CBSE Class–12 Mathematics

NCERT solution

Chapter - 13

Probability - Exercise 13.3

1. An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red?

Ans. Case (i) : S_1 = {5 red balls, 5 black balls}

$\Rightarrow n(S_1) = 10$

Let us draw a red balls first, i.e., A_1 = {5 red balls}

$\Rightarrow n(A_1) = 5$

$P(A_1) = \frac{n(A_1)}{n(S_1)} = \frac{5}{10} = \frac{1}{2}$

Now after adding 2 balls of the same colour, i.e., when the first draw gives a red ball, two additional red balls are put in the urn so that its contents are 7 (5 + 2) red and 5 balck balls. When the first draw gives a black ball, two addtional black balls are put in the urn so that its contents are 5 red and 7 (5 + 2) black balls.

Total balls = S_2 = {7 red balls, 5 black balls}

$\Rightarrow n(S_2) = 12$

Let us draw a red balls first, i.e., A_2 = {7 red balls}

$\Rightarrow n(A_2) = 7$

P (A_2) = $\frac{n(A_2)}{n(S_2)} = \frac{7}{12}$

$\therefore$ P (a red ball is drawn) = $\frac{1}{2} \times \frac{7}{12} = \frac{7}{24}$

Case (ii) : When a black ball is drawn, i.e., A_2 = {5 red balls}

$\Rightarrow n(A_2)$ = 5

P (A_1) = $\frac{n(A_2)}{n(S_1)} = \frac{5}{12}$

Now after adding 2 balls of the same colour, i.e.,

S_2 = {5 red balls, 7 black balls}

$\Rightarrow n(S_2)$ = 12

Let us draw a red balls first, i.e., A_2 = {5 red balls}

$\Rightarrow n(A_2)$ = 5

P (A_2) = $\frac{n(A_2)}{n(S_2)} = \frac{5}{12}$

$\because$ P (a red ball is drawn) = $\frac{1}{2} \times \frac{5}{12} = \frac{5}{24}$

Therefore, required probability in both cases

= Probability that first ball is red and then second ball after two red are added in the urn is also red + Probability that first ball is black and second is red = $\frac{7}{24} + \frac{5}{24} = \frac{12}{24} = \frac{1}{2}$

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2. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls.

One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

Ans. Let A be the event hat ball drawn is red and let E1 and E2 be the events that the ball drawn is from the first bag and second bag respectively.

P $(E_1) = \frac{1}{2}$, P $(E_2) = \frac{1}{2}$,

$P\left(A|E_1\right)$ = P (drawing a red ball from bag I) = $\frac{4}{4+4} = \frac{4}{8} = \frac{1}{2}$

$P\left(A|E_2\right)$ = P (drawing a red ball from bag II) = $\frac{2}{4+4} = \frac{2}{8} = \frac{1}{4}$

Therefore, by Bayes' theorem,

$P\left(E_1|A\right)$ = P (red ball drawn from bag I) = $\frac{P(E_1)P\left(A|E_1\right)}{P(E_1)P\left(A|E_1\right)+P(E_2)P\left(A|E_2\right)}$

$$= \frac{\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{1}{4}+\frac{1}{8}} = \frac{1}{4}\times\frac{8}{3} = \frac{2}{3}$$

3. Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is a hostler?

Ans. Let E_1 = the examinee knows the answer, E_2 = the examinee guesses the answer and

A = student who attain grade A, P $(E_1) = \frac{60}{100}$, P $(E_2) = \frac{40}{100}$,

$P\left(A|E_1\right) = \frac{30}{100}$, $P\left(A|E_2\right) = \frac{20}{100}$

Therefore, by Bayes' theorem,

$$P\left(E_1|A\right) = \frac{P(E_1)P\left(A|E_1\right)}{P(E_1)P\left(A|E_1\right)+P(E_2)P\left(A|E_2\right)}$$

$$= \frac{\frac{60}{100}\times\frac{30}{100}}{\frac{60}{100}\times\frac{30}{100}+\frac{40}{100}\times\frac{20}{100}} = \frac{1800}{1800+800} = \frac{1800}{2600} = \frac{9}{13}$$

4. In answering a question on a multiple choice test a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses the answer, will be correct with probability $\frac{1}{4}$. What is the probability that a student knows the answer given that he answered it correctly?

Ans. Let E_1 = students residing in the hostel, E_2 = day scholars (not residing in the hostel) and

A = the examinee answers correctly

Now P (E_1) = $\frac{3}{4}$, P (E_2) = $\frac{1}{4}$,

Since E_1 and E_2 are mutually exclusive events and exhaustive events, and if E_2 has already occurred, then the examinee guesses, therefore the probability that he answers correctly given that he has made a guess is $\frac{1}{4}$ i.e., $P\left(A|E_2\right) = \frac{1}{4}$

And $P\left(A|E_1\right)$ = P (answers correctly given that he knew the answer) = 1

Therefore, by Bayes' theorem,

$P(E_1|A) = \dfrac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$

$= \dfrac{\frac{3}{4}\times 1}{\frac{3}{4}\times 1+\frac{1}{4}\times\frac{1}{4}} = \dfrac{\frac{3}{4}}{\frac{3}{4}+\frac{1}{16}} = \dfrac{\frac{3}{4}}{\frac{13}{16}} = \dfrac{12}{13}$

5. A laboratory blood test is 99% effective in detecting a certain disease when it is, in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i.e., if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive?

Ans. Let E_1 = The person selected is suffering from certain disease, E_2 = The person selected is not suffering from certain disease and A = The doctor diagnoses correctly

Now P (E_1) = 0.1% = $\dfrac{1}{1000}$ = 0.001, P (E_2) = $1-\dfrac{1}{1000}=\dfrac{999}{1000}=0.999$,

$P(A|E_1)$ = 99% = $\dfrac{99}{100}=0.99$ $P(A|E_2)$ = 0.005%

Therefore, by Bayes' theorem,

$P(E_1|A) = \dfrac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$

$= \dfrac{0.01\times 0.99}{.001\times 0.99+0.999\times 0.005} = \dfrac{990}{990+4995} = \dfrac{22}{133}$

6. There are three coins. One is a two headed coin, another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows head, what is the probability that it was the two headed

coin?

Ans. Let E_1 = a two headed coin, E_2 = a biased coin, E_3 = an unbiased coin and A = A head is shown

Now P (E_1) = $\frac{1}{3}$, P (E_2) = $\frac{1}{3}$, P (E_3) = $\frac{1}{3}$

$P\left(A|E_1\right)$ = 1, $P\left(A|E_2\right)$ = $\frac{75}{100}=\frac{3}{4}$ and $P\left(A|E_3\right)$ = $\frac{1}{2}$

Therefore, by Bayes' theorem,

$$P\left(E_1|A\right) = \frac{P(E_1)P\left(A|E_1\right)}{P(E_1)P\left(A|E_1\right)+P(E_2)P\left(A|E_2\right)+P(E_3)P\left(A|E_3\right)}$$

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1+\frac{1}{3}\times\frac{3}{4}+\frac{1}{3}\times\frac{1}{2}} = \frac{1}{1+\frac{3}{4}+\frac{1}{2}} = \frac{4}{4+3+2} = \frac{4}{9}$$

7. An insurance company insured 2000 scooter driver, 4000 car drivers and 6000 truck drivers. The probability of accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

Ans. Let E_1 = Person chosen is a scooter driver, E_2 = Person chosen is a car driver, E_3 = Person chosen is a truck driver and A = Person meets with an accident

Since there are 12000 persons, therefore,

Now P (E_1) = $\frac{2000}{12000}=\frac{1}{6}$, P ($E_2$) = $\frac{4000}{12000}=\frac{1}{3}$, P ($E_3$) = $\frac{6000}{12000}=\frac{1}{2}$

It is given that $P\left(A|E_1\right)$ = P (a person meets with an accident, he is a scooter driver) = 0.01

Similarly, $P\left(A|E_2\right)$ = 0.03 and $P\left(A|E_3\right)$ = 0.15

To find: P (person meets with an accident that he was a scooter driver)

Therefore, by Bayes' theorem,

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)}$$

$$= \frac{\frac{1}{6}\times 0.01}{\frac{1}{6}\times 0.01+\frac{1}{3}\times 0.03+\frac{1}{2}\times 0.15} = \frac{1}{1+6+45} = \frac{1}{52}$$

8. A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B?

Ans. Given: P (A) = $\frac{60}{100}$; P (B) = $\frac{40}{100}$

Let D denotes a defective item:

$\therefore\ P(D|A) = \frac{2}{100}$ and $P(D|B) = \frac{1}{100}$

$$P(B|D) = \frac{P(B)P(D|B)}{P(A)P(D|A)+P(B)P(D|B)} =$$

$$\frac{\frac{40}{100}\times\frac{1}{100}}{\frac{60}{100}\times\frac{2}{100}+\frac{40}{100}\times\frac{1}{100}} = \frac{40}{120+40} = \frac{40}{160} = \frac{1}{4}$$

9. Two groups are competing for the position on the Board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability if 0.3, if the second group wins. Find

the probability that the new product introduced was by the second group.

Ans. Given: P (G_1) = 0.6, P (G_2) = 0.4

Let P denotes the launching of new product.

$\therefore$ P $(P|G_1)$ = 0.7, P $(P|G_2)$ = 0.3

$$P(G_1|P) = \frac{P(G_2)P(P|G_2)}{P(G_1)P(P|G_1)+P(G_2)P(P|G_2)} = \frac{0.4\times0.3}{0.6\times0.7+0.4\times0.3} = \frac{12}{54} = \frac{2}{9}$$

10. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4 she tosses a coin once and noted whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 and 4 with the die?

Ans. Let E_1 = 5 or 6 appears on a die, E_2 = 1, 2, 3 or 4 appears on a die and A = A head appears on the coin.

Now P (E_1) = $\frac{2}{6} = \frac{1}{3}$, P (E_2) = $\frac{4}{6} = \frac{2}{3}$

Now $P(A|E_1)$ Probability of getting a head on tossing a coin three times,

when E_1 has already occurs = P (HTT) or P (THT) or P (TTH)

$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{3}{8}$$

$P(A|E_2)$ = Probability of getting a head on tossing a coin once,

when E_2 has already occurred = $\frac{1}{2}$

$\therefore$ P (there is exactly one head given that 1, 2, 3 or 4 appears on a die)

$$P(E_2|A) = \frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$$

$$= \frac{\frac{2}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{3}{8}+\frac{2}{3}\times\frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{8}+\frac{1}{3}} = \frac{24}{3\times11} = \frac{8}{11} = \frac{\frac{2}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{3}{8}+\frac{2}{3}\times\frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{8}+\frac{1}{3}} = \frac{8}{3+8} = \frac{8}{11}$$

11. A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, whereas the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B on the job for 30% of the time and C on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A?

Ans. Let E_1 = the item is manufactured by the operator A, E_2 = the item is manufactured by the operator B, E_3 = the item is manufactured by the operator C and A = the item is defective

Now $P(E_1) = \frac{50}{100}$, $P(E_2) = \frac{30}{100}$, $P(E_3) = \frac{20}{100}$

Now $P(A|E_1)$ = P (item drawn is manufactured by operator A) = $\frac{1}{100}$

Similarly, $P(A|E_2) = \frac{5}{100}$ and $P(A|E_3) = \frac{7}{100}$

Now Required probability = Probability that the item is manufactured by operator A given that the item drawn is defective

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)}$$

$$= \frac{\frac{50}{100}\times\frac{1}{100}}{\frac{50}{100}\times\frac{1}{100}+\frac{30}{100}\times\frac{5}{100}+\frac{20}{100}\times\frac{7}{100}} = \frac{50}{50+150+140} = \frac{5}{34}$$

12. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

Ans. Let E_1 = the missing card is a diamond, E_2 = the missing card is a spade, E_3 = the missing card is a club, E_4 = the missing card is a heart and A = drawing of two heart cards from the remaining cards.

Now P (E_1) = $\frac{13}{52} = \frac{1}{4}$, P ($E_2$) = $\frac{13}{52} = \frac{1}{4}$, P ($E_3$) = $\frac{13}{52} = \frac{1}{4}$, P ($E_4$) = $\frac{13}{52} = \frac{1}{4}$

$P\left(A|E_1\right)$ = P (drawing 2 heart cards given that one diamond card is missing) = $\frac{C(12,2)}{C(51,2)}$

Similarly, $P\left(A|E_2\right) = \frac{C(13,2)}{C(51,2)}$, $P\left(A|E_3\right) = \frac{C(13,2)}{C(51,2)}$ and $P\left(A|E_4\right) = \frac{C(13,2)}{C(51,2)}$

By Bayes' theorem,

$$P\left(E_1|A\right) = \frac{P(E_1)P\left(A|E_1\right)}{P(E_1)P\left(A|E_1\right)+P(E_2)P\left(A|E_2\right)+P(E_3)P\left(A|E_3\right)+P(E_4)P\left(A|E_4\right)}$$

$$= \frac{\frac{1}{4}\times\frac{C(12,2)}{C(51,2)}}{\frac{1}{4}\times\frac{C(12,2)}{C(51,2)}+\frac{1}{4}\times\frac{C(13,2)}{C(51,2)}+\frac{1}{4}\times\frac{C(13,2)}{C(51,2)}+\frac{1}{4}\times\frac{C(13,2)}{C(51,2)}}$$

$$= \frac{66}{66+78+78+78} = \frac{11}{50}$$

13. Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed. A report that a head appears. The probability that actually there was head is:

(A) $\frac{4}{5}$

(B) $\frac{1}{2}$

(C) $\frac{1}{5}$

(D) $\frac{2}{5}$

Ans. Let A be the event that the man reports that head occurs in tossing a coin and let E_1 be the event that head occurs and E_2 be the event head does not occur.

$P(E_1) = \frac{1}{2}$, $P(E_2) = \frac{1}{2}$

$P(A|E_1)$ = P (A reports that head occurs when head had actually occur red on the coin) = $\frac{4}{5}$

$P(A|E_2)$ = P (A reports that head occurs when head had not occur red on the coin) = $1-\frac{4}{5}=\frac{1}{5}$

By Bayes' theorem,

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)} = \frac{\frac{1}{2}\times\frac{4}{5}}{\frac{1}{2}\times\frac{4}{5}+\frac{1}{2}\times\frac{1}{5}} = \frac{4}{4+1} = \frac{4}{5}$$

Hence, option (A) is correct.

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14. If A and B are two events such that $A \subset B$ and $P(B) \neq 0$, then which of the following is correct:

(A) $\mathbf{P}\left(A \middle| B\right) = \dfrac{P(A)}{P(B)}$

(B) $\mathbf{P}\left(A \middle| B\right) < \mathbf{P(A)}$

(C) $\mathbf{P}\left(A \middle| B\right) \geq \mathbf{P(A)}$

(D) None of these

Ans. $A \subset B \Rightarrow A \cap B = A$ and $P(B) \neq 0 \Rightarrow P\left(A \middle| B\right) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A)}{P(B)}$

Since $P(B) \neq 0$

$\therefore \dfrac{P(A)}{P(B)} < 1 \Rightarrow P(A) < P(B) \Rightarrow P\left(A \middle| B\right) \geq P(A)$

Hence, option (C) is correct.