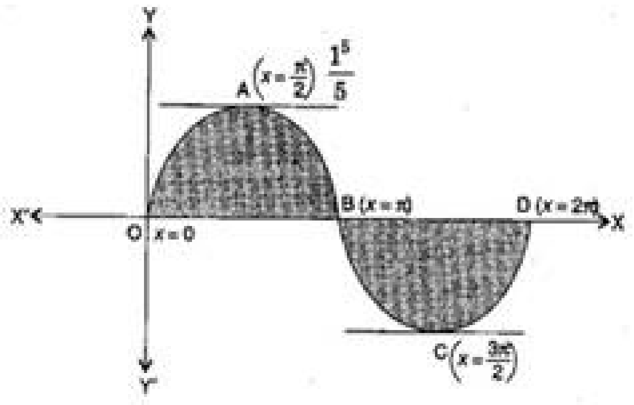
= $18 - \frac{18}{2} = 18 - 9 = 9$ sq. units

5. Find the area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$.

Ans. Equation of the curve is $y = \sin x$ ....(i)

$\because$ $y = \sin x \geq 0$ for $0 \leq x \leq \pi$ i.e., graph is in first and second quadrant.

And $y = \sin x \leq 0$ for $\pi \leq x \leq 2\pi$ i.e., graph is in third and fourth quadrant.



If tangent is parallel to $x-$ axis, then $\frac{dy}{dx} = 0$

$\Rightarrow \cos x = 0$

$\Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

Table of values for curve $y = \sin x$ between $x = 0$ and $x = 2\pi$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	0	1	0	-1	0

Now Required shaded area = Area OAB + Area BCD

= $\left|\int_0^{\pi} y\ dx\right| + \left|\int_{\pi}^{2\pi} y\ dx\right|$

= $\left|\int_0^{\pi} \sin x\ dx\right| + \left|\int_{\pi}^{2\pi} \sin x\ dx\right|$

= $\left|-(\cos x)_0^{\pi}\right| + \left|-(\cos x)_{\pi}^{2\pi}\right|$

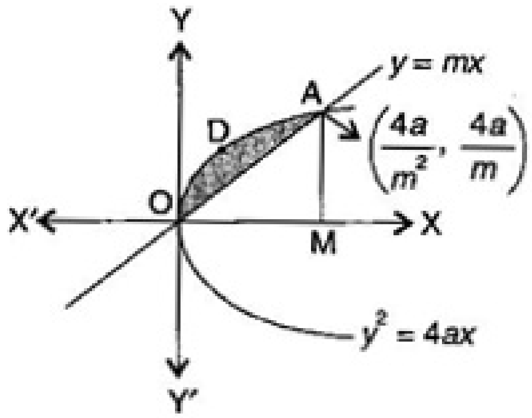
= $\left|-(\cos \pi - \cos 0)\right| + \left|-(\cos 2\pi - \cos \pi)\right|$

= $\left|-1(-1-1)\right| + \left|-(1+1)\right|$

= 2 + 2 = 4 sq. units

6. Find the area enclosed by the parabola $y^2 = 4ax$ and the line $y = mx$.

Ans. Equation of parabola is $y^2 = 4ax$ ....(i)



The area enclosed between the parabola $y^2 = 4ax$ and line $y = mx$ is represented by shaded area OADO.

Here Points of intersection of curve (i) and line $y = mx$ are O (0, 0) and A $\left(\frac{4a}{m^2}, \frac{4a}{m}\right)$.

Now Area ODAMO = Area of parabola and $x-$ axis

= $\left|\int_0^{\frac{4a}{m^2}} 2\sqrt{a}\, x^{\frac{1}{2}}\, dx\right|$

= $2\sqrt{a}\dfrac{\left(x^{\frac{3}{2}}\right)_0^{\frac{4a}{m^2}}}{\frac{3}{2}}$

= $\dfrac{4\sqrt{a}}{3}\left(\dfrac{4a}{m^2}\right)^{\frac{3}{2}}$

= $\dfrac{4\sqrt{a}}{3}.\dfrac{4a}{m^2}\sqrt{\dfrac{4a}{m^2}}$

= $\dfrac{32a^2}{3m^3}$ ....(ii)

Again Area of ΔOAM = Area between line $y = mx$ and $x-$ axis

= $\left|\int_0^{\frac{4a}{m^2}} mx\,dx\right|$ = $m\left(\dfrac{x^2}{2}\right)_0^{\frac{4a}{m^2}}$

= $\dfrac{m}{2}\left(\left(\dfrac{4a}{m^2}\right)^2-0\right)$

= $\dfrac{m}{2}.\dfrac{16a^2}{m^4}$ = $\dfrac{8a^2}{m^3}$ ....(ii)

$\therefore$ Requires shaded area = Area ODAMO – Area of ΔOAM

= $\dfrac{32a^2}{3m^3}-\dfrac{8a^2}{m^3}$

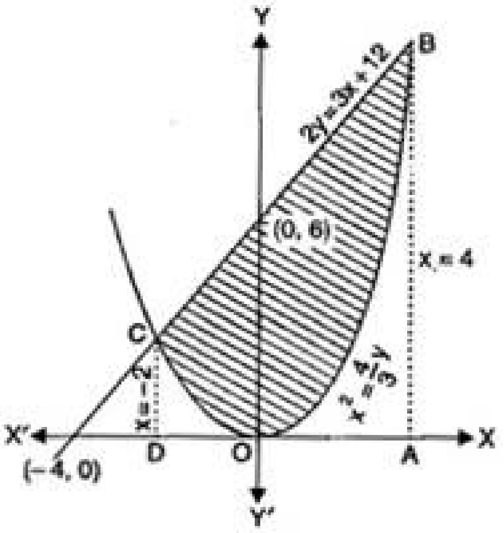
= $\dfrac{a^2}{m^3}\left(\dfrac{32}{3}-8\right)$

= $\frac{8a^2}{3m^3}$

7. Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$.

Ans. Equation of the parabola is

$4y = 3x^2$ ....(i)



$\Rightarrow x^2 = \frac{4}{3}y$

Equation of the line is $2y = 3x + 12$ ......(ii) $\Rightarrow$ $y = \frac{3x+12}{2}$ = $\frac{3x}{2} + 6$

In the graph, points of intersection are B (4, 12) and C $(-2, 3)$.

Now, Area ABCD = $\left| \int_{-2}^{4} \left(\frac{3}{2}x + 6 \right) dx \right|$

= $\left[\frac{3}{4}x^2 + 6x \right]_{-2}^{4}$

= $(12 + 24) - (3 - 12)$

= 45 sq. units

Again, Area CDO + Area OAB = $\left|\int_{-2}^{4}\left(\frac{3}{4}x^2\right)dx\right|$

= $\left[\frac{3}{4}.\frac{x^3}{3}\right]_{-2}^{4}$

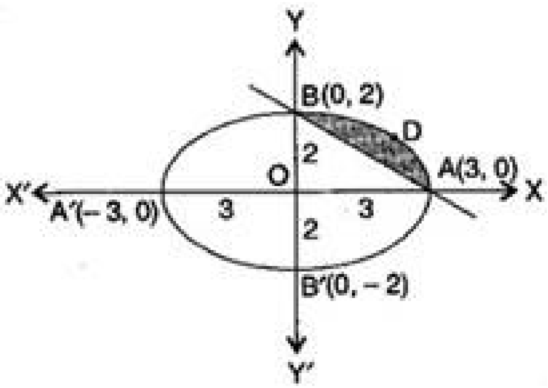
= $\frac{1}{4}\left[64-(-8)\right]$ = 18 sq. units

$\therefore$ Required area = Area ABCD – (Area CDO + Area OAB)

= 45 – 18 = 27 sq. units

8. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$.

Ans. Equation of the ellipse is



$\frac{x^2}{9}+\frac{y^2}{4}=1$ ....(i)

Here intersection of ellipse (i) with $x-$ axis are

A (3, 0) and A' $(-3, 0)$ and intersection of ellipse (i) with $y-$ axis are B (0, 2) and B' $(0, -2)$.

Also, the points of intersections of ellipse (i) and line $\frac{x}{3}+\frac{y}{2}=1$ are A (3, 0) and B (0, 2).

$\therefore$ Area OADB = Area between ellipse (i) (arc AB of it) and $x-$axis

= $\left|\int_0^3 \frac{2}{3}\sqrt{9-x^2}\ dx\right|$

= $\left|\int_0^3 \frac{2}{3}\sqrt{3^2-x^2}\ dx\right|$

= $\frac{2}{3}\left|\left[\frac{x}{2}\sqrt{3^2-x^2}+\frac{3^2}{2}\sin^{-1}\frac{x}{3}\right]\right|$

= $\frac{2}{3}\left[\frac{3}{2}\sqrt{9-9}+\frac{9}{2}\sin^{-1}1-\left(0+\frac{9}{2}\sin^{-1}0\right)\right]$

= $\frac{2}{3}\left[0+\frac{9}{2}.\frac{\pi}{2}-0\right]$

= $\frac{2}{3}.\frac{9\pi}{4}$ = $\frac{3\pi}{2}$ sq. units..........(ii)

Again Area of triangle OAB = Area bounded by line AB and $x-$axis

= $\left|\int_0^3 \frac{2}{3}\,(3-x)\,dx\right|$

= $\frac{2}{3}\left|\left(3x-\frac{x^2}{2}\right)_0^3\right|$

= $\frac{2}{3}\left\{\left(9-\frac{9}{2}\right)-0\right\}$

= $\frac{2}{3}.\frac{9}{2}=3$ sq. units ....(iii)

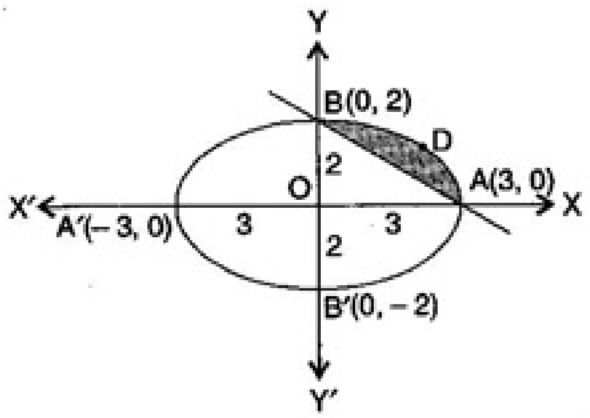
Now Required shaded area = Area OADB – Area OAB

= $\frac{3\pi}{2}-3$

= $3\left(\frac{\pi}{2}-1\right)=\frac{3}{2}(\pi-2)$ sq. units

9. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1$.

Ans. Equation of ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ……..(i)



Area between arc AB of the ellipse and $x-$ axis

= $\left|\int_0^a \frac{b}{a}\sqrt{a^2-x^2}\,dx\right|$

= $\frac{b}{a}\left|\int_0^a \sqrt{a^2-x^2}\,dx\right|$

= $\frac{b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$

$= \frac{b}{a}\left[0+\frac{a^2}{2}\sin^{-1}1-(0+0)\right]$

$= \frac{b}{a}.\frac{a^2}{2}.\frac{\pi}{2}=\frac{\pi ab}{4}$ ....(ii)

Also Area between chord AB and $x-$axis

$= \left|\int_0^a \frac{b}{a}(a-x)\ dx\right|$

$= \frac{b}{a}\left|\int_0^a (a-x)\ dx\right|$

$= \frac{b}{a}\left[ax-\frac{x^2}{2}\right]_0^a$

$= \frac{b}{a}\left(a^2-\frac{a^2}{2}\right)$

$= \frac{b}{a}.\frac{a^2}{2}=\frac{1}{2}ab$

Now Required area

= Area between arc AB of the ellipse and $x-$axis – Area between chord AB and $x-$axis

$= \frac{\pi ab}{4}=\frac{ab}{2}=\frac{ab}{4}(\pi-2)$ sq. units

10.Find the area of the region enclosed by the parabola $x^2=y,$ the line $y=x+2$ and $x-$ axis.

Ans. Equation of parabola is $x^2=y$ ....(i)

Equation of line is $y = x + 2$ ....(ii)

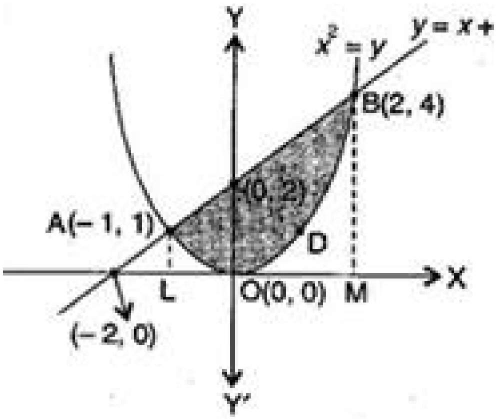
Here the two points of intersections of parabola (i) and line (ii) are A$\left(-1,1\right)$ and B (2, 4).

Area ALODBM = Area bounded by parabola (i) and $x-$ axis

= $\left|\int_{-1}^{2} x^2 \, dx\right|$ = $\left(\frac{x^3}{3}\right)_{-1}^{2}$

= $\frac{8}{3} - \left(\frac{-1}{3}\right)$

= $\frac{8}{3} + \frac{1}{3}$ = $\frac{9}{3}$ = 3 sq. units



AlsoArea of trapezium ALMB = Area bounded by line (ii) and $x-$ axis

= $\left|\int_{-1}^{2} (x+2) \, dx\right|$ = $\left(\frac{x^2}{2} + 2x\right)_{-1}^{2}$

= $2 + 4 - \left(\frac{1}{2} - 2\right)$

= $6 - \frac{1}{2} + 2$

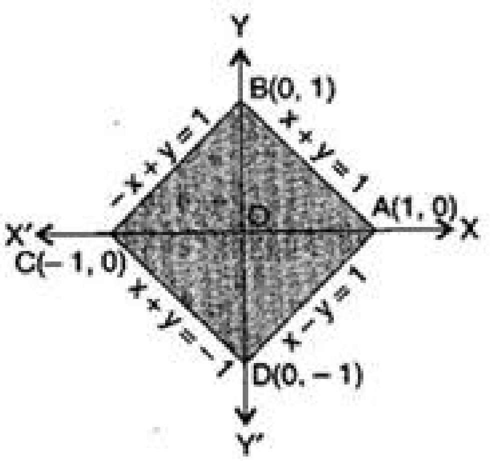
$= \frac{15}{2}$ sq. units

Now Required area = Area of trapezium ALMB – Area ALODBM

$= \frac{15}{2} - 3 = \frac{9}{2}$ sq. units

11.Using the method of integration, find the area enclosed by the curve $|x| + |y| = 1$.

Ans. Equation of the curve (graph) is



$|x| + |y| = 1$ ....(i)

The area bounded by the curve (i) is represented by the shaded region ABCD.

The curve intersects the axes at points A (1, 0), B (0, 1), C $(-1, 0)$ and D $(0, -1)$.

It is observed clearly that given curve is symmetrical about $x-$ axis and $y-$ axis.

$\therefore$ Area bounded by the curve

= Area of square ABCD

= 4 x ΔOAB

$= 4\left|\int_0^1 (1-x)\ dx\right|$

$= 4\left(x-\frac{x^2}{2}\right)_0^1$

$= 4\left[\left(1-\frac{1}{2}\right)-0\right]$

$= 4\times\frac{1}{2}$ = 2 sq. units

12.Find the area bounded by the curves $\left|(x, y): y \geq x^2 \text{ and } y=|x|\right|$.

Ans. The area bounded by the curves $\left\{(x, y): y \geq x^2 \text{ and } y=|x|\right\}$ is represented by the shaded region.

It is clearly observed that the required area is symmetrical about $y-$axis.

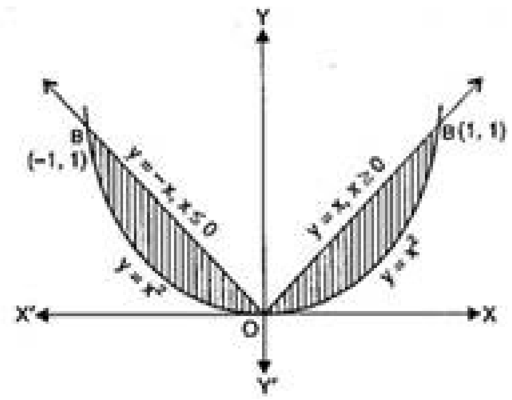
$\therefore$ Required area

= Area between parabola $y=x^2$ and $x-$axis between limits $x=0$ and $x=1$

$= \int_0^1 y\ dx = \int_0^1 x^2\ dx$

$= \left(\frac{x^3}{3}\right)_0^1 = \frac{1}{3}$ ....(i)

And Area of ray $y=x$ and $x-$axis,



$= \int_0^1 y\ dx = \int_0^1 x\ dx = \left(\frac{x^2}{2}\right)_0^1 = \frac{1}{2}$ ....(ii)

$\therefore$ Required shaded area in first quadrant

= Area between ray $y = x$ for $x \geq 0$ and $x-$ axis – Area between parabola $y = x^2$ and $x-$ axis in first quadrant

= Area given by eq. (ii) – Area given by eq. (i) = $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$ sq. units

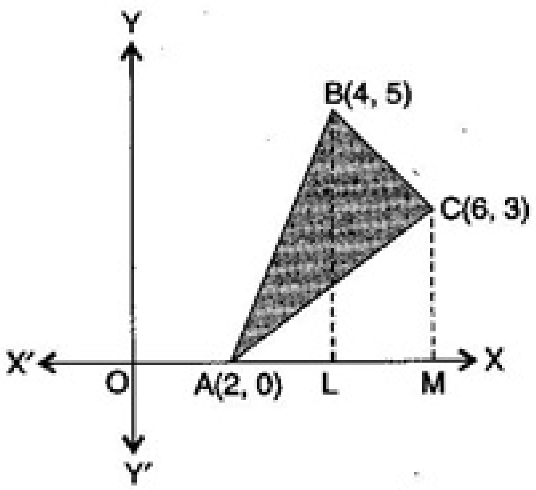
Similarly, shaded area in second quadrant = $\frac{1}{6}$ sq. units

Therefore, Total area of shaded region in the above figure

= $\frac{1}{6} + \frac{1}{6} = 2 \times \frac{1}{6} = \frac{1}{3}$ sq. units

13.Using the method of integration, find the area of the triangle ABC whose vertices are A (2, 0), B (4, 5) and C (6, 3).

Ans. Vertices of the given triangle are A (2, 0), B (4, 5) and C (6, 3).



Equation of side AB is $y - 0 = \frac{5-0}{4-2}(x-2)$

$y = \frac{5}{2}(x-2)$

Equation of side BC is $y-5=\frac{3-5}{6-4}(x-4)$

$y=9-x$

Equation of side AC is $y-0=\frac{3-0}{6-2}(x-2)$

$y=\frac{3}{4}(x-2)$

Now, Required shaded area = Area ΔALB + Area of trapezium BLMC – Area ΔAMC

= $\left|\int_2^4 \frac{5}{2}(x-2)\,dx\right|+\left|\int_4^6 (9-x)\,dx\right|-\left|\int_2^6 \frac{3}{4}(x-2)\,dx\right|$

= $\frac{5}{2}\left(\frac{x^2}{2}-2x\right)_2^4+\left|\left(9x-\frac{x^2}{2}\right)_4^6\right|-\frac{3}{4}\left(\frac{x^2}{2}-2x\right)_2^6$

= $\left[\frac{5}{2}(8-8)-(2-4)\right]+\left|54-18-(36-8)\right|-\left[\frac{3}{4}\{18-12-(2-4)\}\right]$

= $\frac{5}{2}(0+2)+\left|36-36+8\right|-\frac{3}{4}(6+2)$

= $\frac{5}{2}\times 2+8-\frac{3}{4}\times 8$

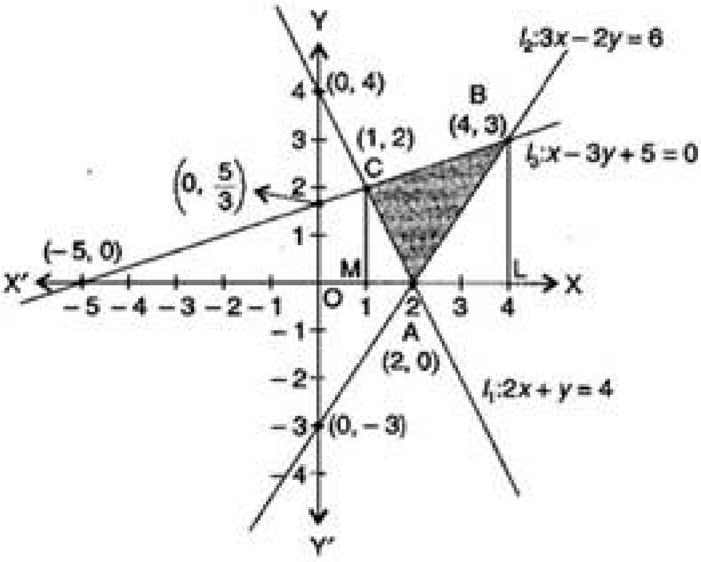
= 5 + 8 – 6 = 7 sq. units

14.Using the method of integration, find the area of the region bounded by the lines: $2x+y=4$, $3x-2y=6$ and $x-3y+5=0$.

Ans. Equation of one line l_1 is $2x+y=4$,

Equation of second line l_2 is $3x-2y=6$

And Equation of third line l_3 is $x-3y+5=0$.



Here, vertices of triangle ABC are A (2, 0), B (4, 3) and C (1, 2).

Now, Required area of triangle

= Area of trapezium CLMB – Area ΔACM $-$ Area ΔABL

= $\left|\int_1^4 \frac{1}{3}(x+5)\ dx\right| - \left|\int_1^2 (4-2x)\ dx\right| - \left|\int_2^4 \frac{3}{2}(x-2)\ dx\right|$

= $\frac{1}{3}\left|\left(\frac{x^2}{2}+5x\right)_1^4\right| - \left|\left(4x-\frac{2x^2}{2}\right)_1^2\right| - \frac{3}{2}\left|\left(\frac{x^2}{2}-2x\right)_2^4\right|$

= $\frac{1}{3}\left[8+20-\left(\frac{1}{2}+5\right)\right] - \{(8-4)-(4-1)\} - \frac{3}{2}\left|(8-8)-(2-4)\right|$

= $\frac{1}{3}\left(28-\frac{11}{2}\right)-(4-3)-\frac{3}{2}\times 2$

= $\frac{1}{3}\times\frac{45}{2}-1-3$

= $\frac{15}{2}-1-3=\frac{7}{2}$ sq. units

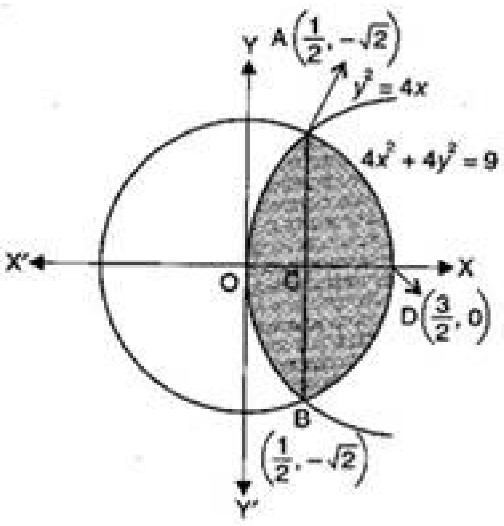
15.Find the area of the region $\left\{(x,y): y^2 \le 4x \text{ and } 4x^2+4y^2 \le 9\right\}$.

Ans. Equation of parabola is $y^2=4x$ ......(i)

And equation of circle is $4x^2+4y^2=9$ ......(ii)

Here, the two points of intersection of parabola (i) and circle (ii) are A $\left(\frac{1}{2},\sqrt{2}\right)$ and B $\left(\frac{1}{2},-\sqrt{2}\right)$

Required shaded area OADBO (Area of the circle which is interior to the parabola)



= 2 x Area OADO = 2 [Area OAC + Area CAD]

$$= 2\left[\left|\int_0^{\frac{1}{2}} 2\sqrt{x}\,dx + \left|\int_{\frac{1}{2}}^{\frac{3}{2}} \sqrt{\frac{9}{4}-x^2}\,dx\right|\right|\right]$$

$$= 2\left[\left\{2.\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right\}_0^{\frac{1}{2}} + \left\{\frac{x\sqrt{\frac{9}{4}-x^2}}{2} + \frac{9/4}{2}\sin^{-1}\frac{x}{3/2}\right\}_{\frac{1}{2}}^{\frac{3}{2}}\right]$$

$$=2\left[\frac{4}{3}\times\frac{1}{2\sqrt{2}}+\frac{9}{8}\sin^{-1}1-\frac{\frac{1}{2}\sqrt{2}}{2}-\frac{9}{8}\sin^{-1}\frac{1}{3}\right]$$

$$= 2\left[\frac{\sqrt{2}}{3}+\frac{9}{8}.\frac{\pi}{2}-\frac{\sqrt{2}}{4}-\frac{9}{8}\sin^{-1}\frac{1}{3}\right]$$

$$= \left(\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}+\frac{\sqrt{2}}{6}\right) \text{ sq. units}$$

16.Choose the correct answer:

Area bounded by the curve $y = x^3$ the $x-$ axis and the ordinate $x = -2$ and $x = 1$ is:

(A) -9

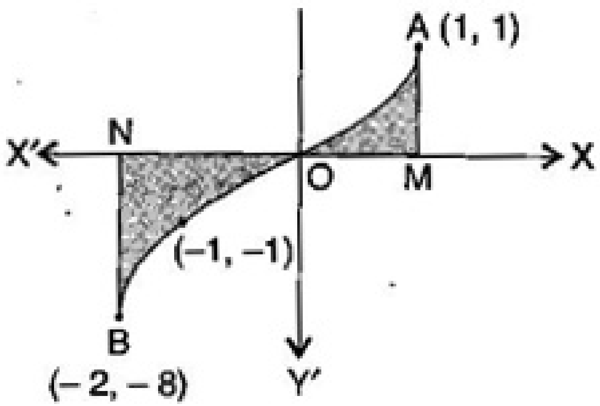
(B) $\frac{-15}{4}$

(C) $\frac{15}{4}$

(D) $\frac{17}{4}$

Ans. Equation of the curve is $y = x^3$

To find: Area OBN ($y = x^3$ for $-2 \le x \le 0$) and Area OAM ($y = x^3$ for $0 \le x \le 1$)



$\therefore$ Required area = Area OBN + Area OAM

= $\left|\int_{-2}^{0} x^3\, dx\right| + \left|\int_{0}^{1} x^3\, dx\right|$

= $\left|\left(\frac{x^4}{4}\right)_{-2}^{0}\right| + \left|\left(\frac{x^4}{4}\right)_{0}^{1}\right|$

= $\left|0 - \frac{16}{4}\right| + \left|\frac{1}{4} - 0\right|$

= $4 + \frac{1}{4}$ = $\frac{17}{4}$ sq. units

Therefore, option (D) is correct,

17.Choose the correct answer:

The area bounded by the curve $y = x|x|$, $x-$ axis and the ordinates $x = -1$ and $x = 1$ is given by:

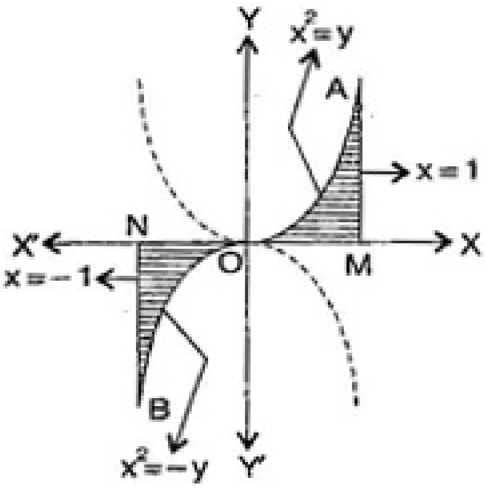
(A) 0

(B) $\frac{1}{3}$

(C) $\frac{2}{3}$

(D) $\frac{4}{3}$

Ans. Equation of the curve is



$y = x|x| = x(x) = x^2$ if $x \geq 0$ ....(i)

And $y = x|x| = x(-x) = -x^2$ if $x \leq 0$ ....(ii)

Required area = Area ONBO + Area OAMO

= $\left|\int_{-1}^{0} -x^2 \; dx\right| + \left|\int_{0}^{1} x^2 \; dx\right|$

= $\left|\left(\frac{-x^3}{3}\right)_{-1}^{0}\right| + \left|\left(\frac{x^3}{3}\right)_{0}^{1}\right|$

= $0 - \left(\frac{-1}{3}\right) + \frac{1}{3} - 0 = \frac{2}{3}$ sq. units

Therefore, option (C) is correct.

18.Choose the correct answer:

The area of the circle $x^2+y^2=16$ exterior to the parabola $y^2=6x$.

(A) $\frac{4}{3}\left(4\pi-\sqrt{3}\right)$

(B) $\frac{4}{3}\left(4\pi+\sqrt{3}\right)$

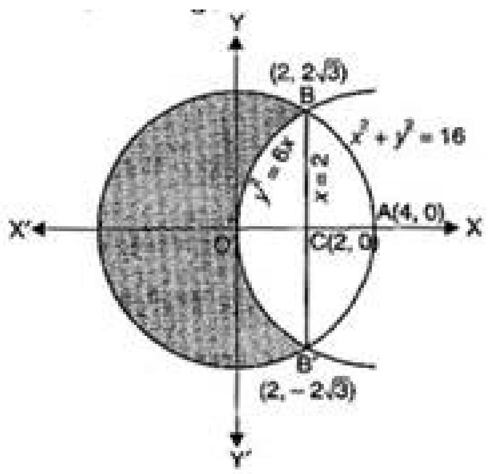
(C) $\frac{4}{3}\left(8\pi-\sqrt{3}\right)$

(D) $\frac{4}{3}\left(8\pi+\sqrt{3}\right)$

Ans. Equation of the circle is $x^2+y^2=16$ .....(i)

This circle is symmetrical about $x-$axis and $y-$ axis.

Here two points of intersection are B$\left(2,2\sqrt{3}\right)$ and B'$\left(2,-2\sqrt{3}\right)$.



Required area = Area of circle – Area of circle interior to the parabola

= πr^2- Area OBAB'O

= $16\pi-$ 2 x Area OBACO $\left[\because r=4\right]$

= $16\pi-$ 2[Area OBCO + Area BACB]

$$= 16\pi - 2\left[\left|\int_0^2 \sqrt{6x}\ dx\right| + \left|\int_2^4 \sqrt{16-x^2}\ dx\right|\right]$$

$$= 16\pi - 2\left[\sqrt{6}.\left(\frac{x^{3/2}}{3/2}\right)_0^2 + \left(\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\frac{\pi}{4}\right)\right]_2^4$$

$$= 16\pi - 2\left[\frac{2}{3}\sqrt{6}\left(2\sqrt{2}\right) + 8\sin^{-1}1 - \sqrt{12} - 8\sin^{-1}\frac{1}{2}\right]$$

$$= 16\pi - 2\left[\frac{8}{\sqrt{3}} + 8.\frac{\pi}{2} - 2\sqrt{3} - 8.\frac{\pi}{6}\right]$$

$$= 16\pi - 2\left[\frac{8}{\sqrt{3}} - 2\sqrt{3} + 8\pi\left(\frac{1}{2} - \frac{1}{6}\right)\right]$$

$$= 16\pi - 2\left[\frac{8-6}{\sqrt{3}} + 8\pi\left(\frac{3-1}{6}\right)\right]$$

$$= 16\pi - 2\left[\frac{2}{\sqrt{3}} + \frac{8\pi}{3}\right]$$

$$= 16\pi - \frac{4}{\sqrt{3}} - \frac{16\pi}{3}$$

$$= 16\pi\left(1 - \frac{1}{3}\right) - \frac{4}{\sqrt{3}}$$

$$= \frac{32\pi}{3} - \frac{4}{\sqrt{3}}$$

$$= \frac{32\pi}{3} - \frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

= $\frac{4}{3}\left(8\pi-\sqrt{3}\right)$ sq. units

19.Choose the correct answer:

The area bounded by the $y-$axis, $y=\cos x$ and $y=\sin x$ when $0\le x\le\frac{\pi}{2}$ is:

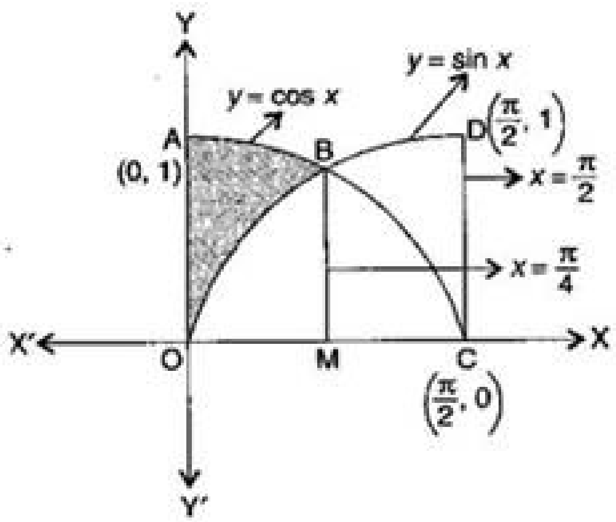
(A) $2\left(\sqrt{2}-1\right)$

(B) $\sqrt{2}-1$

(C) $\sqrt{2}+1$

(D) $\sqrt{2}$

Ans. Here both graphs intersect at the point B $\left(\frac{\pi}{4},\frac{1}{\sqrt{2}}\right)$.



Required shaded area = Area OABC – Area OBC

= Area OABC – (Area OBM + Area BCM)

= $\left|\int_0^{\pi/2}\cos x\,dx\right|-\left(\left|\int_0^{\pi/4}\sin x\,dx\right|+\left|\int_{\pi/4}^{\pi/2}\cos x\,dx\right|\right)$

= $\left|(\sin x)_0^{\pi/2}\right| - \left(\left|(-\cos x)_0^{\pi/4}\right| + (\sin x)_{\pi/4}^{\pi/2}\right)$

= $\left(\sin\frac{\pi}{2} - \sin 0°\right) - \left(\left|-\cos\frac{\pi}{4} + \cos 0°\right| + \left|\sin\frac{\pi}{2} - \sin\frac{\pi}{4}\right|\right)$

= $1 - 0 - \left(\frac{-1}{\sqrt{2}} + 1 + 1 - \frac{1}{\sqrt{2}}\right)$

= $1 + \frac{1}{\sqrt{2}} - 1 - 1 + \frac{1}{\sqrt{2}}$

= $\frac{2}{\sqrt{2}} - 1$ = $\left(\sqrt{2} - 1\right)$ sq. units

Therefore, option (B) is correct.