CBSE Class–12 Mathematics

NCERT solution

Chapter - 10

Vector Algebra - Exercise 10.3

1. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitude $\sqrt{3}$ and 2 respectively having $\vec{a}.\vec{b}=\sqrt{6}$.

Ans. Given: $\left|\vec{a}\right|=\sqrt{3}$, $\left|\vec{b}\right|=2$ and $\vec{a}.\vec{b}=\sqrt{6}$

Let θ be the angle between the vector $\vec{a}$ and $\vec{b}$.

We know that $\cos\theta=\dfrac{\vec{a}.\vec{b}}{\left|\vec{a}\right|.\left|\vec{b}\right|}$

$$\Rightarrow \cos\theta=\frac{\sqrt{6}}{\sqrt{3}.2}$$

$$=\frac{\sqrt{3}.\sqrt{2}}{\sqrt{3}.\sqrt{2}.\sqrt{2}}=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos\theta=\cos\frac{\pi}{4}$$

$$\Rightarrow \theta=\frac{\pi}{4}$$

2. Find the angle between the vectors $\hat{i}-2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$.

Ans. Given: Let $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}-2\hat{j}+\hat{k}$

$$\Rightarrow \left|\vec{a}\right|=\sqrt{1+4+9}=\sqrt{14} \text{ and } \left|\vec{b}\right|=\sqrt{9+4+1}=\sqrt{14}$$

$\left[\because x\hat{i}+y\hat{j}+z\hat{k}=\sqrt{x^2+y^2+z^2}\right]$

Also $\vec{a}.\vec{b}$

= Product of coefficients of $\hat{i}$ + Product of coefficients of $\hat{j}$ + Product of coefficients $\hat{k}$

= $1(3)+(-2)(-2)+3(1)=3+4+3=10$

Let θ be the angle between the vector $\vec{a}$ and $\vec{b}$.

We know that $\cos\theta=\dfrac{\vec{a}.\vec{b}}{\left|\vec{a}\right|.\left|\vec{b}\right|}$

$\Rightarrow \cos\theta=\dfrac{10}{\sqrt{14}.\sqrt{14}}$

= $\dfrac{10}{14}$ = $\dfrac{5}{7}$

$\Rightarrow \cos\theta=\dfrac{5}{7}$

$\Rightarrow \theta=\cos^{-1}\dfrac{5}{7}$

3. Find the projection of the vector $\hat{i}-\hat{j}$ on the vector $\hat{i}+\hat{j}$.

Ans. Let $\vec{a}=\hat{i}-\hat{j}=\hat{i}-\hat{j}+0\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}=\hat{i}+\hat{j}+0\hat{k}$

Projection of vector $\vec{a}$ and $\vec{b}$ = $\dfrac{\vec{a}.\vec{b}}{\left|\vec{b}\right|}$

= $\dfrac{(1)(1)+(-1)(1)+0(0)}{\sqrt{(1)^2+(1)^2+(0)^2}}$

$= \frac{1-1+0}{\sqrt{2}} = \frac{0}{\sqrt{2}} = 0$

If projection of vector $\vec{a}$ and $\vec{b}$ is zero, then vector $\vec{a}$ is perpendicular to $\vec{b}$.

4. Find the projection of the vector $\hat{i}+3\hat{j}+7\hat{k}$ on the vector $7\hat{i}-\hat{j}+8\hat{k}$.

Ans. Let $\vec{a}=\hat{i}+3\hat{j}+7\hat{k}$ and $\vec{b}=7\hat{i}-\hat{j}+8\hat{k}$

Projection of vector $\vec{a}$ and $\vec{b}$ = $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

$= \frac{(1)(7)+(3)(-1)+7(8)}{\sqrt{(7)^2+(-1)^2+(8)^2}}$

$= \frac{7-3+56}{\sqrt{49+1+64}} = \frac{60}{\sqrt{114}}$

5. Show that each of the given three vectors is a unit vector:

$\frac{1}{7}\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$, $\frac{1}{7}\left(3\hat{i}-6\hat{j}+2\hat{k}\right)$, $\frac{1}{7}\left(6\hat{i}+2\hat{j}-3\hat{k}\right)$

Also show that they are mutually perpendicular to each other.

Ans. Let $\vec{a}=\frac{1}{7}\left(2\hat{i}+3\hat{j}+6\hat{k}\right)=\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}+\frac{6}{7}\hat{k}$ ....(i)

$\vec{b}=\frac{1}{7}\left(3\hat{i}-6\hat{j}+2\hat{k}\right)=\frac{3}{7}\hat{i}-\frac{6}{7}\hat{j}+\frac{2}{7}\hat{k}$ ....(ii)

$\vec{c}=\frac{1}{7}\left(6\hat{i}+2\hat{j}-3\hat{k}\right)=\frac{6}{7}\hat{i}+\frac{2}{7}\hat{j}-\frac{3}{7}\hat{k}$ ....(iii)

$$\Rightarrow |\vec{a}| = \sqrt{\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2} = \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}} = \sqrt{\frac{49}{49}} = \sqrt{1} = 1$$

$$|\vec{b}| = \sqrt{\left(\frac{3}{7}\right)^2 + \left(\frac{-6}{7}\right)^2 + \left(\frac{2}{7}\right)^2} = \sqrt{\frac{9}{49} + \frac{36}{49} + \frac{4}{49}} = \sqrt{\frac{49}{49}} = \sqrt{1} = 1$$

$$|\vec{c}| = \sqrt{\left(\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2 + \left(\frac{-3}{7}\right)^2} = \sqrt{\frac{36}{49} + \frac{4}{49} + \frac{9}{49}} = \sqrt{\frac{49}{49}} = \sqrt{1} = 1$$

$\therefore$ Each of the three given vectors $\vec{a}, \vec{b}, \vec{c}$ is a unit vector.

From eq. (i) and (ii),

$$\vec{a}.\vec{b} = \left(\frac{2}{7}\right).\left(\frac{3}{7}\right) + \left(\frac{3}{7}\right).\left(\frac{-6}{7}\right).\left(\frac{6}{7}\right).\left(\frac{2}{7}\right) \quad \left[\because \vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3\right]$$

$$= \frac{6}{49} - \frac{18}{49} + \frac{12}{49} = \frac{6-18+12}{49} = \frac{0}{49} = 0$$

$\therefore$ $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

From eq. (ii) and eq. (iii),

$$\vec{b}.\vec{c} = \left(\frac{3}{7}\right).\left(\frac{6}{7}\right) + \left(\frac{-6}{7}\right).\left(\frac{2}{7}\right).\left(\frac{2}{7}\right).\left(\frac{-3}{7}\right) \quad \left[\because \vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3\right]$$

$$= \frac{18}{49} - \frac{12}{49} - \frac{6}{49} = \frac{18-12-6}{49} = \frac{0}{49} = 0$$

$\therefore$ $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

From eq. (i) and (iii),

$$\vec{a}.\vec{c} = \left(\frac{2}{7}\right).\left(\frac{6}{7}\right) + \left(\frac{3}{7}\right).\left(\frac{2}{7}\right).\left(\frac{6}{7}\right).\left(\frac{-3}{7}\right) \quad \left[\because \vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3\right]$$

$= \frac{12}{49} + \frac{6}{49} - \frac{18}{49} = \frac{12+6-18}{49} = \frac{0}{49} = 0$

$\therefore$ $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

Hence, $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors.

6. Find $|\vec{a}|$ and $|\vec{b}|$, if $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 8$ and $|\vec{a}| = 8|\vec{b}|$.

Ans. Given: $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 8$ and $|\vec{a}| = 8|\vec{b}|$ ....(i)

$\Rightarrow \vec{a}.\vec{a} - \vec{a}.\vec{b} + \vec{b}.\vec{a} - \vec{b}.\vec{b} = 8$

$\Rightarrow |\vec{a}|^2 - \vec{a}.\vec{b} + \vec{a}.\vec{b} - |\vec{b}|^2 = 8$

$\Rightarrow |\vec{a}|^2 - |\vec{b}|^2 = 8$ ....(ii)

Putting $|\vec{a}| = 8|\vec{b}|$ in eq. (ii),

$64|\vec{b}|^2 - |\vec{b}|^2 = 8$

$\Rightarrow (64-1)|\vec{b}|^2 = 8$

$\Rightarrow 63|\vec{b}|^2 = 8$

$\Rightarrow |\vec{b}|^2 = \frac{8}{63}$

$\Rightarrow |\vec{b}| = \sqrt{\frac{8}{63}} = \frac{2\sqrt{2}}{3\sqrt{7}}$

Putting $|\vec{b}| = \frac{2\sqrt{2}}{3\sqrt{7}}$ in eq (i),

$\left|\vec{a}\right| = 8\left(\frac{2\sqrt{2}}{3\sqrt{7}}\right) = \frac{16}{3}\sqrt{\frac{2}{7}}$

7. Evaluate the product $\left(3\vec{a}-5\vec{b}\right).\left(2\vec{a}+7\vec{b}\right)$.

Ans. Given: $\left(3\vec{a}-5\vec{b}\right).\left(2\vec{a}+7\vec{b}\right)$ = $\left(3\vec{a}\right).\left(2\vec{a}\right)+\left(3\vec{a}\right).\left(7\vec{b}\right)-\left(5\vec{b}\right).\left(2\vec{a}\right)-\left(5\vec{b}\right).\left(7\vec{b}\right)$

= $6\vec{a}.\vec{a}+21\vec{a}.\vec{b}-10\vec{b}.\vec{a}-35\vec{b}.\vec{b}$

= $6\left|\vec{a}\right|^2+21\vec{a}.\vec{b}-10\vec{a}.\vec{b}-35\left|\vec{b}\right|^2$

= $6\left|\vec{a}\right|^2+11\vec{a}.\vec{b}-35\left|\vec{b}\right|^2$

8. Find the magnitude of two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that the angle between them is 60° and their scalar product is $\frac{1}{2}$.

Ans. Given: $\left|\vec{a}\right| = \left|\vec{b}\right|$, angle θ (say) between $\vec{a}$ and $\vec{b}$ is 60° and their scalar (i.e., dot) product = $\frac{1}{2}$

$\Rightarrow \vec{a}.\vec{b} = \frac{1}{2} \quad \Rightarrow \left|\vec{a}\right|.\left|\vec{b}\right|\cos\theta = \frac{1}{2}$

Putting $\left|\vec{a}\right| = \left|\vec{b}\right|$ and $\theta = 60^\circ$, we have $\left|\vec{a}\right|.\left|\vec{a}\right|\cos 60^\circ = \frac{1}{2}$

$\Rightarrow \left|\vec{a}\right|^2.\left(\frac{1}{2}\right) = \frac{1}{2}$

$\Rightarrow \left|\vec{a}\right|^2 = 1$

$\Rightarrow \left|\vec{a}\right| = 1$

$\therefore \left|\vec{b}\right| = \left|\vec{a}\right| = 1$

$\therefore \left|\vec{a}\right| = 1$ and $\left|\vec{b}\right| = 1$

9. Find $\left|\vec{x}\right|$, if for a unit vector $\vec{a}$, $\left(\vec{x}-\vec{a}\right).\left(\vec{x}+\vec{a}\right) = 12$.

Ans. Given: $\vec{a}$ is a unit vector $\Rightarrow \left|\vec{a}\right| = 1$ ....(i)

$\left(\vec{x} - \vec{a}\right)\left(\vec{x} + \vec{a}\right) = 12$

$\Rightarrow \vec{x}.\vec{x} + \vec{x}.\vec{a} - \vec{a}.\vec{x} - \vec{a}.\vec{a} = 12$

$\Rightarrow \left|\vec{x}\right|^2 + \vec{x}.\vec{a} - \vec{a}.\vec{x} - \left|\vec{a}\right|^2 = 12$

$\Rightarrow \left|\vec{x}\right|^2 + \vec{a}.\vec{x} - \vec{a}.\vec{x} - \left|\vec{a}\right|^2 = 12$

$\Rightarrow \left|\vec{x}\right|^2 - \left|\vec{a}\right|^2 = 12$

Putting $\left|\vec{a}\right| = 1$ from eq. (i), $\left|\vec{x}\right|^2 - 1 = 12$

$\Rightarrow \left|\vec{x}\right|^2 = 13 \Rightarrow \left|\vec{x}\right| = \sqrt{13}$

10. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ.

Ans. Given: $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$

Now $\vec{a} + \lambda\vec{b} = 2\hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(-\hat{i} + 2\hat{j} + \hat{k}\right) =$

$= 2\hat{i} + 2\hat{j} + 3\hat{k} - \lambda\hat{i} + 2\lambda\hat{j} + \lambda\hat{k}$

$\Rightarrow \vec{a} + \lambda\vec{b} = (2-\lambda)\hat{i} + (2+2\lambda)\hat{j} + (3+\lambda)\hat{k}$

Again, $\vec{c} = 3\hat{i} + \hat{j}$ = $3\hat{i} + \hat{j} + 0\hat{k}$

Since, $(\vec{a} + \lambda\vec{b})$ is perpendicular to $\vec{c}$, therefore, $(\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0$

$\Rightarrow (2-\lambda)3 + (2+2\lambda)1 + (3+\lambda)0 = 0$

$\Rightarrow 6 - 3\lambda + 2 + 2\lambda = 0 \Rightarrow -\lambda + 8 = 0$

$\Rightarrow -\lambda = -8 \Rightarrow \lambda = 8$

11. Show that $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$, for any two non-zero vectors $\vec{a}$ and $\vec{b}$.

Ans. Let $\vec{c} = |\vec{a}|\vec{b} + |\vec{b}|\vec{a} = l\vec{b} + m\vec{a}$, where $l = |\vec{a}|$ and $m = |\vec{b}|$

Let $\vec{d} = |\vec{a}|\vec{b} - |\vec{b}|\vec{a} = l\vec{b} - m\vec{a}$

Now $|\vec{c}|.|\vec{d}| = (l\vec{b} + m\vec{a})(l\vec{b} - m\vec{a})$

= $l^2\vec{b}.\vec{b} - lm\vec{b}.\vec{a} + lm\vec{a}.\vec{b} - m^2\vec{a}.\vec{a}$

= $l^2|\vec{b}|^2 - lm\vec{a}.\vec{b} + lm\vec{a}.\vec{b} - m^2|\vec{a}|^2$

= $l^2|\vec{b}|^2 - m^2|\vec{a}|^2$

Putting, $l = |\vec{a}|$ and $m = |\vec{b}|$,

= $|\vec{a}|^2|\vec{b}|^2 - |\vec{b}|^2|\vec{a}|^2$ = 0

$\Rightarrow \left|\vec{c}\right|.\left|\vec{d}\right| = 0$

Therefore, vectors $\vec{c}$ and $\vec{d}$ are perpendicular ot each other.

12. If $\vec{a}.\vec{a} = 0$ and $\vec{a}.\vec{b} = 0$, then what can be concluded about the vector $\vec{b}$?

Ans. Given: $\vec{a}.\vec{a} = 0 \Rightarrow \left|\vec{a}\right|^2 = 0$

$\Rightarrow \left|\vec{a}\right| = 0$

Again $\vec{a}.\vec{b} = 0 \Rightarrow \left|\vec{a}\right|.\left|\vec{b}\right|\cos\theta = 0$

$\Rightarrow 0.\left|\vec{b}\right|\cos\theta = 0 \quad \left[\because \left|\vec{a}\right| = 0\right]$

$\Rightarrow$ 0 = 0 for all (any vector $\vec{b}$.)

Therefore, $\vec{b}$ can be any vector.

13. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$.

Ans. Since, $\vec{a}, \vec{b}, \vec{c}$ are unit vectors.

Therefore, $\left|\vec{a}\right| = 1, \left|\vec{b}\right| = 1$ and $\left|\vec{c}\right| = 1$ ....(i)

Also given $\vec{a}+\vec{b}+\vec{c} = 0$

$\Rightarrow \left(\vec{a}+\vec{b}+\vec{c}\right)^2 = 0$

$\Rightarrow \left(\vec{a}\right)^2 + \left(\vec{b}\right)^2 + \left(\vec{c}\right)^2 + 2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right) = 0$

$\left[\because (a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca\right]$

$\Rightarrow\ \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + \left|\vec{c}\right|^2 + 2\left(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}\right) = 0$

Putting the values from eq. (i), we get

$\Rightarrow\ 1 + 1 + 1 + 2\left(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}\right) = 0$

$\Rightarrow\ 2\left(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}\right) = -3$

$\Rightarrow\ \left(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}\right) = \frac{-3}{2}$

14. If either vector $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, then $\vec{a}.\vec{b} = 0$. But the converse need not be true. Justify your answer with an example.

Ans. Case I: Vector $\vec{a} = \vec{0}$. Therefore by definition of zero vector, $\left|\vec{a}\right| = 0$ ....(i)

$\therefore\ \vec{a}.\vec{b} = \left|\vec{a}\right|.\left|\vec{b}\right|\cos\theta$

= $0.\left|\vec{b}\right|\cos\theta$ [From eq. (i)]

$\Rightarrow\ \vec{a}.\vec{b} = 0$

Case II: Vector $\vec{b} = \vec{0}$. Therefore by definition of zero vector, $\left|\vec{b}\right| = 0$ ....(ii)

$\therefore\ \vec{a}.\vec{b} = \left|\vec{a}\right|.\left|\vec{b}\right|\cos\theta$

= $\vec{a}.0.\cos\theta$ [From eq. (ii)]

$\Rightarrow\ \vec{a}.\vec{b} = 0$

But the converse is not true.

Justification: Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

Therefore, $\left|\vec{a}\right| = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3} \neq 0$

Therefore, $\vec{a} \neq \vec{0}$

Again let $\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$

$\therefore \ \left|\vec{b}\right| = \sqrt{(1)^2 + (1)^2 + (-2)^2} = \sqrt{6} \neq 0$

Therefore, $\vec{b} \neq \vec{0}$

But $\vec{a}.\vec{b} = 1(1) + 1(1) + 1(-2) = 1 + 1 + -2 = 0$

Hence, here $\vec{a}.\vec{b} = 0$, but $\vec{a} \neq \vec{0}$ and $\vec{b} \neq \vec{0}$.

15. If the vertices A, B, C of a triangle ABC are $(1, 2, 3), (-1, 0, 0)$ and (0, 1, 2) respectively, then find $\angle ABC$.

Ans. Vertices A, B, C of a triangle are A (1, 2,3), B $(-1, 0, 0)$ and C (0, 1, 2) respectively.

$\therefore$ Position vector of point A = $\overrightarrow{OA} = (1, 2, 3) = \hat{i} + 2\hat{j} + 3\hat{k}$

Position vector of point B = $\overrightarrow{OB} = (-1, 0, 0) = -\hat{i} + 0\hat{j} + 0\hat{k}$

Position vector of point C = $\overrightarrow{OC} = (0, 1, 2) = 0\hat{i} + 1\hat{j} + 2\hat{k}$

Now $\overrightarrow{BA}$ = Position vector of point A – Position vector of point B

= $\hat{i} + 2\hat{j} + 3\hat{k} - \left(-\hat{i} + 0\hat{j} + 0\hat{k}\right)$

= $\hat{i} + 2\hat{j} + 3\hat{k} + \hat{i} - 0\hat{j} - 0\hat{k} = 2\hat{i} + 2\hat{j} + 3\hat{k}$ ....(i)

And $\overrightarrow{BC}$ = Position vector of point C – Position vector of point B

= $0\hat{i} + \hat{j} + 2\hat{k} - \left(-\hat{i} + 0\hat{j} + 0\hat{k}\right)$

= $0\hat{i}+\hat{j}+2\hat{k}+\hat{i}-0\hat{j}-0\hat{k}=\hat{i}+\hat{j}+2\hat{k}$ ....(ii)

Let θ be the angle between the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$.

$\therefore$ $\cos\theta=\dfrac{\overrightarrow{BA}.\overrightarrow{BC}}{\left|\overrightarrow{BA}\right|.\left|\overrightarrow{BC}\right|}$

= $\dfrac{2(1)+2(1)+3(2)}{\sqrt{4+4+9}\sqrt{1+1+4}}$ [Using eq. (i) and (ii)]

$\Rightarrow \cos\theta=\dfrac{10}{\sqrt{17}\sqrt{6}}=\dfrac{10}{\sqrt{102}}$

$\Rightarrow \theta=\cos^{-1}\dfrac{10}{\sqrt{102}}$

16. Show that the points A (1, 2, 7), B (2, 6, 3) and C $(3,10,-1)$ are collinear.

Ans. Vertices A, B, C of a triangle are A (1, 2, 7), B (2, 6, 3) and C $(3,10,-1)$ respectively.

$\therefore$ Position vector of point A = $\overrightarrow{OA}=(1,2,7)=\hat{i}+2\hat{j}+7\hat{k}$

Position vector of point B = $\overrightarrow{OB}=(2,6,3)=2\hat{i}+6\hat{j}+3\hat{k}$

Position vector of point C = $\overrightarrow{OC}=(3,10,-1)=3\hat{i}+10\hat{j}-\hat{k}$

Now $\overrightarrow{AB}$ = Position vector of point B – Position vector of point A

= $2\hat{i}+6\hat{j}+3\hat{k}-\left(\hat{i}+2\hat{j}+7\hat{k}\right)$

= $2\hat{i}+6\hat{j}+3\hat{k}-\hat{i}-2\hat{j}-7\hat{k}=\hat{i}+4\hat{j}-4\hat{k}$ ....(i)

And $\overrightarrow{AC}$ = Position vector of point C – Position vector of point A

= $3\hat{i}+10\hat{j}-\hat{k}-\left(\hat{i}+2\hat{j}+7\hat{k}\right)$

= $3\hat{i}+10\hat{j}-\hat{k}-\hat{i}-2\hat{j}-7\hat{k}=2\hat{i}+8\hat{j}-8\hat{k}=2\left(\hat{i}+4\hat{j}-4\hat{k}\right)$ ....(ii)

$\Rightarrow$ $\overrightarrow{AC}$ = 2.$\overrightarrow{AB}$ [Using eq. (i)]

$\Rightarrow$ Vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are collinear and parallel. $\left[\because \vec{a}=m\vec{b}\right]$

Thus, points A, B and C are collinear.

And also vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ have a common point A and hence can't be parallel.

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17. Show that the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$ form the vertices of a right angled triangle.

Ans. Let the given position vectors be A, B, C.

$\therefore$ Position vector of point A is $2\hat{i}-\hat{j}+\hat{k}$, Position vector of point B is $\hat{i}-3\hat{j}-5\hat{k}$ and Position vector of point C is $3\hat{i}-4\hat{j}-4\hat{k}$.

$\therefore$ $\overrightarrow{AB}$ = Position vector of B – Position vector of A

= $\hat{i}-3\hat{j}-5\hat{k}-\left(2\hat{i}-\hat{j}+\hat{k}\right)$

= $\hat{i}-3\hat{j}-5\hat{k}-2\hat{i}+\hat{j}-\hat{k}$ = $-\hat{i}-2\hat{j}-6\hat{k}$ ....(i)

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

= $3\hat{i}-4\hat{j}-4\hat{k}-\left(\hat{i}-3\hat{j}-5\hat{k}\right)$

= $3\hat{i}-4\hat{j}-4\hat{k}-\hat{i}+3\hat{j}+5\hat{k}$ = $2\hat{i}-\hat{j}+\hat{k}$ ....(ii)

$\overrightarrow{AC}$ = Position vector of C – Position vector of A

= $3\hat{i}-4\hat{j}-4\hat{k}-\left(2\hat{i}-\hat{j}+\hat{k}\right)$

= $3\hat{i}-4\hat{j}-4\hat{k}-2\hat{i}+\hat{j}-\hat{k}$ = $\hat{i}-3\hat{j}-5\hat{k}$ ....(iii)

Adding eq. (i) and (ii),

$\overrightarrow{AB}$ + $\overrightarrow{BC}$ = $-\hat{i}-2\hat{j}-6\hat{k}$ + $2\hat{i}-\hat{j}+\hat{k}$ = $\hat{i}-3\hat{j}-5\hat{k}$ = $\overrightarrow{AC}$ [Using eq. (iii)]

Therefore, by Triangle law of addition of vectors, points A, B, C are the vertices of a triangle ABC.

Now from eq. (i) and (ii),

$\overrightarrow{AB}\cdot\overrightarrow{BC}$ = $(-1)(2)+(-2)(-1)+(-6)(1)$ = $-2+2-6=-6\neq 0$

Again from eq. (ii) and (iii),

$\overrightarrow{BC}\cdot\overrightarrow{AC}$ = $(2)(1)+(-1)(-3)+(1)(-5)$ = 2 + 3 – 5 = 0

$\Rightarrow$ $\overrightarrow{BC}$ is perpendicular to $\overrightarrow{AC}$.

$\Rightarrow$ Angle C is 90°. Therefore ΔABC is a right angled at C.

Thus, A, B, C are the vertices of a right angled triangle.

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18. If $\vec{a}$ is a non-zero vector of magnitude 'a' and λ is a non-zero scalar, then $\lambda\vec{a}$ is a unit vector if:

(A) $\lambda=1$

(B) $\lambda=-1$

(C) $a=|\lambda|$

(D) $a=\dfrac{1}{|\lambda|}$

Ans. Given: $\vec{a}$ is a non-zero vector of magnitude a

$\Rightarrow \left|\vec{a}\right| = 1$

Also given $\lambda \neq 0$ and $\lambda\vec{a}$ is a unit vector.

$\Rightarrow \left|\lambda\vec{a}\right| = 1$

$\Rightarrow |\lambda|\left|\vec{a}\right| = 1$

$\Rightarrow |\lambda| a = 1$

$\Rightarrow a = \dfrac{1}{|\lambda|}$

Therefore, option (D) is correct.