CBSE Class–12 Mathematics

NCERT solution

Chapter - 7

Integrals - Exercise 7.2

Integrate the functions in Exercise 1 to 8.

1. $\frac{2x}{1+x^2}$

Ans. Putting $1+x^2=t$

$\Rightarrow 2x=\frac{dt}{dx}$

$\Rightarrow 2x\,dx=dt$

$\therefore \int\frac{2x}{1+x^2}\,dx=\int\frac{dt}{t}$

= $\int\frac{1}{t}\,dt=\log|t|+c$

= $\log|1+x^2|+c$

2. $\frac{(\log x)^2}{x}$

Ans. Putting $\log x=t$

$\Rightarrow \frac{1}{x}=\frac{dt}{dx}$

$\Rightarrow \dfrac{dx}{x} = dt$

$\therefore \int \dfrac{(\log x)^2}{x}\ dx$

= $\int t^2\ dt$

= $\dfrac{t^3}{3} + c$

= $\dfrac{1}{3}(\log x)^3 + c$

3. $\dfrac{1}{x + x\log x}$

Ans. Putting $1 + \log x = t$

$\Rightarrow \dfrac{1}{x} = \dfrac{dt}{dx}$

$\Rightarrow \dfrac{dx}{x} = dt$

$\therefore \int \dfrac{1}{x + x\log x}\ dx$

= $\int \dfrac{1}{1+\log x}\ \dfrac{dx}{x}$

= $\int \dfrac{1}{t}\ dt = \log|t| + c$

= $\log|1 + \log x| + c$

4. $\sin x \sin(\cos x)$

Ans. Putting $\cos x = t$

$\Rightarrow -\sin x = \frac{dt}{dx}$

$\Rightarrow -\sin x\, dx = dt$

$\therefore \int \sin x \sin(\cos x)\, dx$

= $-\int \sin(\cos x)(-\sin x\, dx)$

= $-\int \sin t\, dt$

= $-(-\cos t) + c$

= $\cos t + c$ = $\cos(\cos x) + c$

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5. $\sin(ax+b)\cos(ax+b)$

Ans. $\int \sin(ax+b)\cos(ax+b)\, dx$

= $\frac{1}{2}\int 2\sin(ax+b)\cos(ax+b)\, dx$

= $\frac{1}{2}\int \sin 2(ax+b) dx$

= $\frac{1}{2}\int \sin(2ax+2b)\, dx$

= $\frac{1}{2}\left[\frac{-\cos(2ax+2b)}{2a}\right] + c$ [because $\int \text{Sin}(ax+b)dx = -\frac{1}{a}\text{Cos}(ax+b)$]

= $\frac{-1}{4a}\cos 2(ax+b) + c$

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6. $\sqrt{ax+b}$

Ans. $\int \sqrt{ax+b}\ dx$

= $\int (ax+b)^{\frac{1}{2}}\ dx$

Using $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$ We have

$\int (ax+b)^{\frac{1}{2}}\ dx$ = $\dfrac{(ax+b)^{\frac{3}{2}}}{\frac{3}{2}a} + c$

= $\frac{2}{3a}(ax+b)^{\frac{3}{2}} + c$

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7. $x\sqrt{x+2}$

Ans. $\int x\sqrt{x+2}\ dx$

= $\int \{(x+2)-2\}\sqrt{x+2}\ dx$

= $\int \left\{(x+2)(x+2)^{\frac{1}{2}} - 2(x+2)^{\frac{1}{2}}\right\} dx$

= $\int \left\{(x+2)^{\frac{3}{2}} - 2(x+2)^{\frac{1}{2}}\right\} dx$

= $\int (x+2)^{\frac{3}{2}}\ dx - 2\int (x+2)^{\frac{1}{2}}\ dx$

Using $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$

= $\dfrac{(x+2)^{\frac{5}{2}}}{\frac{5}{2}} - 2\dfrac{(x+2)^{\frac{3}{2}}}{\frac{3}{2}} + c$

= $\frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + c$

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8. $x\sqrt{1+2x^2}$

Ans. Let I = $\int x\sqrt{1+2x^2}\ dx$

= $\frac{1}{4}\int \sqrt{1+2x^2}\,(4x\ dx)$ ....(i)

Putting $1+2x^2 = t$

$\Rightarrow 4x = \frac{dt}{dx}$

$\Rightarrow 4x\ dx = dt$

∴ From eq. (i),

I = $\frac{1}{4}\int \sqrt{t}\ dt$

= $\frac{1}{4}\int t^{\frac{1}{2}}\ dt$

= $\frac{1}{4}\dfrac{t^{3/2}}{3/2} + c$

= $\frac{1}{4}.\frac{2}{3}t^{3/2} + c$

= $\frac{1}{6}\left(1+2x^2\right)^{3/2}+c$

Integrate the functions in Exercise 9 to 17.

9. $(4x+2)\sqrt{x^2+x+1}$

Ans. Let I = $\int(4x+2)\sqrt{x^2+x+1}\ dx$

= $\int 2(2x+1)\sqrt{x^2+x+1}\ dx$

= $\int 2\sqrt{x^2+x+1}(2x+1)\ dx$ .....(i)

Putting $x^2+x+1=t$

$\Rightarrow (2x+1)=\frac{dt}{dx}$

$\Rightarrow (2x+1)\ dx=dt$

$\therefore$ From eq. (i), I = $\int 2\sqrt{t}\ dt$

= $2\int t^{\frac{1}{2}}\ dt$

= $2\frac{t^{3/2}}{3/2}+c$

= $\frac{4}{3}t^{3/2}+c$

= $\frac{4}{3}\left(x^2+x+1\right)^{3/2}+c$

10. $\frac{1}{x-\sqrt{x}}$

Ans. Let I = $\int \frac{1}{x-\sqrt{x}}\ dx$ ....(i)

Putting $\sqrt{x}=t$

$\Rightarrow\ x=t^2$

$\Rightarrow\ \frac{dx}{dt}=2t$

$\Rightarrow\ dx=2t\ dt$

$\therefore$ From eq. (i),

I = $\int \frac{1}{t^2-t}2t\ dt$

= $2\int \frac{t}{t(t-1)}\ dt$

= $2\int \frac{1}{(t-1)}\ dt$

= $2\log|t-1|+c$

= $2\log\left|\sqrt{x}-1\right|+c$

11. $\frac{x}{\sqrt{x+4}}, x>0$

Ans. $\int \frac{x}{\sqrt{x+4}}\ dx$

$= \int \frac{x+4-4}{\sqrt{x+4}}\,dx$

$= \int \frac{x+4}{\sqrt{x+4}} - \frac{4}{\sqrt{x+4}}\,dx$

$= \int \sqrt{x+4}\,dx - 4\int \frac{1}{\sqrt{x+4}}\,dx$

$= \int (x+4)^{\frac{1}{2}}\,dx - 4\int (x+4)^{\frac{-1}{2}}\,dx$

$= \frac{(x+4)^{\frac{3}{2}}}{\frac{3}{2}(1)} - \frac{4(x+4)^{\frac{1}{2}}}{\frac{1}{2}(1)} + c$ Using $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$

$= \frac{2}{3}(x+4)^{\frac{3}{2}} - 8(x+4)^{\frac{1}{2}} + c$

$= 2\sqrt{x+4}\left(\frac{x+4}{3} - 4\right) + c$

$= 2\sqrt{x+4}\left(\frac{x+4-12}{3}\right) + c$

$= \frac{2}{3}\sqrt{x+4}(x-8) + c$ ans.

12. $(x^3-1)^{\frac{1}{3}} x^5$

Ans. Let I = $\int (x^3-1)^{\frac{1}{3}} x^5\,dx$

$= \int (x^3-1)^{\frac{1}{3}} x^3 x^2\,dx$

$= \frac{1}{3}\int (x^3-1)^{\frac{1}{3}} x^3 (3x^2\,dx)$ ....(i)

Putting $x^3 - 1 = t$

$\Rightarrow x^3 = t + 1$

$\Rightarrow 3x^2 = \frac{dt}{dx}$

$\Rightarrow 3x^2 \; dx = dt$

$\therefore$ From eq. (i), I = $\frac{1}{3}\int t^{\frac{1}{3}}(t+1) \; dt$

= $\frac{1}{3}\int\left(t^{\frac{4}{3}} + t^{\frac{1}{3}}\right) \; dt$

= $\frac{1}{3}\left(\int t^{\frac{4}{3}} \; dt + \int t^{\frac{1}{3}} \; dt\right)$

= $\frac{1}{3}\left(\frac{t^{\frac{7}{3}}}{\frac{7}{3}} + \frac{t^{\frac{4}{3}}}{\frac{4}{3}}\right) + c$

= $\frac{1}{3}\left(\frac{3}{7}t^{\frac{7}{3}} + \frac{3}{4}t^{\frac{4}{3}}\right) + c$

= $\frac{1}{7}t^{\frac{7}{3}} + \frac{1}{4}t^{\frac{4}{3}} + c$

= $\frac{1}{7}\left(x^3 - 1\right)^{\frac{7}{3}} + \frac{1}{4}\left(x^3 - 1\right)^{\frac{4}{3}} + c$

13. $\frac{x^2}{\left(2+3x^3\right)^3}$

Ans. Let I = $\int \frac{x^2}{\left(2+3x^3\right)^3}\ dx$

= $\frac{1}{9}\int \frac{9x^2}{\left(2+3x^3\right)^3}\ dx$ ....(i)

Putting $2+3x^3 = t$

$\Rightarrow 9x^2 = \frac{dt}{dx}$

$\Rightarrow 9x^2\ dx = dt$

$\therefore$ From eq. (i), I = $\frac{1}{9}\int t^{-3}\ dt$

= $\frac{1}{9}.\frac{t^{-2}}{-2}+c$

= $\frac{-1}{18t^2}+c$

= $\frac{-1}{18\left(2+3x^3\right)^2}+c$

14. $\frac{1}{x\left(\log x\right)^m}, x > 0$

Ans. Let I = $\int \frac{1}{x\left(\log x\right)^m}\ dx$

= $\int \frac{\frac{1}{x}\ dx}{\left(\log x\right)^m}$ ....(i)

Putting $\log x = t$

$$\Rightarrow \frac{1}{x} = \frac{dt}{dx}$$

$$\Rightarrow \frac{dx}{x} = dt$$

$\therefore$ From eq. (i), I = $\int \frac{dt}{t^m}$ = $\int t^{-m}\ dt$

= $\frac{t^{-m+1}}{-m+1} + c$

= $\frac{(\log x)^{1-m}}{1-m} + c$

15. $\frac{x}{9-4x^2}$

Ans. Let I = $\int \frac{x}{9-4x^2}\ dx$

= $\frac{-1}{8}\int \frac{-8x}{9-4x^2}\ dx$ ....(i)

Putting $9-4x^2 = t$

$$\Rightarrow -8x = \frac{dt}{dx}$$

$$\Rightarrow -8x\ dx = dt$$

$\therefore$ From eq. (i), I = $\frac{-1}{8}\int \frac{dt}{t}$ = $\frac{-1}{8}\int \frac{1}{t}\ dt$

= $\frac{-1}{8}\log|t| + c$

= $\frac{-1}{8}\log\left|9-4x^2\right|+c$

16. e^{2x+3}

Ans. $\int e^{2x+3}\,dx$

= $\frac{1}{2}e^{2x+3}+c$ Using $\int e^{ax+b}dx = \frac{e^{ax+b}}{a}+c$

17. $\frac{x}{e^{x^2}}$

Ans. Let I = $\int \frac{x}{e^{x^2}}\,dx$

= $\frac{1}{2}\int \frac{2x}{e^{x^2}}\,dx$..........(i

Putting $x^2=t$

$\Rightarrow 2x=\frac{dt}{dx}$

$\Rightarrow 2x\,dx=dt$

$\therefore$ From eq. (i), I = $\frac{1}{2}\int\frac{dt}{e^t}$ = $\frac{1}{2}\int e^{-t}\,dt$

using $\int e^{ax+b}dx = \frac{e^{ax+b}}{a}+c$

We have $\frac{1}{2}\int e^{-t}\,dt = \frac{1}{2}\left(\frac{e^{-t}}{-1}\right)+c$

= $\frac{-1}{2\left(e^t\right)}+c$

= $\frac{-1}{2\left(e^{x^2}\right)}+c$

Integrate the functions in Exercise 18 to 26.

18. $\frac{e^{\tan^{-1}x}}{1+x^2}$

Ans. Let I = $\int \frac{e^{\tan^{-1}x}}{1+x^2}\ dx$ ....(i)

Putting $\tan^{-1}x = t$

$\Rightarrow \frac{1}{1+x^2} = \frac{dt}{dx}$

$\Rightarrow \frac{dx}{1+x^2} = dt$

$\therefore$ From eq. (i), I = $\int e^t\ dt$

= $e^t + c$

= $e^{\tan^{-1}x} + c$

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19. $\frac{e^{2x}-1}{e^{2x}+1}$

Ans. Let I = $\int \frac{e^{2x}-1}{e^{2x}+1}\ dx$

= $\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\ dx$ [Multiplying each term by e^{-x}]

Putting $e^x + e^{-x} = t$

$\Rightarrow e^{x}+e^{-x}\frac{d}{dx}(-x)=\frac{dt}{dx}$

$\Rightarrow \left(e^{x}-e^{-x}\right)dx=dt$

$\therefore$ From eq. (i), I = $\int\frac{dt}{t}$ = $\int\frac{1}{t}\,dt$

= $\log|t|+c$

= $\log\left|e^{x}+e^{-x}\right|+c$

= $\log\left(e^{x}+e^{-x}\right)+c$

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20. $\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

Ans. Let I = $\int\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\,dx$

= $\frac{1}{2}\int\frac{2\left(e^{2x}-e^{-2x}\right)}{e^{2x}+e^{-2x}}\,dx$ ....(i)

Putting $e^{2x}+e^{-2x}=t$

$\Rightarrow e^{2x}\frac{d}{dx}2x+e^{-2x}\frac{d}{dx}(-2x)=\frac{dt}{dx}$

$\Rightarrow e^{2x}.2-2e^{-2x}=\frac{dt}{dx}$

$\Rightarrow 2\left(e^{2x}-e^{-2x}\right)dx=dt$

$\therefore$ From eq. (i), I = $\frac{1}{2}\int\frac{dt}{t}$

= $\frac{1}{2}\log|t|+c$

= $\frac{1}{2}\log\left|e^{2x}+e^{-2x}\right|+c$

= $\frac{1}{2}\log\left(e^{2x}+e^{-2x}\right)+c$

21. $\tan^2(2x-3)$

Ans. $\int \tan^2(2x-3)\ dx$

= $\int\left\{\sec^2(2x-3)-1\right\}\ dx$

= $\int \sec^2(2x-3)\ dx-\int 1\ dx$

Using $\int \sec^2(ax+b)dx=\frac{\tan(ax+b)}{a}+c$

$=\frac{\tan(2x-3)}{2}-x+c$

22. $\sec^2(7-4x)$

Ans. $\int \sec^2(7-4x)\ dx$

Using $\int \sec^2(ax+b)dx=\frac{\tan(ax+b)}{a}+c$

= $\frac{-1}{4}\tan(7-4x)+c$

23. $\frac{\sin^{-1}x}{\sqrt{1-x^2}}$

Ans. Let I = $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\, dx$ ....(i)

Putting $\sin^{-1} x = t$

$\Rightarrow \frac{1}{\sqrt{1-x^2}} = \frac{dt}{dx}$

$\Rightarrow \frac{dx}{\sqrt{1-x^2}} = dt$

$\therefore$ From eq. (i), I = $\int t\, dt$

= $\frac{t^2}{2} + c$

= $\frac{1}{2}\left(\sin^{-1} x\right)^2 + c$

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24. $\frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}$

Ans. Let I = $\int \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}\, dx$

= $\int \frac{2\cos x - 3\sin x}{2(2\sin x + 3\cos x)}\, dx$

= $\frac{1}{2}\int \frac{2\cos x - 3\sin x}{2\sin x + 3\cos x}\, dx$ ....(i)

Putting $2\sin x + 3\cos x = t$

$\Rightarrow 2\cos x - 3\sin x = \frac{dt}{dx}$

$\Rightarrow \left(2\cos x - 3\sin x\right) dx = dt$

$\therefore$ From eq. (i), I = $\frac{1}{2}\int \frac{dt}{t}$ = $\frac{1}{2}\log|t|+c$

= $\frac{1}{2}\log|2\sin x+3\cos x|+c$

25. $\frac{1}{\cos^2 x(1-\tan x)^2}$

Ans. Let I = $\int \frac{1}{\cos^2 x(1-\tan x)^2}\ dx$

= $\int \frac{\sec^2 x}{(1-\tan x)^2}\ dx$

= $-\int \frac{-\sec^2 x}{(1-\tan x)^2}\ dx$..........(i)

Putting $1-\tan x=t$

$\Rightarrow -\sec^2 x=\frac{dt}{dx}$

$\Rightarrow\ -\sec^2 x\ dx=dt$

$\therefore$ From eq. (i), I = $-\int \frac{dt}{t^2}$ = $-\int t^{-2}\ dt$

= $\frac{-t^{-1}}{-1}+c$

= $\frac{1}{t}+c$

= $\frac{1}{1-\tan x}+c$

26. $\dfrac{\cos\sqrt{x}}{\sqrt{x}}$

Ans. Let I = $\int \dfrac{\cos\sqrt{x}}{\sqrt{x}}\,dx$ ....(i)

Putting $\sqrt{x} = t$

$\Rightarrow x = t^2$

$\Rightarrow \dfrac{dx}{dt} = 2t$

$\Rightarrow dx = 2t\,dt$

$\therefore$ From eq. (i), I = $\int \dfrac{\cos t}{t} 2t\,dt$

= $2\int \cos t\,dt$

= $2\sin t + c$

= $2\sin\sqrt{x} + c$

Integrate the functions in Exercise 27 to 37.

27. $\sqrt{\sin 2x}\cos 2x$

Ans. Let I = $\int \sqrt{\sin 2x}\cos 2x\,dx$

= $\dfrac{1}{2}\int \sqrt{\sin 2x}\left(2\cos 2x\,dx\right)$ ....(i)

Putting $\sin 2x = t$

$\Rightarrow \cos 2x\dfrac{d}{dx}(2x) = \dfrac{dt}{dx}$

$\Rightarrow 2\cos 2x\, dx = dt$

$\therefore$ From eq. (i), I = $\frac{1}{2}\int \sqrt{t}\ dt$

= $\frac{1}{2}\int t^{\frac{1}{2}}\ dt$

= $\frac{1}{2}\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1}+c$

= $\frac{1}{2}\cdot\frac{t^{3/2}}{3/2}+c$

= $\frac{1}{3}(\sin 2x)^{\frac{3}{2}}+c$

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28. $\frac{\cos x}{\sqrt{1+\sin x}}$

Ans. Let I = $\int \frac{\cos x}{\sqrt{1+\sin x}}\ dx$ ....(i)

Putting $1+\sin x = t$

$\Rightarrow \cos x = \frac{dt}{dx}$

$\Rightarrow \cos x\, dx = dt$

$\therefore$ From eq. (i), I = $\int \frac{dt}{\sqrt{t}}$

= $\int t^{\frac{-1}{2}}\ dt$

= $\dfrac{t^{\frac{-1}{2}+1}}{\frac{-1}{2}+1}+c$

= $\dfrac{t^{\frac{1}{2}}}{\frac{1}{2}}+c$

= $2\sqrt{t}+c$

= $2\sqrt{1+\sin x}+c$

29. $\cot x \log \sin x$

Ans. Let I = $\int \cot x \log \sin x \, dx$ ....(i)

Putting $\log \sin x = t$

$\Rightarrow \dfrac{1}{\sin x}\dfrac{d}{dx}(\sin x)=\dfrac{dt}{dx}$

$\Rightarrow \dfrac{1}{\sin x}\cos x=\dfrac{dt}{dx}$

$\Rightarrow \cot x \, dx = dt$

$\therefore$ From eq. (i), I = $\int t \, dt$

= $\dfrac{t^2}{2}+c$

= $\dfrac{1}{2}(\log \sin x)^2+c$

30. $\dfrac{\sin x}{1+\cos x}$

Ans. Let I = $\int \frac{\sin x}{1+\cos x}\, dx$

= $-\int \frac{-\sin x}{1+\cos x}\, dx$ ....(i)

Putting $1+\cos x = t$

$\Rightarrow -\sin x = \frac{dt}{dx}$

$\Rightarrow -\sin x\, dx = dt$

$\therefore$ From eq. (i), I = $-\int \frac{dt}{t}$

= $-\log|t| + c$

= $-\log|1+\cos x| + c$

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31. $\frac{\sin x}{(1+\cos x)^2}$

Ans. Let I = $\int \frac{\sin x}{(1+\cos x)^2}\, dx$

= $-\int \frac{-\sin x}{(1+\cos x)^2}\, dx$ ....(i)

Putting $1+\cos x = t$

$\Rightarrow -\sin x = \frac{dt}{dx}$

$\Rightarrow -\sin x\, dx = dt$

$\therefore$ From eq. (i), I = $-\int \frac{dt}{t^2}$

= $-\int t^{-2}\ dt$

= $\frac{-t^{-1}}{-1} + c$

= $\frac{1}{t} + c$

= $\frac{1}{1+\cos x} + c$

32. $\frac{1}{1+\cot x}$

Ans. Let I = $\int \frac{1}{1+\cot x}\ dx$

= $\int \frac{1}{1+\frac{\cos x}{\sin x}}\ dx$

= $\int \frac{1}{\left(\frac{\sin x+\cos x}{\sin x}\right)}\ dx$

= $\int \frac{\sin x}{\sin x+\cos x}\ dx$

= $\frac{1}{2}\int \frac{2\sin x}{\sin x+\cos x}\ dx$

= $\frac{1}{2}\int \frac{\sin x+\sin x}{\sin x+\cos x}\ dx$

Adding and subtracting $\cos x$ in the numerator,

= $\frac{1}{2}\int \frac{\sin x+\cos x-\cos x+\sin x}{\sin x+\cos x}\ dx$

= $\frac{1}{2}\int \frac{(\sin x+\cos x)-(\cos x-\sin x)}{\sin x+\cos x}\ dx$

= $\frac{1}{2}\int \left(\frac{\sin x+\cos x}{\sin x+\cos x}-\frac{\cos x-\sin x}{\sin x+\cos x}\right) dx$

= $\frac{1}{2}\int \left(1-\frac{\cos x-\sin x}{\sin x+\cos x}\right)\ dx$

= $\frac{1}{2}\left[\int 1\ dx-\int \frac{\cos x-\sin x}{\sin x+\cos x}\ dx\right]$ = $\frac{1}{2}\left[x-I_1\right]$

where $I_1=\int \frac{\cos x-\sin x}{\sin x+\cos x}\ dx$ ....(i)

Putting $\sin x+\cos x=t$

$\Rightarrow \cos x-\sin x=\frac{dt}{dx}$

$\Rightarrow (\cos x-\sin x)\ dx=dt$

$\therefore I_1$ = $\int \frac{dt}{t}$ = $\log|t|$

= $\log|\sin x+\cos x|$

Putting this value in eq. (i), we get required integral,

= $\frac{1}{2}\left[x-\log|\sin x+\cos x|\right]+c$

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33. $\frac{1}{1-\tan x}$

Ans. Let I = $\int \frac{1}{1-\tan x}\,dx$

= $\int \frac{1}{1-\frac{\sin x}{\cos x}}\,dx$

= $\int \frac{1}{\left(\frac{\cos x-\sin x}{\cos x}\right)}\,dx$

= $\int \frac{\cos x}{\cos x-\sin x}\,dx$

= $\frac{1}{2}\int \frac{2\cos x}{\cos x-\sin x}\,dx$

= $\frac{1}{2}\int \frac{\cos x+\cos x}{\cos x-\sin x}\,dx$

Adding and subtracting $\sin x$ in the numerator,

= $\frac{1}{2}\int \frac{\cos x-\sin x+\sin x+\cos x}{\cos x-\sin x}\,dx$

= $\frac{1}{2}\int \frac{(\cos x-\sin x)+(\sin x+\cos x)}{\cos x-\sin x}\,dx$

= $\frac{1}{2}\int \frac{\cos x-\sin x}{\cos x-\sin x}+\frac{\sin x+\cos x}{\cos x-\sin x}dx$

= $\frac{1}{2}\int \left(1+\frac{\sin x+\cos x}{\cos x-\sin x}\right)dx$

= $\frac{1}{2}\left[\int 1\,dx-\int \frac{-\sin x-\cos x}{\cos x-\sin x}\,dx\right]$

= $\frac{1}{2}\left[x-\log\left|\cos x-\sin x\right|\right]+c$

34. $\dfrac{\sqrt{\tan x}}{\sin x \cos x}$

Ans. Let I = $\int \dfrac{\sqrt{\tan x}}{\sin x \cos x}\, dx$

= $\int \dfrac{\sqrt{\tan x}}{\frac{\sin x}{\cos x} \cos x . \cos x}\, dx$

= $\int \dfrac{\sqrt{\tan x}}{\tan x \cos^2 x}\, dx$

= $\int \dfrac{\sec^2 x}{\sqrt{\tan x}}\, dx$ .....(i)

Putting $\tan x = t$

$\Rightarrow \sec^2 x = \dfrac{dt}{dx}$

$\Rightarrow \sec^2 x\, dx = dt$

$\therefore$ From eq. (i), I = $\int \dfrac{dt}{\sqrt{t}}$

= $\int t^{\frac{-1}{2}}\, dt$

= $\dfrac{t^{1/2}}{1/2} + c$

= $2\sqrt{t} + c$

= $2\sqrt{\tan x} + c$

35. $\dfrac{(1+\log x)^2}{x}$

Ans. Let I = $\int \frac{(1+\log x)^2}{x} dx$ ....(i)

Putting $1+\log x = t$

$\Rightarrow \frac{1}{x} = \frac{dt}{dx}$

$\Rightarrow \frac{dx}{x} = dt$

$\therefore$ From eq. (i), I = $\int t^2 \ dt$

= $\frac{t^3}{3} + c$

= $\frac{1}{3}(1+\log x)^3 + c$

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36. $\frac{(x+1)(x+\log x)^2}{x}$

Ans. Let I = $\int \frac{(x+1)(x+\log x)^2}{x} dx$ ....(i)

Putting $x+\log x = t$

$\Rightarrow 1+\frac{1}{x} = \frac{dt}{dx}$

$\Rightarrow \frac{x+1}{x} = \frac{dt}{dx}$

$\Rightarrow \left(\frac{x+1}{x}\right) dx = dt$

$\therefore$ From eq. (i), I = $\int t^2 \ dt$

= $\frac{t^3}{3}+c$

= $\frac{1}{3}(x+\log x)^3+c$

37. $\frac{x^3 \sin\left(\tan^{-1}x^4\right)}{1+x^8}$

Ans. Let I = $\int \frac{x^3 \sin\left(\tan^{-1}x^4\right)}{1+x^8}\,dx$

= $\frac{1}{4}\int \sin\left(\tan^{-1}x^4\right).\frac{4x^3}{1+x^8}\,dx$ ....(i)

Putting $\tan^{-1}x^4=t$

$\Rightarrow \frac{1}{1+\left(x^4\right)^2}\frac{d}{dx}x^4=\frac{dt}{dx}$

$\Rightarrow \frac{4x^3}{1+x^8}\,dx=dt$

$\therefore$ From eq. (i), I = $\frac{1}{4}\int \sin t\;dt$

= $\frac{-1}{4}\cos t+c$

= $\frac{-1}{4}\cos\left(\tan^{-1}x^4\right)+c$

Choose the correct answer in Exercise 38 and 39.

38. $\int \frac{10x^9+10^x\log_e 10\;dx}{x^{10}+10^x}$ **equals**

(A) $10^x - x^{10} + C$

(B) $10^x + x^{10} + C$

(C) $\left(10^x - x^{10}\right)^{-1} + C$

(D) $log(10^x + x^{10}) + c$

Ans. Let I = $\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x}\, dx$ ....(i)

Putting $x^{10} + 10^x = t$

$\Rightarrow \left(10x^9 + 10^x \log_e 10\right)\, dx = dt$

$\therefore$ From eq. (i), I = $\int \frac{dt}{t}$

= $\log|t| + c$

= $\log\left|x^{10} + 10^x\right| + c$

Therefore, option (D) is correct.

39. $\int \frac{dx}{\sin^2 x \cos^2 x}$ **equals**

(A) $\tan x + \cot x + C$

(B) $\tan x - \cot x + C$

(C) $\tan x \cot x + C$

(D) $\tan x - \cot 2x + C$

Ans. $\int \frac{dx}{\sin^2 x \cos^2 x}$

$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}\, dx$

$= \int \frac{\sin^2 x}{\sin^2 x \cos^2 x} + \frac{\cos^2 x}{\sin^2 x \cos^2 x}\, dx$

$= \int \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\, dx$

$= \int \left(\sec^2 x + \operatorname{cosec}^2 x\right) dx$

$= \int \sec^2 x\, dx + \int \operatorname{cosec}^2 x\, dx$

$= \tan x - \cot x + c$

Therefore, option (B) is correct.