CBSE Class–12 Mathematics

NCERT solution

Chapter - 5

Continuity & Differentiability - Exercise 5.4

Differentiate the functions with respect to x in Exercise 1 to 10.

1. $\frac{e^x}{\sin x}$

Ans. Let $y = \frac{e^x}{\sin x}$

$\therefore \quad \frac{dy}{dx} = \frac{\sin x \frac{d}{dx} e^x - e^x \frac{d}{dx} \sin x}{\sin^2 x}$ [By quotient rule]

$= \frac{\sin x . e^x - e^x \cos x}{\sin^2 x}$

$= e^x \frac{(\sin x - \cos x)}{\sin^2 x}$

2. $e^{\sin^{-1} x}$

Ans. Let $y = e^{\sin^{-1} x}$

$\therefore \quad \frac{dy}{dx} = e^{\sin^{-1} x} . \frac{d}{dx} \sin^{-1} x$

$= e^{\sin^{-1} x} . \frac{1}{\sqrt{1-x^2}} \quad \left[\because \frac{d}{dx} e^{f(x)} = e^{f(x)} \frac{d}{dx} f(x)\right]$

3. e^{x^3}

Ans. Let $y = e^{x^3} = e^{\left(x^3\right)}$

$$\therefore \frac{dy}{dx} = e^{\left(x^3\right)} \frac{d}{dx} x^3$$

$$= e^{\left(x^3\right)} .3x^2 = 3x^2 .e^{\left(x^3\right)} \left[\because \frac{d}{dx} e^{f(x)} = e^{f(x)} \frac{d}{dx} f(x)\right]$$

4. $\sin\left(\tan^{-1} e^{-x}\right)$

Ans. Let $y = \sin\left(\tan^{-1} e^{-x}\right)$

$$\therefore \frac{dy}{dx} = \cos\left(\tan^{-1} e^{-x}\right) \frac{d}{dx}\left(\tan^{-1} e^{-x}\right) \left[\because \frac{d}{dx} \sin f(x) = \cos f(x) \frac{d}{dx} f(x)\right]$$

$$= \cos\left(\tan^{-1} e^{-x}\right) \frac{1}{1+\left(e^{-x}\right)^2} \frac{d}{dx} e^{-x} \left[\because \frac{d}{dx} \tan^{-1} f(x) = \frac{1}{\left(f(x)\right)^2} \frac{d}{dx} f(x)\right]$$

$$= \cos\left(\tan^{-1} e^{-x}\right) \frac{1}{1+e^{-2x}} e^{-x} \frac{d}{dx}(-x)$$

$$= -\frac{e^{-x} \cos\left(\tan^{-1} e^{-x}\right)}{1+e^{-2x}}$$

5. $\log\left(\cos e^{x}\right)$

Ans. Let $y = \log\left(\cos e^{x}\right)$

$$\therefore \frac{dy}{dx} = \frac{1}{\cos e^{x}} \frac{d}{dx}\left(\cos e^{x}\right) \left[\because \frac{d}{dx} \log f(x) = \frac{1}{f(x)} \frac{d}{dx} f(x)\right]$$

$$= \frac{1}{\cos e^{x}}\left(-\sin e^{x}\right) \frac{d}{dx} e^{x} \left[\because \frac{d}{dx} \cos f(x) = -\sin f(x) \frac{d}{dx} f(x)\right]$$

= $-\left(\tan e^x\right)e^x = -e^x\left(\tan e^x\right)$

6. $e^x + e^{x^2} + \ldots\ldots + e^{x^5}$

Ans. Let $y = e^x + e^{x^2} + \ldots\ldots + e^{x^5}$ = $e^x + e^{x^2} + e^{x^3} + e^{x^4} + e^{x^5}$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}e^x + \frac{d}{dx}e^{x^2} + \frac{d}{dx}e^{x^3} + \frac{d}{dx}e^{x^4} + \frac{d}{dx}e^{x^5}$$

$$= e^x + e^{x^2}\frac{d}{dx}x^2 + e^{x^3}\frac{d}{dx}x^3 + e^{x^4}\frac{d}{dx}x^4 + e^{x^5}\frac{d}{dx}x^5$$

$$= e^x + e^{x^2}.2x + e^{x^3}.3x^2 + e^{x^4}.4x^3 + e^{x^5}.5x^4$$

$$= e^x + 2xe^{x^2} + 3x^2e^{x^3} + 4x^3.e^{x^4} + 5x^4.e^{x^5}$$

7. $\sqrt{e^{\sqrt{x}}}, x > 0$

Ans. Let $y = \sqrt{e^{\sqrt{x}}}$ = $\left(e^{\sqrt{x}}\right)^{\frac{1}{2}}$

$$\therefore \frac{dy}{dx} = \frac{1}{2}\left(e^{\sqrt{x}}\right)^{\frac{-1}{2}}\frac{d}{dx}e^{\sqrt{x}} \quad \left[\because \frac{d}{dx}\left(f(x)\right)^n = n\left(f(x)\right)^{n-1}\frac{d}{dx}f(x)\right]$$

$$= \frac{1}{2\sqrt{e^{\sqrt{x}}}}e^{\sqrt{x}}\frac{d}{dx}\sqrt{x}$$

$$= \frac{1}{2\sqrt{e^{\sqrt{x}}}}e^{\sqrt{x}}\frac{1}{2\sqrt{x}}$$

$$= \frac{e^{\sqrt{x}}}{4\sqrt{x}\sqrt{e^{\sqrt{x}}}}$$

$$= \frac{e^{\sqrt{x}}}{4\sqrt{xe^{\sqrt{x}}}}$$

8. $\log(\log x), x > 1$

Ans. Let $y = \log(\log x)$

$$\therefore \frac{dy}{dx} = \frac{1}{\log x}\frac{d}{dx}(\log x)$$

$$= \frac{1}{\log x}.\frac{1}{x} = \frac{1}{x\log x}$$

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9. $\frac{\cos x}{\log x}, x > 0$

Ans. Let $y = \frac{\cos x}{\log x}$

$$\therefore \frac{dy}{dx} = \frac{\log x\frac{d}{dx}(\cos x) - \cos x\frac{d}{dx}(\log x)}{(\log x)^2}$$ [By quotient rule]

$$= \frac{\log x(-\sin x) - \cos x\frac{1}{x}}{(\log x)^2}$$

$$= \frac{-\left(\sin x\log x + \frac{\cos x}{x}\right)}{(\log x)^2}$$

$$= \frac{-(x\sin x\log x + \cos x)}{x(\log x)^2}$$

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10. $\cos\left(\log x + e^x\right), x > 0$

Ans. Let $y = \cos\left(\log x + e^x\right)$

$\therefore \qquad \frac{dy}{dx} = -\sin\left(\log x + e^x\right)\frac{d}{dx}\left(\log x + e^x\right)$

$= -\sin\left(\log x + e^x\right).\left(\frac{1}{x} + e^x\right)$