CBSE Class–12 Mathematics

NCERT solution

Chapter - 2

Inverse Trigonometric Functions - Miscellaneous Exercise

Find the value of the following:

1. $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

Ans. $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

$= \cos^{-1}\left(\cos\frac{12\pi+\pi}{6}\right)$

$= \cos^{-1}\left[\cos\left(2\pi+\frac{\pi}{6}\right)\right]$

$= \cos^{-1}\left(\cos\frac{\pi}{6}\right)$

$= \cos^{-1}\left(\cos\frac{\pi}{6}\right) = \frac{\pi}{6}$

2. $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

Ans. $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

$= \tan^{-1}\left(\tan\frac{6\pi+\pi}{6}\right)$

$= \tan^{-1}\left[\tan\left(\pi + \frac{\pi}{6}\right)\right]$

$= \tan^{-1}\left(\tan\frac{\pi}{6}\right)$

$= \tan^{-1}\left(\tan\frac{\pi}{6}\right) = \frac{\pi}{6}$

3. Prove that: $2\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{24}{7}$

Ans. Let $\sin^{-1}\frac{3}{5} = \theta$ so that $\sin\theta = \frac{3}{5}$

$\therefore\ \cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

$\therefore\ \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3}{4}$

Since $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\times\frac{3}{4}}{1-\frac{9}{16}} = \frac{\frac{3}{2}}{\frac{7}{16}} = \frac{24}{7}$

$\Rightarrow\ 2\theta = \tan^{-1}\frac{24}{7}$

$\Rightarrow\ 2\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{24}{7}$

4. Prove that: $\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{77}{36}$

Ans. Let $\sin^{-1}\frac{8}{17}=\theta$ so that $\sin\theta=\frac{8}{17}$

$\therefore\ \cos\theta=\sqrt{1-\sin^2\theta}=\sqrt{1-\frac{64}{289}}=\sqrt{\frac{225}{289}}=\frac{15}{17}$

$\therefore\ \tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{8}{15}$

Again, Let $\sin^{-1}\frac{3}{5}=\phi$ so that $\sin\phi=\frac{3}{5}$

$\therefore\ \cos\phi=\sqrt{1-\sin^2\phi}=\sqrt{1-\frac{9}{25}}=\sqrt{\frac{16}{25}}=\frac{4}{5}$

$\therefore\ \tan\phi=\frac{\sin\phi}{\cos\phi}=\frac{3}{4}$

Since $\tan(\theta+\phi)=\frac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}$

$=\frac{\frac{8}{15}+\frac{3}{4}}{1-\frac{8}{15}\times\frac{3}{4}}$

$=\frac{32+45}{60-24}=\frac{77}{36}$

$\Rightarrow\ \theta+\phi=\tan^{-1}\frac{77}{36}$

$\Rightarrow\ \sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{77}{36}$

5. Prove that: $\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}=\cos^{-1}\frac{33}{65}$

Ans. Let $\cos^{-1}\frac{4}{5}=\theta$ so that $\cos\theta=\frac{4}{5}$

$\therefore\ \sin\theta=\sqrt{1-\cos^2\theta}=\sqrt{1-\frac{16}{25}}=\sqrt{\frac{9}{25}}=\frac{3}{5}$

Again, Let $\cos^{-1}\frac{12}{13}=\phi$ so that $\cos\phi=\frac{12}{13}$

$\therefore\ \sin\phi=\sqrt{1-\cos^2\phi}=\sqrt{1-\frac{144}{169}}=\sqrt{\frac{25}{169}}=\frac{5}{13}$

Since $\cos(\theta+\phi)=\cos\theta\cos\phi-\sin\theta\sin\phi$ = $\frac{4}{5}\times\frac{12}{13}-\frac{3}{5}\times\frac{5}{13}$

= $\frac{48-15}{65}=\frac{33}{65}$

$\Rightarrow\ \theta+\phi=\cos^{-1}\frac{33}{65}$

$\Rightarrow\ \cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}=\cos^{-1}\frac{33}{65}$

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6. Prove that: $\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{56}{65}$

Ans. Let $\cos^{-1}\frac{12}{13}=\theta$ so that $\cos\theta=\frac{12}{13}$

$\therefore\ \sin\theta=\sqrt{1-\cos^2\theta}=\sqrt{1-\frac{144}{169}}=\sqrt{\frac{25}{169}}=\frac{5}{13}$

Again, Let $\sin^{-1}\frac{3}{5}=\phi$ so that $\sin\phi=\frac{3}{5}$

$\therefore\ \cos\phi = \sqrt{1-\sin^2\phi} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

Since $\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$ = $\frac{5}{13} \times \frac{4}{5} + \frac{12}{13} \times \frac{3}{5}$

= $\frac{20+36}{65} = \frac{56}{65}$

$\Rightarrow\ \theta+\phi = \sin^{-1}\frac{56}{65}$

$\Rightarrow\ \cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$

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7. Prove that: $\tan^{-1}\frac{63}{16} = \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}$

Ans. Let $\sin^{-1}\frac{5}{13} = \theta$ so that $\sin\theta = \frac{5}{13}$

$\therefore\ \cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{25}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$

$\therefore\ \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{5}{12}$

Again, Let $\cos^{-1}\frac{3}{5} = \phi$ so that $\cos\phi = \frac{3}{5}$

$\therefore\ \sin\phi = \sqrt{1-\cos^2\phi} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

$\therefore\ \tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{4}{3}$

Since $\tan(\theta+\phi)=\dfrac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}$ = $\dfrac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}\times\frac{4}{3}}$

= $\dfrac{\frac{21}{12}}{\frac{4}{9}}=\dfrac{63}{16}$

$\Rightarrow\ \theta+\phi=\tan^{-1}\dfrac{63}{16}$

$\Rightarrow\ \tan^{-1}\dfrac{63}{16}=\sin^{-1}\dfrac{5}{13}+\cos^{-1}\dfrac{3}{5}$

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8. Prove that: $\tan^{-1}\dfrac{1}{5}+\tan^{-1}\dfrac{1}{7}+\tan^{-1}\dfrac{1}{3}+\tan^{-1}\dfrac{1}{8}=\dfrac{\pi}{4}$

Ans. L.H.S. = $\left(\tan^{-1}\dfrac{1}{5}+\tan^{-1}\dfrac{1}{7}\right)+\left(\tan^{-1}\dfrac{1}{3}+\tan^{-1}\dfrac{1}{8}\right)$

= $\tan^{-1}\left(\dfrac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}\times\frac{1}{7}}\right)+\tan^{-1}\left(\dfrac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{3}\times\frac{1}{8}}\right)$

$\left[\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\dfrac{x+y}{1-xy}\right]$

= $\tan^{-1}\left(\dfrac{\frac{12}{35}}{\frac{34}{35}}\right)+\tan^{-1}\left(\dfrac{\frac{11}{24}}{\frac{23}{24}}\right)$

= $\tan^{-1}\dfrac{6}{17}+\tan^{-1}\dfrac{11}{23}$

$$= \tan^{-1}\left(\frac{\frac{6}{17}+\frac{11}{23}}{1-\frac{6}{17}\times\frac{11}{23}}\right)$$

$$= \tan^{-1}\left(\frac{138+187}{391-66}\right)$$

$$= \tan^{-1}\frac{325}{325} = \tan^{-1}1 = \frac{\pi}{4}$$

R.H.S.

9. Prove that: $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in [0,1]$

Ans. Let $\tan^{-1}\sqrt{x} = \theta$ so that $\tan\theta = \sqrt{x}$

$\Rightarrow\ x = \tan^2\theta$

$$\therefore\ \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$$

$$= \frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$= \frac{1}{2}\cos^{-1}\cos 2\theta$$

$$= \frac{1}{2}\times 2\theta = \theta$$

$$= \tan^{-1}\sqrt{x}$$

10. Prove that: $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$

Ans. We know that $1+\sin x=\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\cos\frac{x}{2}\sin\frac{x}{2}$ = $\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2$

Again, $1-\sin x=\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-2\cos\frac{x}{2}\sin\frac{x}{2}$

= $\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2$

$\therefore$ L.H.S. = $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$

= $\cot^{-1}\left[\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)+\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)-\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}\right]$

= $\cot^{-1}\left(\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right)$

= $\cot^{-1}\cot\frac{x}{2}=\frac{x}{2}$

11. Prove that: $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x, \frac{-1}{\sqrt{2}}\leq x\leq 1$

Ans. Putting $x=\cos 2\theta$ so that $\theta=\frac{1}{2}\cos^{-1}x$

L.H.S. = $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

$= \tan^{-1}\left(\dfrac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$

$= \tan^{-1}\left(\dfrac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right)$

$= \tan^{-1}\left(\dfrac{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}\right)$

Dividing every term by $\sqrt{2}\cos\theta$,

$= \tan^{-1}\left(\dfrac{1-\tan\theta}{1+\tan\theta}\right)$

$= \tan^{-1}\left(\dfrac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}\right)$

$= \tan^{-1}\tan\left(\dfrac{\pi}{4}-\theta\right) = \dfrac{\pi}{4}-\theta$

$= \dfrac{\pi}{4}-\dfrac{1}{2}\cos^{-1}x$ = R.H.S.

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12. Prove that: $\dfrac{9\pi}{8}-\dfrac{9}{4}\sin^{-1}\dfrac{1}{3}=\dfrac{9}{4}\sin^{-1}\dfrac{2\sqrt{2}}{3}$

Ans. L.H.S. = $\dfrac{9\pi}{8}-\dfrac{9}{4}\sin^{-1}\dfrac{1}{3}$

$= \dfrac{9}{4}\left(\dfrac{\pi}{2}-\sin^{-1}\dfrac{1}{3}\right)$

= $\frac{9}{4}\cos^{-1}\frac{1}{3}$ ...(i) $\left[\because \sin^{-1}\theta+\cos^{-1}\theta=\frac{\pi}{2}\right]$

Now, let $\theta=\cos^{-1}\frac{1}{3}$ so that $\cos\theta=\frac{1}{3}$

$\therefore\ \sin\theta=\sqrt{1-\cos^2\theta}=\sqrt{1-\frac{1}{9}}=\sqrt{\frac{8}{9}}=\frac{2\sqrt{2}}{3}$

$\Rightarrow\quad \theta=\sin^{-1}\frac{2\sqrt{2}}{3}$

$\therefore$ From eq. (i), $\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ = R.H.S.

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13. Solve the equation: $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$

Ans. $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$

$\Rightarrow\ \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right)=\tan^{-1}\left(\frac{2}{\sin x}\right)$

$\Rightarrow\ \frac{2\cos x}{1-\cos^2 x}=\frac{2}{\sin x}$

$\Rightarrow\ \frac{\cos x}{\sin x}=1$

$\Rightarrow\ \cot x=1$

$\Rightarrow\ x=\frac{\pi}{4}$

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14. Solve the equation: $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x, (x>0)$

Ans. Putting $x=\tan\theta$

$$\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x$$

$$\Rightarrow \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)=\frac{1}{2}\tan^{-1}\tan\theta$$

$$\Rightarrow \tan^{-1}\left(\frac{\tan\frac{\pi}{4}-\tan\theta}{\tan\frac{\pi}{4}+\tan\theta}\right)=\frac{1}{2}\theta$$

$$\Rightarrow \tan^{-1}\tan\left(\frac{\pi}{4}-\theta\right)=\frac{\theta}{2}$$

$$\Rightarrow \frac{\pi}{4}-\theta=\frac{\theta}{2}$$

$$\Rightarrow \theta+\frac{\theta}{2}=\frac{\pi}{4}$$

$$\Rightarrow \frac{3\theta}{2}=\frac{\pi}{4}$$

$$\Rightarrow 12\theta=2\pi$$

$$\Rightarrow \theta=\frac{\pi}{6}$$

$$\therefore x=\tan\theta=\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$$

15. $\sin\left(\tan^{-1}x\right), |x|<1$ **is equal to:**

(A) $\frac{x}{\sqrt{1-x^2}}$

(B) $\frac{1}{\sqrt{1-x^2}}$

(C) $\frac{1}{\sqrt{1+x^2}}$

(D) $\frac{x}{\sqrt{1+x^2}}$

Ans. Let $\sin\left(\tan^{-1}x\right)=\sin\theta$ where $\theta=\tan^{-1}x$ so that $x=\tan\theta$

$$\Rightarrow \sin\left(\tan^{-1}x\right)=\frac{1}{\operatorname{cosec}\theta}=\frac{1}{\sqrt{1+\cot^2\theta}}$$

Putting $\cot\theta=\frac{1}{\tan\theta}=\frac{1}{x}$

$$\Rightarrow \sin\left(\tan^{-1}x\right)=\frac{1}{\sqrt{1+\frac{1}{x^2}}}=\frac{x}{\sqrt{x^2+1}}$$

Therefore, option (D) is correct.

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16. $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$, **then** x **is equal to:**

(A) $0, \frac{1}{2}$

(B) $1, \frac{1}{2}$

(C) 0

(D) $\frac{1}{2}$

Ans. Putting $\sin^{-1}x=\theta$ so that $x=\sin\theta$

$\therefore\ \sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$

$\Rightarrow\ \sin^{-1}(1-x)-2\theta=\frac{\pi}{2}$

$\Rightarrow\ \sin^{-1}(1-x)=\frac{\pi}{2}+2\theta$

$\Rightarrow\ 1-x=\sin\left(\frac{\pi}{2}+2\theta\right)$

$\Rightarrow 1-x=\cos 2\theta$

$\Rightarrow\ 1-x=1-2\sin^2\theta$

$\Rightarrow\ 1-x=1-2x^2\ [x=\sin\theta]$

$\Rightarrow\ -x=-2x^2$

$\Rightarrow 2x^2-x=0$

$\Rightarrow\ x(2x-1)=0$

$\Rightarrow\ x=0$ or $2x-1=0$

$\Rightarrow\ x=0$ or $x=\frac{1}{2}$

But $x=\frac{1}{2}$ does not satisfy the given equation.

Therefore, option (C) is correct.

17. $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)$ **is equal to:**

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$

(D) $-\frac{3\pi}{4}$

Ans. $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)$

$= \tan^{-1}\left[\frac{\frac{x}{y}-\left(\frac{x-y}{x+y}\right)}{1+\frac{x}{y}\left(\frac{x-y}{x+y}\right)}\right]$

$= \tan^{-1}\left[\frac{x(x+y)-y(x-y)}{y(x+y)+x(x-y)}\right]$

$= \tan^{-1}\left(\frac{x^2+xy-xy+y^2}{xy+y^2+x^2-xy}\right)$

$= \tan^{-1}\left(\frac{x^2+y^2}{x^2+y^2}\right)$

$= \tan^{-1}1$

$= \tan^{-1}\tan\frac{\pi}{4}=\frac{\pi}{4}$

Therefore, option (C) is correct.