CBSE Class–12 Mathematics

NCERT solution

Chapter - 5

Continuity & Differentiability - Exercise 5.2

Differentiate the functions with respect to x in Exercise 1 to 8.

1. $\sin\left(x^2+5\right)$

Ans. Let $y=\sin\left(x^2+5\right)$

$\therefore \dfrac{dy}{dx}= \cos\left(x^2+5\right)\dfrac{d}{dx}\left(x^2+5\right)$

= $\cos\left(x^2+5\right)\left(2x+0\right)$

= $2x\cos\left(x^2+5\right)$

2. $\cos\left(\sin x\right)$

Ans. Let $y=\cos\left(\sin x\right)$

$\therefore \dfrac{dy}{dx}= -\sin\left(\sin x\right)\dfrac{d}{dx}\sin x$

= $-\sin\left(\sin x\right)\cos x$

$= -cosx.\, sin(sinx)$

3. $\sin\left(ax+b\right)$

Ans. Let $y=\sin\left(ax+b\right)$

$\therefore \ \frac{dy}{dx} = \cos(ax+b)\frac{d}{dx}(ax+b)$

$= \cos(ax+b)(a+0)$

$= a\cos(ax+b)$

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4. $\sec\left(\tan\sqrt{x}\right)$

Ans. Let $y = \sec\left(\tan\sqrt{x}\right)$

$\therefore \ \frac{dy}{dx} = \sec\left(\tan\sqrt{x}\right)\tan\left(\tan\sqrt{x}\right)\frac{d}{dx}\left(\tan\sqrt{x}\right)$

$= \sec\left(\tan\sqrt{x}\right)\tan\left(\tan\sqrt{x}\right)\sec^2\sqrt{x}\frac{d}{dx}\sqrt{x}$

$= \sec\left(\tan\sqrt{x}\right)\tan\left(\tan\sqrt{x}\right)\sec^2\sqrt{x}.\frac{1}{2}x^{\frac{1}{2}-1}$

$= \sec\left(\tan\sqrt{x}\right)\tan\left(\tan\sqrt{x}\right)\sec^2\sqrt{x}.\frac{1}{2\sqrt{x}}$

$= \frac{sec(tan\sqrt{x}).tan(tan\sqrt{x}).sec^2\sqrt{x}}{2\sqrt{x}}$

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5. $\frac{\sin(ax+b)}{\cos(cx+d)}$

Ans. Let $y = \frac{\sin(ax+b)}{\cos(cx+d)}$

Using quotient rule,

$$\therefore \frac{dy}{dx} = \frac{\cos(cx+d)\frac{d}{dx}\sin(ax+b) - \sin(ax+b)\frac{d}{dx}\cos(cx+d)}{\cos^2(cx+d)}$$

$$= \frac{\cos(cx+d)\cos(ax+b)\frac{d}{dx}(ax+b) - \sin(ax+b)\{-\sin(cx+d)\}\frac{d}{dx}(cx+d)}{\cos^2(cx+d)}$$

$$= \frac{\cos(cx+d)\cos(ax+b)(a) + \sin(ax+b)\sin(cx+d)(c)}{\cos^2(cx+d)}$$

$$= \frac{a.cos(cx+d)cos(ax+b)}{cos^2(cx+d)} + \frac{c.sin(ax+b)sin(cx+d)}{cos^2(cx+d)}$$

$$= a.cos(ax+b).sec(cx+d) + c.sin(ax+b).tan(cx+d).sec(cx+d)$$

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6. $\cos x^3 \sin^2(x^5)$

Ans. Let $y = \cos x^3 . \sin^2(x^5)$

$$\therefore \frac{dy}{dx} = \cos x^3 \frac{d}{dx}\sin^2(x^5) + \sin^2(x^5)\frac{d}{dx}\cos x^3$$

$$= \cos x^3 .2\sin(x^5)\frac{d}{dx}\sin(x^5) + \sin^2(x^5)(-\sin x^3)\frac{d}{dx}x^3$$

$$= cosx^3 .2sin(x^5).cos(x^5)\frac{d}{dx}(x^5) + sin^2(x^5)(-sinx^3)3x^2$$

$$= \cos x^3 .2\sin(x^5)\cos(x^5)(5x^4) - \sin^2(x^5)\sin x^3 .3x^2$$

$$= 10x^4 \cos x^3 \sin(x^5)\cos(x^5) - 3x^2 \sin^2(x^5)\sin x^3$$

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7. $2\sqrt{\cot(x^2)}$

Ans. Let $y = 2\sqrt{\cot(x^2)}$

$\therefore \ \frac{dy}{dx} = 2.\frac{1}{2}\left\{\cot\left(x^2\right)\right\}^{\frac{-1}{2}}.\frac{d}{dx}\cot\left(x^2\right)$

$= \frac{1}{\sqrt{\cot\left(x^2\right)}}.\left\{-\cos ec\left(x^2\right)\right\}\frac{d}{dx}x^2$

$= \frac{1}{\sqrt{\cot\left(x^2\right)}}.\left\{-\cos ec\left(x^2\right)\right\}(2x)$

$= \frac{-2x\cos ec\left(x^2\right)}{\sqrt{\cot\left(x^2\right)}}$

8. $\cos\left(\sqrt{x}\right)$

Ans. Let $y = \cos\left(\sqrt{x}\right)$

$\therefore \ \frac{dy}{dx} = -\sin\sqrt{x}\frac{d}{dx}\sqrt{x}$

$= -\sin\sqrt{x}.\frac{1}{2}(x)^{\frac{-1}{2}}$

$= \frac{-\sin\sqrt{x}}{2\sqrt{x}}$

9. Prove that the function f given by $f(x) = |x-1|, x \in \mathrm{R}$ is not differentiable at $x = 1$.

Ans. Given: $f(x) = |x-1|$

$\therefore \ f(1) = |1-1| = 0$

$f'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$

= $\lim_{h\to 0}\frac{|1+h-1|-0}{h}$

= $\lim_{h\to 0}\frac{|h|}{h}$ = $\lim_{h\to 0}\frac{h}{h}$ = 1

And L $f'(1)=\lim_{h\to 0}\frac{f(1-h)-f(1)}{-h}$

= $\lim_{h\to 0}\frac{|1-h-1|-0}{-h}$

= $\lim_{h\to 0}\frac{|-h|}{-h}$

= $\lim_{h\to 0}\frac{-h}{h}=-1$

Since R $f'(1)\neq$ L $f'(1)$

Therefore, $f(x)$ is not differentiable at $x=1$.

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10. Prove that the greatest integer function defined by $f(x)=[x], 0<x<3$ is not differentiable at $x=1$ and $x=2$.

Ans. Given: $f(x)=[x], 0<x<3$

R $f'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$

= $\lim_{h\to 0}\frac{|1+h|-1}{h}$ = $\lim_{h\to 0}\frac{1-1}{h}$

= $\lim_{h \to 0} \frac{0}{h}$ = 0

And L $f'(1) = \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h}$

= $\lim_{h \to 0} \frac{|1-h|-1}{-h}$

= $\lim_{h \to 0} \frac{0-1}{-h}$ = ∞

Since R $f'(1) \neq$ L $f'(1)$

Therefore, $f(x) = [x]$ is not differentiable at $x = 1$.

Similarly, $f(x) = [x]$ is not differentiable at $x = 2$.