CBSE Class–12 Mathematics

NCERT solution

Chapter - 1

Relations & Functions - Exercise 1.4

1. Determine whether or not each of the definition of * given below gives a binary operation. In the event that * is not a binary operation, given justification for this.

(i) On Z^+, define * by $a * b = a - b$

(ii) On Z^+, define * by $a * b = ab$

(iii) On R, define * by $a * b = ab^2$

(iv) On Z^+, define * by $a * b = |a - b|$

(v) On Z^+, define * by $a * b = a$

Ans. (i) On Z^+ = {1, 2, 3, ...}, $a * b = a - b$

Let $a = 1, b = 3$ $\therefore a * b = 1 - 3 = -2 \notin Z^+$

Therefore, operation * is not a binary operation on Z^+.

(ii) On Z^+ = {1, 2, 3, ...}, $a * b = ab$

Let $a = 2, b = 4$ $\therefore$ $a * b = 2 \times 4 = 8 \in Z^+$

Therefore, operation * is a binary operation on Z^+.

(iii) on R (set of real numbers) $a * b = ab^2$

Let $a = 5.2, b = 3$ $\therefore a * b = 5.2(3)^2 = 36.8 \in$ R

Therefore, operation * is a binary operation on R.

(iv) On Z^+ = {1, 2, 3, ...}, $a * b = |a - b|$

Let $a=3, b=7 \therefore a*b=|3-7|=|-4|=4 \in Z^+$

Therefore, operation * is a binary operation on Z^+.

(v) On Z^+ = {1, 2, 3, …..}, $a*b=a$

Let $a=5, b=7 \therefore a*b=5\times 7=35 \in Z^+$

Therefore, operation * is a binary operation on Z^+.

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2. For each binary operation * defined below, determine whether * is commutative or associative:

(i) On Z, define $a*b=a-b$

(ii) On Q, define $a*b=ab+1$

(iii) On Q, define $a*b=\frac{ab}{2}$

(iv) On Z, define $a*b=2^{ab}$

(v) On Z, define $a*b=a^b$

(vi) On R – {– 1}, define $a*b=\frac{a}{b+1}$

Ans. (i) For commutativity: $a*b=a-b$ and $b*a=b-a$ = $-(a-b)\neq a*b$

For associativity: $a*(b*c)=a*(b-c)$ = $a-(b-c)=(a-b+c)$

Also, $(a*b)*c=(a-b)*c$ = $(a-b-c)$

$\therefore a*(b*c)\neq(a*b)*c$

Therefore, the operation * is neither commutative nor associative.

(ii) For commutativity: $a*b=ab+1$ and $b*a=ba+1=ab+1=a*b$

For associativity: $a*(b*c)=a*(bc+1) = a(bc+1)+1=abc+a+1$

Also, $(a*b)*c=(ab+1)c+1 = abc+c+1$

$\therefore\ a*(b*c)\neq(a*b)*c$

Therefore, the operation * is commutative but not associative.

(iii) For commutativity: $a*b=\frac{ab}{2}$ and $b*a=\frac{ba}{2}=\frac{ab}{2} = a*b$

For associativity: $a*(b*c)=a*\left(\frac{bc}{2}\right) = \frac{abc/2}{2}=\frac{abc}{4}$

Also, $(a*b)*c=\left(\frac{ab}{2}\right)*c = \frac{abc/2}{2}=\frac{abc}{4}$

$\therefore\ a*(b*c)=(a*b)*c$

Therefore, the operation * is commutative and associative.

(iv) For commutativity: $a*b=2^{ab}$ and $b*a=2^{ba}=2^{ab}=a*b$

For associativity: $a*(b*c)=a*2^{bc} = (2)$

Also, $(a*b)*c=\left(2^{ab}\right)*2 = 2^{ab}\times c$

$\therefore\ a*(b*c)\neq(a*b)*c$

Therefore, the operation * is commutative but not associative.

(v) For commutativity: $a*b=a^{b}$ and $b*a=b^{a}$

$\Rightarrow\ a*b\neq b*a$

For associativity: $a*(b*c)=a*b^{c} = (a)^{b^c}$

Also, $(a*b)*c=(a^b)*c = a^{bc}$

$\therefore\ a*(b*c)\neq(a*b)*c$

Therefore, the operation * is neither commutative nor associative.

(vi) For commutativity: $a*b=\dfrac{a}{b+1}$ and $b*a=\dfrac{b}{a+1}$ $\Rightarrow$ $a*b\neq b*a$

For associativity: $a*(b*c)=a*\left(\dfrac{b}{c+1}\right)$ = $\dfrac{a}{\frac{b}{c+1}+1}$ $=$ $\dfrac{a(c+1)}{b+c+1}$

Also, $(a*b)*c=\left(\dfrac{a}{b+1}\right)*c$ = $\dfrac{a/b+1}{c+1/c}=\dfrac{a}{(b+1)(c+1)}$

$\therefore\ a*(b*c)\neq(a*b)*c$

Therefore, the operation * is neither commutative nor associative.

3. Consider the binary operation $\wedge$ on the set {1, 2, 3, 4, 5} defined by $a\wedge b=\min\{a,b\}$. Write the operation table of the operation $\wedge$.

Ans. Let A = {1, 2, 3, 4, 5} defined by $a\wedge b=\min\{a,b\}$ i.e., minimum of a and b.

$\wedge$	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

4. Consider a binary operation * on the set {1, 2, 3, 4, 5} given by the following multiplication table (table 1.2).

(i) Compute (2 * 3) * 4 and 2 * (3 * 4)

(ii) Is * commutative?

(iii) Compute (2 * 3) * (4 * 5)

(Hint: Use the following table)

Table 1.2

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

Ans. (i) 2 * 3 = 1 and 3 * 4 = 1

Now (2 * 3) * 4 = 1 * 4 = 1 and 2 * (3 * 4) = 2 * 1 = 1

(ii) 2 * 3 = 1 and 3 * 4 = 1

2 * 3 = 3 * 2 and other element of the given set.

Hence the operation is commutative.

(iii) (2 * 3) * (4 * 5) = 1 * 1 = 1

5. Let *' be the binary operation on the set {1, 2, 3, 4, 5} defined by $a *' b =$ H.C.F. of a and b. Is the operation *' same as the operation * defined in Exercise 4 above? Justify your answer.

Ans. Let A = {1, 2, 3, 4, 5} and $a *' b =$ H.C.F. of a and b.

*'	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1

5	1	1	1	1	5

We observe that the operation *' is the same as the operation * in Exp.4.

6. Let * be the binary operation on N given by $a*b=$ L.C.M. of a and b. Find:

(i) 5 * 7, 20 * 16

(ii) Is * commutative?

(iii) Is * associative?

(iv) Find the identity of * in N.

(v) Which elements of N are invertible for the operation *?

Ans. $a*b=$ L.C.M. of a and b

(i) 5 * 7 = L.C.M. of 5 and 7 = 35

20 * 16 = L.C.M. of 20 and 16 = 80

(ii) $a*b=$ L.C.M. of a and b = L.C.M. of b and a = $b*a$

Therefore, operation * is commutative.

(iii) $a*(b*c)=a*(\text{L.C.M. of } b \text{ and } c)$ = $\text{L.C.M. of } (a \text{ and L.C.M. of } b \text{ and } c)$

= $\text{L.C.M. of } a, b \text{ and } c$

Similarly, $(a*b)*c=\text{L.C.M. of } a, b \text{ and } c$

Thus, $a*(b*c)=(a*b)*c$

Therefore, the operation is associative.

(iv) Identity of * in N = 1 because $a*1$ = L.C.M. of a and 1 = a

(v) Only the element 1 in N is invertible for the operation * because $1*\frac{1}{1}=1$

7. Is * defined on the set {1, 2, 3, 4, 5} by $a * b =$ L.C.M. of a and b a binary operation? Justify your answer.

Ans. Let A = {1, 2, 3, 4, 5} and $a *' b =$ L.C.M. of a and b.

*	1	2	3	4	5
1	1	2	3	4	5
2	2	2	6	4	10
3	3	x	3	12	15
4	4	4	12	4	20
5	5	x	15	20	5

Here, 2 * 3 = 6 $\notin$ A

Therefore, the operation * is not a binary operation.

8. Let * be the binary operation on N defined by $a * b =$ H.C.F. of a and b. Is * commutative? Is * associative? Does there exist identity for this binary operation on N?

Ans. $a * b =$ H.C.F. of a and b.

(i) $a * b =$ H.C.F. of a and b = H.C.F. of b and a = $b * a$

Therefore, operation * is commutative.

(ii) $(a * b) * c = (\text{H.C.F. of } a \text{ and } b) * c$ = $\text{H.C.F. of } (\text{H.C.F. of } a \text{ and } b) \text{ and } c$

= $\text{H.C.F. of } a, b \text{ and } c = a * (b * c)$

Therefore, the operation is associative.

$1 * a = a * 1 \neq a$

Therefore, there does not exist any identity element.

9. Let * be a binary operation on the set Q of rational numbers as follows:

(i) $a * b = a - b$ **(ii)** $a * b = a^2 + b^2$

(iii) $a * b = a + ab$ **(iv)** $a * b = (a-b)^2$

(v) $a * b = \frac{ab}{4}$ **(vi)** $a * b = ab^2$

Find which of the binary operations are commutative and which are associative.

Ans. (i) $a * b = a - b = -(b-a) = -b * a$ $\therefore$ operation * is not commutative.

$(a*b)*c = (a-b)*c = (a-b)-c = a-b-c$

And $a*(b*c) = a*(b-c) = a-(b-c) = a-b+c$

Here, $(a*b)*c \neq a*(b*c)$ $\therefore$ operation * is not associative.

(ii) $a*b = a^2+b^2 = b^2+a^2 = b*a$ $\therefore$ operation * is commutative.

$(a*b)*c = (a^2+b^2)*c = (a^2+b^2)+c^2 = a^2+b^2+c^2$

And $a*(b*c) = a*(b^2+c^2) = a^2+(b^2+c^2)^2$

Here, $(a*b)*c \neq a*(b*c)$ $\therefore$ operation * is not associative.

(iii) $a*b = a+ab = a(1+b)$ and $b*a = b+ba = b(1+a) \neq a*b$

Therefore, operation * is not commutative.

$(a*b)*c = (a+ab)*c = (a+ab)+(a+ab)c$

And $a*(b*c) = a*(b+bc) = a+a(b+bc)$

Here, $(a*b)*c \neq a*(b*c)$ $\therefore$ operation * is not associative.

(iv) $a*b = (a-b)^2 = (b-a)^2 = b*a$ $\therefore$ operation * is commutative.

$(a*b)*c=(a-b)^2*c=\left[(a-b)^2-c\right]^2$

And $a*(b*c)=a*(b-c)^2=\left[a-(b-c)^2\right]^2$

Here, $(a*b)*c \neq a*(b*c)$ $\therefore$ operation * is not associative.

(v) $a*b=\frac{ab}{4}=\frac{ba}{4}=b*a$ $\therefore$ operation * is commutative.

$(a*b)*c=\frac{ab}{4}*c=\frac{\frac{ab}{4}c}{4}=\frac{abc}{16}$ And $a*(b*c)=a*\frac{bc}{4}=\frac{a\frac{bc}{4}}{4}=\frac{abc}{16}$

Here, $(a*b)*c=a*(b*c)$ $\therefore$ operation * is associative.

(vi) $a*b=ab^2$ and $b*a=ba^2 \neq a*b$ $\therefore$ operation * is not commutative.

$(a*b)*c=(ab^2)*c=(ab^2)c^2=ab^2c^2$

And $a*(b*c)=a*(bc^2)=a(bc^2)^2=ab^2c^4$

Here, $(a*b)*c \neq a*(b*c)$ $\therefore$ operation * is not associative.

10. Show that none of the operations given above the identity.

Ans. Let the identity be I.

(i) $a*\mathrm{I}=a-\mathrm{I}\neq a$

(ii) $a*\mathrm{I}=a^2-\mathrm{I}^2\neq a$

(iii) $a*\mathrm{I}=a+a\mathrm{I}\neq a$

(iv) $a*\mathrm{I}=(a-\mathrm{I})^2\neq a$

(v) $a * \mathrm{I} = \frac{a\mathrm{I}}{4} \neq a$

(vi) $a * \mathrm{I} = a\mathrm{I}^2 \neq a$

Therefore, none of the operations given above has identity.

11. Let A = N x N and * be the binary operation on A defined by $(a,b)*(c,d)=(a+c,b+d)$

Show that * is commutative and associative. Find the identity element for * on A, if any.

Ans. A = N x N and * is a binary operation defined on A.

$(a,b)*(c,d)=(a+c,b+d)=(c+a,d+b)=(c,d)*(a,b)$ $\therefore$ The operation is commutative

Again, $[(a,b)*(c,d)]*(e,f)=(a+c,b+d)*(e,f)=(a+c+e,b+d+f)$

And $(a,b)[(c,d)*(e,f)]=(a,b)*(c+e,e+f)=(a+c+e,b+d+f)$

Here, $[(a,b)*(c,d)]*(e,f)=(a,b)[(c,d)*(e,f)]$ $\therefore$ The operation is associative.

Let identity function be (e,f), then $(a,b)*(e,f)=(a+e,b+f)$

For identity function $a=a+e \Rightarrow e=0$

And for $b+f=b \Rightarrow f=0$

As $0 \neq$ N, therefore, identity-element does not exist.

12. State whether the following statements are true or false. Justify:

(i) For an arbitrary binary operation * on a set N, $a*a=a \forall a \in \mathrm{N}$.

(ii) If * is a commutative binary operation on N, then $a*(b*b)=(c*b)*a$.

Ans. (i) * being a binary operation on N, is defined as $a * a = a \forall a \in \mathrm{N}$.

Hence operation * is not defined, therefore, the given statement is false.

(ii) * being a binary operation on N.

$\therefore\ c * b = b * c \ \therefore\ (c * b) * a = (b * c) * a = a * (b * c)$

Thus, $a * (b * b) = (c * b) * a$, therefore the given statement is true.

13. Consider a binary operation * on N defined as $a * b = a^3 + b^3$. Choose the correct answer:

(A) Is * both associative and commutative?

(B) Is * commutative but not associative?

(C) Is * associative but not commutative?

(D) Is * neither commutative nor associative?

Ans. $a * b = a^3 + b^3 = b^3 + a^3 = b * a$ $\therefore$ The operation * is commutative.

Again, $(a * b) * c = a * (a^3 + b^3) = a^3 (a^3 + b^3)^3$

And $(a * b) * c = (a^3 + b^3) * c = (a^3 + b^3)^3 + c^3 \neq a * (b * c)$

$\therefore$ The operation * is not associative.

Therefore, option (B) is correct.