CBSE Class–12 Mathematics

NCERT solution

Chapter -12

Linear Programming - Miscellaneous Exercise

1. A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A, while each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, atleast 460 units of iron and atmost 300 units of cholesterol. How many packets of each food should be used to maximize the amount of vitamin A in the diet? What is the maximum amount of vitamin A in the diet?

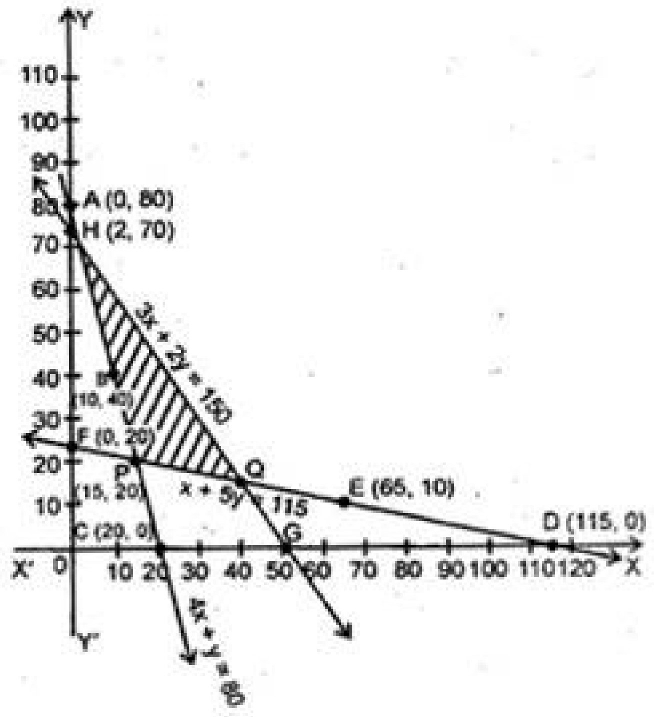
Ans. Let x and y be the number of packets of food P and Q respectively, $x \geq 0, y \geq 0.$ -

We have to maximize Z = $6x+3y$ (vitamin A) subject to the constraints $12x+3y \geq 240$ (constraints on Calcium), i.e., $4x+y \geq 80$ ....(i)

And $4x+50y \geq 460$ (constraints on Iron), i.e., $x+5y \geq 115$ ....(ii)

Also $6x+4y \leq 300$ (constraints on Cholesterol), i.e., $3x+2y \leq 150$ ....(iii)

$x \geq 0, y \geq 0$ ....(iv)



Consider $4x + y \geq 80$

Let $4x + y = 80$

$\Rightarrow \quad y = 80 - 4x$

	A	B	C
x	0	10	20
y	80	40	0

Here, (0, 0) does not satisfy this inequation, therefore the required half plane does not include the point (0, 0)

Again consider $x + 5y \geq 115$

Let $x + 5y = 115$

$\Rightarrow x = 115 - 5y$

	D	E	F
x	115	65	0
y	0	10	23

Here, also (0, 0) does not satisfy this inequation, therefore the required half plane does not include the point (0, 0)

Again consider $3x+2y \leq 150$

Let $3x+2y=150 \;\; \Rightarrow \;\; \frac{x}{50}+\frac{y}{75}=1$

Therefore, G (50, 0) and H (0, 75) satisfy the equation.

As (0, 0) satisfies the inequation $3x+2y=150$, therefore the required half plane contains (0, 0)

The shaded region is the feasible solution and its corners are P (15, 20), Q (40, 15) and R (2, 72).

Now Z = $6x+3y$

At P(15, 20) Z = 6 x 15 + 3 x 20 = 90 + 60 = 150

At Q (40, 15) Z = 6 x 40 + 3 x 15 = 240 + 45 = 285

At R (2, 72) Z = 6 x 2 + 3 x 72 = 12 + 216 = 228

Hence, maximum Z = 285 units of vitamin A at $x=40, y=15.$

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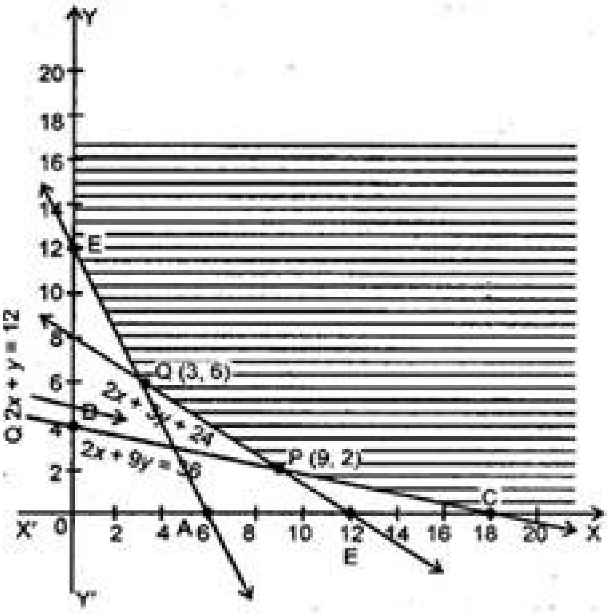
2. A farmer mixes two brands P and Q of cattle feed. Brand P, costing Rs. 250 per bag, contains 3 units of nutritional element A, 2.5 units of element B and 2 units of element C. Brand Q costing Rs. 200 per bag contains 1.5 units of nutritional element A, 11.25 units of element B and 3 units of element C. The minimum requirements of nutrients A, B and C are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag? What is the minimum cost of the mixture per bag?

Ans. Let number of bags of cattle feed brand P = x and number of bags of cattle feed brand Q = y



We have to minimize Z = $250x+200y$ subject to the constraints $3x+1.5y\geq 18$, $2.5x+11.25y\geq 45$, $2x+3y\geq 24$,

$x\geq 0, y\geq 0$

Consider $3x+1.5y\geq 18$

Let $3x+1.5y=18$

$\Rightarrow \quad 2x+y=12$

$\Rightarrow \quad \frac{x}{6}+\frac{y}{12}=1$

Now the points are A (6, 0) and B (0, 12).

Now clearly (0, 0) does not lie in the required half plane as (0, 0) does not satisfy the inequation $3x+1.5y\geq 18$.

Again consider $2.5x+11.25y\geq 45$

Let $2.5x+11.25y=45$

$\Rightarrow 2x+9y=36$

$\Rightarrow \frac{x}{18}+\frac{y}{4}=1$

Now point C (18, 0) and D (0, 4) lie on the line.

Now again (0, 0) does not lie in the required half plane as (0, 0) does not satisfy the inequation $2.5x+11.25y \geq 45$.

Again consider $2x+3y \geq 24$

Let $2x+3y=24 \quad \Rightarrow \frac{x}{12}+\frac{y}{8}=1$

Here, the points E (12, 0) and F (0, 8) lie on the line.

Again also (0, 0) does not lie on the half plane as (0, 0) does not satisfy this inequation.

The feasible region of XCPQEY and the co-ordinates of corners are C (18, 0), P (9, 2), Q (3, 6) and E (0, 12).

Now Z = $250x+200y$

At C (18, 0) Z = 250 x 18 + 200 x 0 = 4500

At P (9, 2) Z = 250 x 9 + 200 x 2 = 2450

At Q (3, 6) Z = 250 x 3 + 200 x 6 = 1950

At E (0, 12) Z = 250 x 0 + 200 x 12 = 2400

Here, minimum cost Z = Rs. 1950 when $x=3, y=6$

Hence, number of bags of brand P = 3 and number of bags of brand Q = 6 and minimum cost of the mixture per bag = Rs. $\frac{1950}{9}$ = Rs. 216.67 per bag.

3. A dietician wishes to mix together two kinds of food X and Y in such a way that the

mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of one kg food is given below:

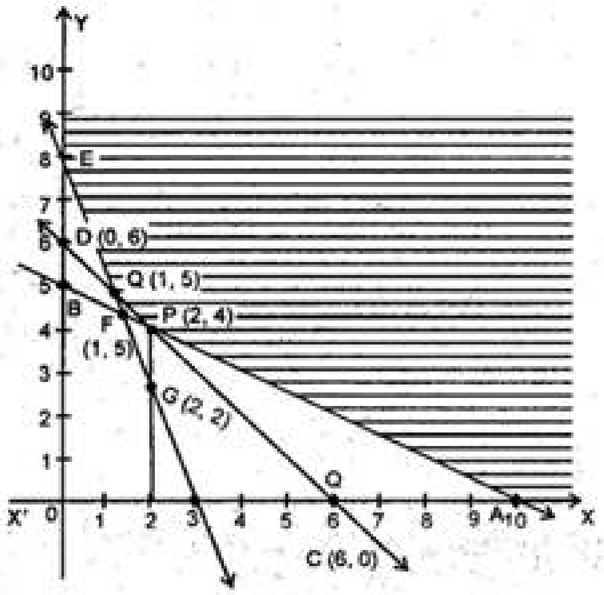
Food	Vitamin A	Vitamin B	Vitamin C
X	1	2	3
Y	2	2	1

One kg of food X costs Rs. 16 and one kg of food Y costs Rs. 20. Find the least cost of the mixture which will produce the required diet?

Ans. Let in the mixture food X weighs = x kg and in the mixture food Y weighs = y kg

We have to minimize Z = $16x+20y$ subject to constraints, $x+2y\geq 10$, $2x+2y\geq 12$, $3x+y\geq 8$, $x\geq 0, y\geq 0$

Consider $x+2y\geq 10$



Let $x+2y=10$

$$\Rightarrow \frac{x}{10}+\frac{y}{5}=1$$

$\therefore$ Points A (10, 0) and B (0, 5) lies on the line.

Here, (0, 0) does not satisfy the inequation $x+2y\geq 10$, therefore the required half plane does not include (0, 0).

Again consider $2x+2y \geq 12$

Let $2x+2y=12$

$\Rightarrow x+y=6$

$\Rightarrow \frac{x}{6}+\frac{y}{6}=1$

$\therefore$ Points C (6, 0) and D (0, 6) lies on the line.

Again consider $3x+y \geq 8$

Let $3x+y=8 \Rightarrow y=8-3x$

Again in the inequation (0, 0) is not included in the required half plane.

The shaded region is our feasible solution A (10, 0), P (2, 4), Q (1, 5), E (0, 8).

	E	F	G
x	0	1	2
y	8	5	2

The corners of the feasible region are A (10, 0), P (2, 4), Q (1, 5), E (0, 8).

Now Z = $16x+20y$

At A (10, 0)   Z = 16 x 10 + 20 x 0 = 160

At P (2, 4)   Z = 16 x 2 + 20 x 4 = 112

At Q (1, 5)   Z = 16 x 1 + 20 x 5 = 116

At E (0, 8)   Z = 16 x 0 + 20 x 8 = 160

Therefore minimum Z = Rs. 112 at $x=2, y=4$

Hence, minimum cost of the mixture = Rs. 112 when he mixes 2 kg of food X and 4 kg of food

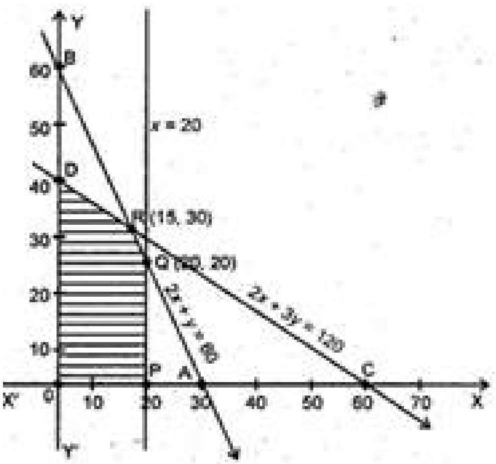
Y.

4. A manufacturer makes two types of toys A and B. Three machines are needed for this purpose and the time (in minutes) requires for each toy on the machines is given below:

Types of Toys	Machines		
	I	II	III
A	12	18	6
B	6	0	9

Each machine is available for a maximum of 6 hours per day. If the profit on each type of type A is Rs. 7.50 and that on each toy of type B is Rs. 5, shows that 15 toys of type A and 30 of type B should be manufactures in a day to get maximum profit.

Ans. Let x units of toys A and y units of toys B are produced by the manufacturer. Time spent on machine I to produce x units of toys A and y units of toys B = $(12x+6y)$ minutes.



Since each machine is available for a maximum of 6 x 60 = 360 minutes.

Therefore, we have $12x+6y \leq 360 \Rightarrow 2x+y \leq 60$

$18x \leq 360$ and $2x+3y \leq 120$

now, the profit Z earned by the manufacturer to produce x units of type A and y units of type B is $7.50x+5y$.

we have to maximize Z = $7.50x+5y$ i.e., 4Z = $3x+2y$ subject to constraints $2x+y\le 6$, $x\le 20$, $2x+3y\le 120$ and $x\ge 0$, $y\ge 0$.

Consider $2x+y\le 6$

Let $2x+y=6$

$$\Rightarrow \frac{x}{30}+\frac{y}{60}=1$$

$\therefore$ Points A (30, 0) and B (0, 60) lies on the line. Also (0, 0) lies in the required half plane.

Again consider $x\le 20$

Let $x=20$

It represent the half plane to the left of $x=20$.

Again consider $2x+3y\le 120$

Let $2x+3y=120 \Rightarrow \frac{x}{60}+\frac{y}{40}=1$

$\therefore$ Points C (60, 0) and D (0, 40) lies on the line. Therefore, (0, 0) lies in the required half plane.

The shaded portion is our feasible region. Its corners are O (0, 0), P (20, 0), Q (20, 20), R (15, 30), D (0, 40).

Now Z = $7.50x+5y$

At O (0, 0) Z = 7.5 x 0 + 5 x 0 = 0

At P (20, 0) Z = 7.5 x 00 + 5 x 0 = 150

At Q (20, 20) Z = 7.5 x 20 + 5 x 20 = 250

At R (15, 30) Z = 7.5 x 15 + 5 x 30 = 262.50

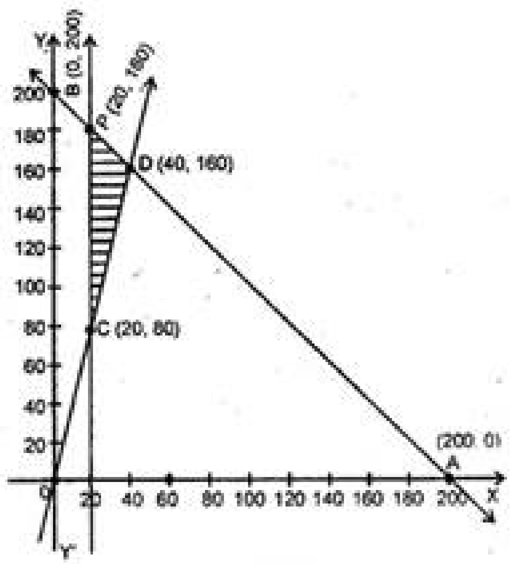
At D (0, 40) Z = 7.5 x 0 + 5 x 40 = 200

Now maximum profit = Z = Rs. 262.50, when he manufacturers 15 toys of types A and 30 of type B in a day.

5. An aeroplane carries a maximum of 200 passengers. A profit of Rs. 1000 is made on each executive class ticket and a profit of Rs. 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class, However at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit?

Ans. Let number of tickets of executive class sold = x and number of tickets of economy class sold = y

We have to maximize = Z = $1000x+600y$ subject to $x+y\leq 200$, $x\geq 20$ and $y\geq 4x$



$x\geq 0, y\geq 0$

Consider $x+y\leq 200$

Let $x+y=200$

$$\Rightarrow \quad \frac{x}{200}+\frac{y}{200}=1$$

$\therefore$ Points A (200, 0) and B(0, 200) are on the line and therefore (0, 0) is included in the required half plane.

Again consider $x \geq 20$

Let $x = 20$

It is the line parallel to $y-$axis at a positive distance 20 and the half plane lies towards right of it.

Again consider $y \geq 4x$

Let $y = 4x$

	O	C	D
x	0	20	40
y	0	80	160

Here, (40, 0) does not satisfy $y \geq 4x$, therefore plane does not include (40, 0).

The shaded portion is the feasible region. Its corners are C (20, 80), D (40, 160) and P (20, 180)

Now Z = $1000x + 600y$

At C (20, 80) Z = 1000 x 20 + 600 x 80 = 20000 + 48000 = 68,000

At D (40, 60) Z = 1000 x 40 + 600 x 60 = 40000 + 96000 = 1,36,000

At P (20, 180) Z = 1000 x 20 + 600 x 180 = 20000 + 108000 = 1,28,000

Hence Maximum profit Z = Rs. 1,36,000 at $x = 40, y = 160.$

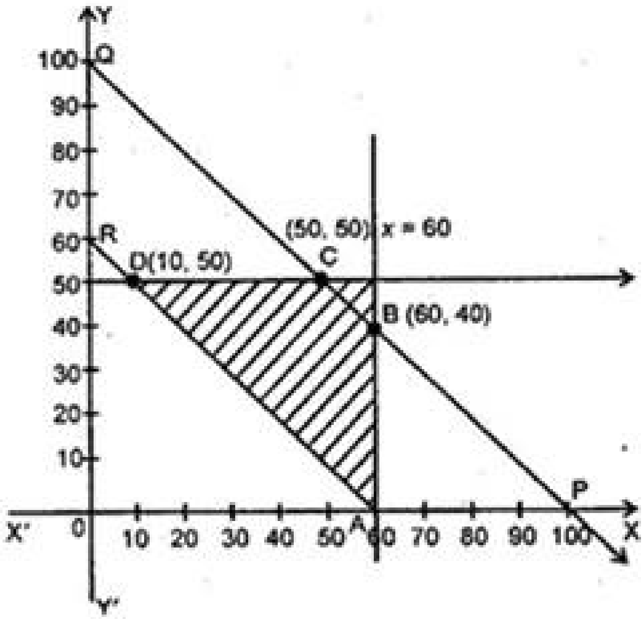
6. Two godowns A and B have grain capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops. D, E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops are given in the following table:

Transportation cost per quintal (in Rs.)		
From / To	**A**	**B**

D	**6**	**4**
E	**3**	**2**
F	**2.50**	**3**

How should the supplies be transported in order that the transportation cost is minimum? What is the minimum cost?

Ans. Let godown A supplies x quintals of grain to the ration shop D and y quintals to ration shop E.



We have to minimize Z = $6x+3y+\frac{5}{2}$

$(100-x-y)+4(60-x)+2(50-y)$

$+3(x+y-10)$

$\Rightarrow$ Z = $\frac{5x}{2}+\frac{3y}{2}+410$

Subject to $x+y\leq100$, $x\leq60$, $y\leq50$ and $x+y\geq60$, $x\geq0, y\geq0.$

Consider $x+y\leq100$

Let $x + y = 100$

$$\Rightarrow \frac{x}{100} + \frac{y}{100} = 1$$

$\therefore$ Points P (100, 0) and Q (0, 100) lie on the line and it represents the half-plane containing (0, 0).

Again we consider $x \le 60$ and $y \le 50$

We draw $x = 60$ and $y = 50$

Again consider $x + y \ge 60$

Let $x + y = 60 \Rightarrow \frac{x}{60} + \frac{y}{60} = 1$

$\therefore$ Points A (60, 0) and R (0, 60) lie on the line and it represents the half-plane containing (0, 0)

The shaded region is the feasible solution. Its corners are A (60, 0), B (60, 40), C (50, 50) and D (10, 50).

Now Z = $\frac{5x}{2} + \frac{3y}{2} + 410$

At A (60, 0) Z = $\frac{5}{2} \times 60 + \frac{3}{2} \times 0 + 410 = 150 + 410 = 560$

At B (60, 40) Z = $\frac{5}{2} \times 60 + \frac{3}{2} \times 40 + 410 = 150 + 60 + 410 = 620$

At C (50, 50) Z = $\frac{5}{2} \times 50 + \frac{3}{2} \times 50 + 410 = 125 + 75 + 410 = 610$

At D (10, 50) Z = $\frac{5}{2} \times 10 + \frac{3}{2} \times 50 + 410 = 25 + 75 + 410 = 510$

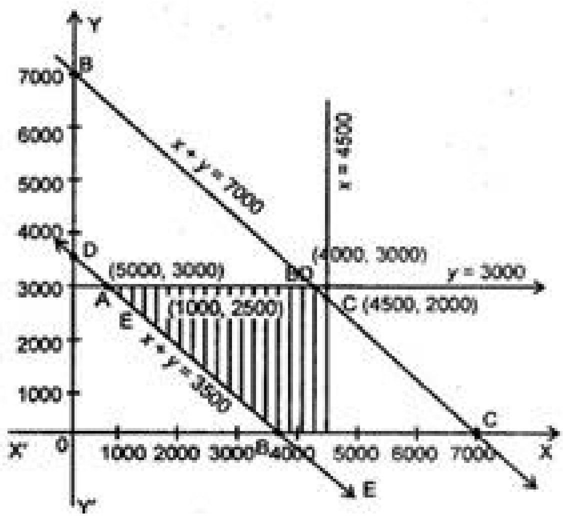
Hence minimum value is Z = 510 at $x = 10, y = 50.$

7. An oil company has two depots A and B with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three petrol pumps. D, E and F whose requirements are 4500 L, 3000 L and 3500 L respectively. The distances (in km) between the depots and the petrol pumps is given in the following table:

Distance in km.		
From / To	**A**	**B**
D	**7**	**3**
E	**6**	**4**
F	**3**	**2**

Assuming that the transportation cost of 10 liters of oil is Re. 1 per km, how should the delivery be scheduled in order that the transportation cost is minimum? What is the minimum cost?

Ans. Let x liters of oil is supplied from depot A to petrol pump D and y liters of oil supplied from depot A to petrol pump E then $7000-(x+y)$ liters of oil will be supplied from depot A to petrol pump F.



$\therefore$ We have $x \geq 0, y \geq 0$ and $7000-(x+y) \geq 0$

$\Rightarrow \quad x+y \leq 700$

Since requirements of oil at petrol pump, D, E and F are $(4500-x)$, $(5000-x)$ and $[3500-(x+y)]$ liters respectively.

$\therefore\ 4500-x\geq 0$

$\Rightarrow\ x\leq 4500$

And $3000-y\geq 0\quad\Rightarrow\ y\leq 3000$

And $3500-\left[7000-(x+y)\right]\geq 0\quad\Rightarrow\ x+y\geq 3500$

$\because$ The cost of transportation per km for 10 liters oil is Re 1

$\therefore$ The cost of transportation per km per liter = Rs. $\frac{1}{10}$

$\therefore$ The cost of transportation

Z =

$$0.7x+0.6y+0.3\left[700-(x+y)\right]+0.3(4500-x)+0.4(3000-y)+\left[(x+y)-3500\right]$$

Z = $0.3x+0.1y+3950$

Therefore, the feasible region is ABECD.

Its corners are A (500, 3000), B (35, 0), E (4500, 0), C (4500, 2500), D (4000, 3000).

Now Z = $0.3x+0.1y+3950$

At A (500, 3000) Z = 0.3 x 500 + 0.1 x 3000 + 3950 = 4400

At B (3500, 0) Z = 0.3 x 3500 + 0.1 x 0 + 3950 = 5000

At E (4500, 0) Z = 0.3 x 4500 + 0.1 x 0 + 3950 = 5300

At C (4500, 2500) Z = 0.3 x 4500 + 0.1 x 2500 + 3950 = 5550

At D (4000, 3000) Z = 0.3 x 4000 + 0.1 x 3000 + 3950 = 5450

Minimum transportation charges are Rs. 4400 at $x=500,\ y=3000$

Hence, 500 liters, 3000 liters and 3500 liters of oil should be transported from depot A to petrol pumps D, E, F and 4000 liters, 0 liter and 0 liter of oil be transported from depot B to petrol pumps D, E and F with minimum cost of transportation of Rs. 4400.

8. A fruit grower can use two types of fertilizer in his garden, brand P and brand Q. The amounts (in kg) of nitrogen, phosphoric acid, potash and chlorine in a bag of each brand are given in the table. Tests indicate that the garden needs at least 240 kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine.

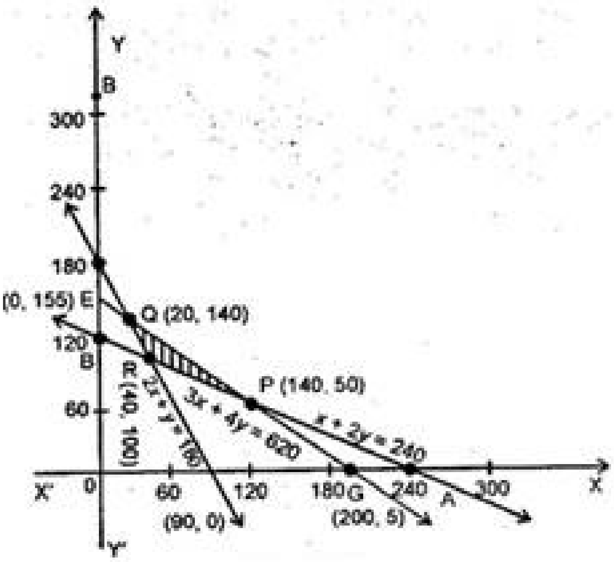
If the grower wants to minimize the amount of nitrogen added to the garden, how many bags of each brand should be used? What is the minimum amount of nitrogen added in the garden?

kg per bag		
	Brand P	**Brand Q**
Nitrogen	**3**	**3.5**
Phosphoric acid	**1**	**2**
Potash	**3**	**1.5**
Chlorine	**1.5**	**2**

Ans. Let amount of Brand P of fertilizer = x bags and amount of Brand Q of fertilizer = y bags

We have to minimize Z = $3x+3.5y$ subject to $x+2y \geq 240$, $3x+1.5y \geq 270$, $1.5x+2y \leq 310$

$x \geq 0, y \geq 0$



Consider $x+2y \geq 240$

Let $x+2y=240$

$$\Rightarrow \frac{x}{240}+\frac{y}{120}=1$$

$\therefore$ Points A (240, 0) and B (0, 120) lie on the line.

And (0, 0) does not lie on the required half-plane of this in equation.

Again consider

$3x+1.5y \geq 270$

Let $3x+1.5y=270$

$\Rightarrow 2x+y=180$

$$\Rightarrow \frac{x}{90}+\frac{y}{180}=1$$

$\therefore$ Points C (90, 0) and D (0, 180) lie on the line.

Here also (0, 0) does not lie on the required half-plane of this inequation.

	E	F	G

x	0	100	200
y	155	80	5

Again consider $1.5x+2y \le 310$

Let $1.5x+2y=310 \Rightarrow 3x+4y=620$

$$\Rightarrow y=\frac{620-3x}{4}$$

Here also (0, 0) does not lie on the required half-plane of this inequation.

The shaded portion is the feasible region. Its corners are P (140, 50), Q (20, 140) and R (40, 100).

Now Z = $3x+3.5y$

At P (140, 50) Z = 3 x 140 + 3.5 x 50 = 420 + 175 = 595

At Q (20, 140) Z = 3 x 20 + 3.5 x 140 = 60 + 490 = 550

At R (40, 100) Z = 3 x 40 + 3.5 x 100 = 120 + 350 = 470

Hence minimum Z = 470 at $x=40, y=100$.

Therefore, minimum amount of nitrogen = 470 kg when 40 bags of brand P and 100 bags of brand Q are used.

9. Refer to Question 8. If the grower wants to maximize the amount of nitrogen added to the garden, how many bags of each brand should be added? What is the maximum amount of nitrogen added?

Ans. We have Z = $3x+3.5y$ and Z is maximum at P (140, 50).

To maximize the amount of nitrogen, 140 bags of brand P and 50 bags of brand Q are required.

Therefore maximum amount of nitrogen required = 595 kg

10. A toy company manufactures two types of dolls, A and B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs.12 and Rs. 16 per doll respectively on dolls A and B, how many of each should be produced weekly in order to maximize the profit?

Ans. Let the number of dolls of type A = x and number of dolls of type B = y

We have to maximize Z = $12x+16y$ subject to $x+y\le 1200$, $y\le\frac{1}{2}x$, $x-3y\le 600$, $x\ge 0, y\ge 0$

Consider, $x+y\le 1200$

Let x + y = 1200

$=> y = 1200 - x$

	A	B	C
x	0	600	1200
y	1200	600	0

Here, (0, 0) is included in the required half plane and satisfies this inequation.

Again consider $y\le\frac{1}{2}x$

Let $y=\frac{1}{2}x$

	O	E	F
x	0	200	600
y	0	100	300

Here, (100, 0) satisfies the inequation $y \le \frac{1}{2}x$, therefore the required half plane includes (100, 0).

Again consider $x - 3y \le 600$

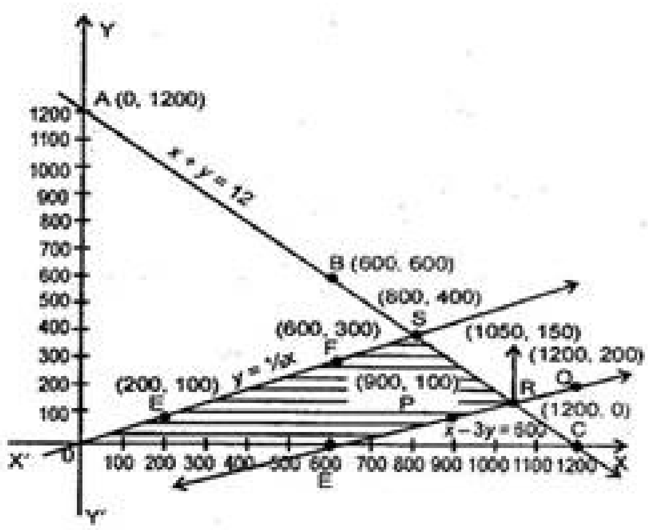
Let $x - 3y = 600$

$=> x = 600 - 3y$

	D	P	Q
x	600	900	1200
y	0	100	200

Here, also (0, 0) is included in the required half-plane.

The shaded region DRSOD is the feasible region whose corners are D(600,0), R(1050,150),



S(800,400) and O(0,0).

Now Z = $12x + 16y$

At D (600, 0) Z = 12 x 600 + 16 x 0 = 7200

At R (1050, 150) Z = 12 x 105 + 16 x 150 = 12600 + 2400 = 15,000

At S (800, 400) Z = 12 x 800 + 16 x 400 = 9600 + 6400 = 16,000

At O (0, 0) Z = 12 x 0 + 16 x 0 = 0

Hence maximum profit Z = Rs. 16,000 at $x = 800, y = 400$.