CBSE Class–12 Mathematics

NCERT solution

Chapter - 2

Inverse Trigonometric Functions - Exercise 2.1

Find the principal values of the following:

1. $\sin^{-1}\left(\frac{-1}{2}\right)$

Ans. Let $\sin^{-1}\left(\frac{-1}{2}\right) = y$

$\Rightarrow \sin y = -\frac{1}{2}$

$\Rightarrow \sin y = -\sin\frac{\pi}{6}$

$\Rightarrow \sin y = \sin\left(-\frac{\pi}{6}\right)$

Since, the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Therefore, Principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is $-\frac{\pi}{6}$.

2. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Ans. Let $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = y$

$\Rightarrow \cos y = \frac{\sqrt{3}}{2}$

$\Rightarrow \cos y = \cos \frac{\pi}{6}$

$\Rightarrow y = \frac{\pi}{6}$

Since, the principal value branch of $\cos^{-1}$ is $[0, \pi]$.

Therefore, Principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{6}$.

3. $\operatorname{cosec}^{-1}(2)$

Ans. Let $\operatorname{cosec}^{-1}(2) = y$

$\Rightarrow \operatorname{cosec} y = 2$

$\Rightarrow \operatorname{cosec} y = \operatorname{cosec} \frac{\pi}{6}$

Since, the principal value branch of $\operatorname{cosec}^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$.

Therefore, Principal value of $\operatorname{cosec}^{-1}(2)$ is $\frac{\pi}{6}$.

4. $\tan^{-1}\left(-\sqrt{3}\right)$

Ans. Let $\tan^{-1}\left(-\sqrt{3}\right) = y$

$\Rightarrow \tan y = -\sqrt{3}$

$\Rightarrow \tan y = -\tan \frac{\pi}{3}$

$\Rightarrow \tan y = \tan\left(-\frac{\pi}{3}\right)$

Since, the principal value branch of $\tan^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Therefore, Principal value of $\tan^{-1}\left(-\sqrt{3}\right)$ is $-\frac{\pi}{3}$.

5. $\cos^{-1}\left(\frac{-1}{2}\right)$

Ans. Let $\cos^{-1}\left(\frac{-1}{2}\right)=y$

$\Rightarrow \cos y=-\frac{1}{2}$

$\Rightarrow \cos y=-\cos\frac{\pi}{3}$

$\Rightarrow \cos y=\cos\left(\pi-\frac{\pi}{3}\right)=\cos\frac{2\pi}{3}$

Since, the principal value branch of $\cos^{-1}$ is $[0,\pi]$.

Therefore, Principal value of $\cos^{-1}\left(\frac{-1}{2}\right)$ is $\frac{2\pi}{3}$.

6. $\tan^{-1}(-1)$

Ans. Let $\tan^{-1}(-1)=y$

$\Rightarrow \tan y=-1$

$\Rightarrow \tan y=-\tan\frac{\pi}{4}$

$\Rightarrow \tan y=\tan\left(-\frac{\pi}{4}\right)$

Since, the principal value branch of $\tan^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Therefore, Principal value of $\tan^{-1}(-1)$ is $-\frac{\pi}{4}$.

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7. $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Ans. Let $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = y$

$\Rightarrow \sec y = \frac{2}{\sqrt{3}}$

$\Rightarrow \sec y = \sec\frac{\pi}{6}$

Since, the principal value branch of $\sec^{-1}$ is $[0, \pi]$.

Therefore, Principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{6}$.

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8. $\cot^{-1}\left(\sqrt{3}\right)$

Ans. Let $\cot^{-1}\left(\sqrt{3}\right) = y$

$\Rightarrow \cot y = \sqrt{3}$

$\Rightarrow \cot y = \cot\frac{\pi}{6}$

Since, the principal value branch of $\cot^{-1}$ is $[0, \pi]$.

Therefore, Principal value of $\cot^{-1}\left(\sqrt{3}\right)$ is $\frac{\pi}{6}$.

9. $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

Ans. Let $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)=y$

$\Rightarrow \cos y=-\frac{1}{\sqrt{2}}$

$\Rightarrow \cos y=-\cos\frac{\pi}{4}$

$\Rightarrow \cos y=\cos\left(\pi-\frac{\pi}{4}\right)=\cos\frac{3\pi}{4}$

Since, the principal value branch of $\cos^{-1}$ is $[0,\pi]$.

Therefore, Principal value of $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$.

10. $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$

Ans. Let $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)=y$

$\Rightarrow \operatorname{cosec} y=-\sqrt{2}$

$\Rightarrow \operatorname{cosec} y=\operatorname{cosec}\frac{-\pi}{4}$

Since, the principal value branch of $\operatorname{cosec}^{-1}$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.

Therefore, Principal value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$ is $\frac{-\pi}{4}$.

Find the value of the following:

11. $\tan^{-1}(1)+\cos^{-1}\left(\frac{-1}{2}\right)+\sin^{-1}\left(\frac{-1}{2}\right)$

Ans. $\tan^{-1}(1)+\cos^{-1}\left(\frac{-1}{2}\right)+\sin^{-1}\left(\frac{-1}{2}\right)$

$= \tan^{-1}\tan\frac{\pi}{4}+\cos^{-1}\left(-\cos\frac{\pi}{3}\right)+\sin^{-1}\left(-\sin\frac{\pi}{6}\right)$

$= \frac{\pi}{4}+\cos^{-1}\left(\cos\left(\pi-\frac{\pi}{3}\right)\right)+\sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right)$

$= \frac{\pi}{4}+\frac{2\pi}{3}-\frac{\pi}{6}$

$= \frac{3\pi+8\pi-2\pi}{12} = \frac{9\pi}{12}=\frac{3\pi}{4}$

12. $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$

Ans. $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$

$= \cos^{-1}\cos\frac{\pi}{3}+2\sin^{-1}\sin\frac{\pi}{6}$

$= \frac{\pi}{3}+2\left(\frac{\pi}{6}\right)$

$= \frac{\pi}{3}+\frac{\pi}{3}=\frac{2\pi}{3}$

13. If $\sin^{-1}x=y,$ **then:**

A) $0\le y\le\pi$

(B) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

(C) $0 < y < \pi$

(D) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Ans. By definition of principal value for $y = \sin^{-1} x$, $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

Therefore, option (B) is correct.

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14. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ **is equal to:**

(A) π

(B) $-\frac{\pi}{3}$

(C) $\frac{\pi}{3}$

(D) $\frac{2\pi}{3}$

Ans. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

$= \tan^{-1}\tan\frac{\pi}{3} - \sec^{-1}\left(-\sec\frac{\pi}{3}\right)$

$= \frac{\pi}{3} - \sec^{-1}\sec\left(\pi - \frac{\pi}{3}\right)$

$= \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$

Therefore, option (B) is correct.