CBSE Class–12 Mathematics

NCERT solution

Chapter - 5

Continuity & Differentiability - Exercise 5.8

1. Verify Rolle's theorem for $f(x)=x^2+2x-8, x\in[-4,2]$.

Ans. Consider $f(x)=x^2+2x-8, x\in[-4,2]$

(i) Function is continuous in $[-4,2]$ as it is a polynomial function and polynomial function is always continuous.

(ii) $f'(x)=2x+2$, $f'(x)$ exists in $[-4,2]$, hence derivable.

(iii) $f(-4)=0$ and $f(2)=0$

$\therefore\ f(-4)=f(2)$

Conditions of Rolle's theorem are satisfied, hence there exists, at least one $c\in(-4,2)$ such that $f'(c)=0$

$\Rightarrow\ 2c+2=0$

$\Rightarrow\ c=-1$

2. Examine if Rolle's theorem is applicable to any of the following functions. Can you say something about the converse of Rolle's theorem from these examples:

(i) $f(x)=[x]$ **for** $x\in[5,9]$

(ii) $f(x)=[x]$ **for** $x\in[-2,2]$

(iii) $f(x)=x^2-1$ **for** $x\in[1,2]$

Ans. (i) Being greatest integer function the given function is not differentiable and continuous

hence Rolle's theorem is not applicable.

(ii) Being greatest integer function the given function is not differentiable and continuous hence Rolle's theorem is not applicable.

(iii) $f(x)=x^2-1 \Rightarrow \quad f(1)=(1)^2-1=1-1=0$

$f(2)=(2)^2-1=4-1=3 \therefore f(1)\neq f(2)$

Hence, Rolle's theorem is not applicable.

3. If $f:[-5,5]\to$ R is a differentiable function and if $f'(x)$ does not vanish anywhere, then prove that $f(-5)\neq f(5)$.

Ans. For, Rolle's theorem, if

(i) f is continuous in $[a,b]$

(ii) f is derivable in $[a,b]$

(iii) $f(a)=f(b)$

Then, $f'(c)=0, c\in(a,b)$

It is given that f is continuous and derivable, but $f'(c)\neq 0$

$\Rightarrow f(a)\neq f(b)$

$\Rightarrow f(-5)\neq f(5)$

4. Verify Mean Value Theorem if $f(x)=x^2-4x-3$ in the interval $[a,b]$ where $a=1$ and $b=4$.

Ans. (i) Function is continuous in [1, 4] as it is a polynomial function and polynomial function is always continuous.

(ii) $f'(x) = 2x - 4$, $f'(x)$ exists in [1, 4], hence derivable. Conditions of MVT theorem are satisfied, hence there exists, at least one $c \in (1, 4)$ such that

$$\frac{f(4) - f(1)}{4 - 1} = f'(c)$$

$$\Rightarrow \frac{-3 - (-6)}{3} = 2c - 4$$

$$\Rightarrow 1 = 2c - 4$$

$$\Rightarrow c = \frac{5}{2}$$

5. Verify Mean Value Theorem if $f(x) = x^3 - 5x^2 - 3x$ in the interval $[a, b]$ where $a = 1$ and $b = 3$. Find all $c \in (1, 3)$ for which $f'(c) = 0$.

Ans. (i) Function is continuous in [1, 3] as it is a polynomial function and polynomial function is always continuous.

(ii) $f'(x) = 3x^2 - 10x - 3$, it exists in [1, 3], hence derivable.

Conditions of MVT theorem are satisfied, hence there exists, at least one $c \in (1, 3)$ such that

$$\Rightarrow \frac{f(3) - f(1)}{3 - 1} = f'(c)$$

$$\Rightarrow \frac{-27 - (-7)}{2} = 3c^2 - 10c - 3$$

$$\Rightarrow -7 = 3c^2 - 10c$$

$$\Rightarrow 3c^2 - 10c + 7 = 0$$

$$\Rightarrow 3c^2 - 7c - 3c + 7 = 0$$

$\Rightarrow c(3c-7)-1(3c-7)=0$

$\Rightarrow (3c-7)(c-1)=0$

$\Rightarrow (3c-7)=0$ or $(c-1)=0$

$\Rightarrow 3c=7$ or $c=1$

$\Rightarrow c=\frac{7}{3}$ or $c=1$

$\therefore\ c=\frac{7}{3}\in(1,3)$ and other value $\in(1,3)$

Since $f(1)\neq f(3)$, therefore the value of 'c' does not exist such that $f(c)=0$.

---

6. Examine the applicability of Mean Value Theorem for all the three functions being given below:

(i) $f(x)=[x]$ **for** $x\in[5,9]$

(ii) $f(x)=[x]$ **for** $x\in[-2,2]$

(iii) $f(x)=x^2-1$ **for** $x\in[1,2]$

Ans. Mean Value Theorem states that for a function $f:[a,b]\to$ R, if

(i) f is continuous on (a,b)

(ii) f is differentiable on (a,b)

Then there exist some $c\in(a,b)$ such that $f'(c)=\frac{f(b)-f(a)}{b-a}$

Therefore, the Mean Value Theorem is not applicable to those functions that do not satisfy any of the two conditions of the hypothesis.

(i) $f(x)=[x]$ for $x\in[5,9]$

It is evident that the given function $f(x)$ is not continuous at $x=5$ and $x=9$.

Therefore,

$f(x)$ is not continuous at $[5,9]$.

Now let n be an integer such that $n\in[5,9]$

$\therefore$ L.H.L. = $\lim\limits_{h\to0^-}\dfrac{f(n+h)-f(n)}{h}=\lim\limits_{h\to0^-}\dfrac{(n+h)-(n)}{h}=\lim\limits_{h\to0^-}\dfrac{n-1-n}{h}=\lim\limits_{h\to0^-}\dfrac{-1}{h}=\infty$

And R.H.L. = $\lim\limits_{h\to0^+}\dfrac{f(n+h)-f(n)}{h}=\lim\limits_{h\to0^+}\dfrac{(n+h)-(n)}{h}=\lim\limits_{h\to0^+}\dfrac{n-n}{h}=\lim\limits_{h\to0^+}0=0$

Since, L.H.L. $\neq$ R.H.L.,

Therefore f is not differentiable at $[5,9]$.

Hence Mean Value Theorem is not applicable for $f(x)=[x]$ for $x\in[5,9]$

(ii) $f(x)=[x]$ for $x\in[-2,2]$

It is evident that the given function $f(x)$ is not continuous at $x=-2$ and $x=2$.

Therefore,

$f(x)$ is not continuous at $[-2,2]$.

Now let n be an integer such that $n\in[-2,2]$

$\therefore$ L.H.L. = $\lim\limits_{h\to0^-}\dfrac{f(n+h)-f(n)}{h}=\lim\limits_{h\to0^-}\dfrac{(n+h)-(n)}{h}=\lim\limits_{h\to0^-}\dfrac{n-1-n}{h}=\lim\limits_{h\to0^-}\dfrac{-1}{h}=\infty$

And R.H.L. = $\lim\limits_{h\to0^+}\dfrac{f(n+h)-f(n)}{h}=\lim\limits_{h\to0^+}\dfrac{(n+h)-(n)}{h}=\lim\limits_{h\to0^+}\dfrac{n-n}{h}=\lim\limits_{h\to0^+}0=0$

Since, L.H.L. $\neq$ R.H.L.,

Therefore f is not differentiable at $[-2, 2]$.

Hence Mean Value Theorem is not applicable for $f(x) = [x]$ for $x \in [-2, 2]$

(iii) $f(x) = x^2 - 1$ for $x \in [1, 2]$ ……….(i)

Here, $f(x)$ is a polynomial function of degree 2.

Therefore, $f(x)$ is continuous and derivable everywhere i.e., on the real time $(-\infty, \infty)$.

Hence $f(x)$ is continuous in the closed interval [1, 2] and derivable in open interval (1, 2).

Therefore, both conditions of Mean Value Theorem are satisfied.

Now, From eq. (i), $f'(x) = 2x$

$\therefore\ f'(c) = 2c$

Again, From eq. (i), $f(a) = f(1) = (1)^2 - 1 = 1 - 1 = 0$

And From eq. (ii), $f(b) = f(2) = (2)^2 - 1 = 4 - 1 = 3$

$$\therefore\ f'c = \frac{f(b) - f(a)}{b - 1}$$

$$\Rightarrow\ 2c = \frac{3 - 0}{2 - 1}$$

$$\Rightarrow\ c = \frac{3}{2} \in (1, 2)$$

Therefore, Mean Value Theorem is verified.