CBSE Class–12 Mathematics

NCERT solution

Chapter - 6

Application of Derivatives - Exercise 6.3

1. Find the slope of tangent to the curve $y = 3x^4 - 4x$ at $x = 4$.

Ans. Given: Equation of the curve $y = 3x^4 - 4x$ ....(i)

Slope of the tangent to the curve = Value of $\frac{dy}{dx}$ at the point (x, y)

$\therefore \frac{dy}{dx} = 3(4x^3) - 4 = 12x^3 - 4$

$\therefore$ Slope of the tangent at point $x = 4$ to the curve (i)

= $12(4)^3 - 4$ = 768 – 4 = 764

2. Find the slope of tangent to the curve $y = \frac{x-1}{x-2}, x \neq 2$ at $x = 10$.

Ans. Given: Equation of the curve $y = \frac{x-1}{x-2}$ ....(i)

$$\therefore \frac{dy}{dx} = \frac{(x-2)\frac{d}{dx}(x-1) - (x-1)\frac{d}{dx}(x-2)}{(x-2)^2}$$

$$= \frac{(x-2)-(x-1)}{(x-2)^2}$$

$$= \frac{-1}{(x-2)^2} \text{ ....(ii)}$$

$\therefore$ Slope of the tangent at point $x=10$ to the curve (i)

= $\dfrac{-1}{(10-2)^2}$

= $\dfrac{-1}{8^2}=\dfrac{-1}{64}$

3. Find the slope of tangent to the curve $y=x^3-x+1$ at the given point whose $x-$ coordinate is 2.

Ans. Given: Equation of the curve $y=x^3-x+1$ ....(i)

$\therefore$ $\dfrac{dy}{dx}=3x^2-1$

$\therefore$ Slope of the tangent at point $x=2$ to the curve (i)

= $3(2)^2-1$ = 12 – 1 = 11

4. Find the slope of tangent to the curve $y=x^3-3x+2$ at the given point whose $x-$ coordinate is 3.

Ans. Given: Equation of the curve $y=x^3-3x+2$ ....(i)

$\therefore$ $\dfrac{dy}{dx}=3x^2-3$

$\therefore$ Slope of the tangent at point $x=3$ to the curve (i)

= $3(3)^2-3$ = 27 – 3 = 24

5. Find the slope of the normal to the curve $x=a\cos^3\theta$, $y=a\sin^3\theta$ at $\theta=\dfrac{\pi}{4}$.

Ans. Given: Equations of the curves are $x=a\cos^3\theta$, $y=a\sin^3\theta$

$\therefore \dfrac{dx}{d\theta} = a\dfrac{d}{d\theta}(\cos\theta)^3$ and $\dfrac{dy}{d\theta} = a\dfrac{d}{d\theta}(\sin\theta)^3$

= $a.3(\cos\theta)^2\dfrac{d}{d\theta}(\cos\theta)$ and $a.3(\sin\theta)^2\dfrac{d}{d\theta}(\sin\theta)$

$\Rightarrow \dfrac{dx}{d\theta} = -3a\cos^2\theta\sin\theta$ and $\dfrac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$

$\therefore \dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = \dfrac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta}$

= $\dfrac{-\sin\theta}{\cos\theta} = -\tan\theta$

$\therefore$ Slope of the tangent at $\theta = \dfrac{\pi}{4}$

= $-\tan\dfrac{\pi}{4} = -1$

And Slope of the normal at $\theta = \dfrac{\pi}{4}$

= $\dfrac{-1}{m} = \dfrac{-1}{-1}$ = 1

6. Find the slope of the normal to the curve $x = 1 - a\sin\theta, y = b\cos^2\theta$ at $\theta = \dfrac{\pi}{2}$.

Ans. Given: Equations of the curves are $x = 1 - a\sin\theta$ and $y = b\cos^2\theta$

$\therefore \dfrac{dx}{d\theta} = 0 - a\cos\theta$ and $\dfrac{dy}{d\theta} = b\dfrac{d}{d\theta}(\cos\theta)^2$

$\Rightarrow \dfrac{dx}{d\theta} = -a\cos\theta$ and $\dfrac{dy}{d\theta} = b.2\cos\theta\dfrac{d}{d\theta}\cos\theta = -2b\cos\theta\sin\theta$

$\therefore \quad \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-2b\cos\theta\sin\theta}{-a\cos\theta}$

= $\frac{2b}{a}\sin\theta$

$\therefore$ Slope of the tangent at $\theta = \frac{\pi}{2}$

= $\frac{2b}{a}\sin\frac{\pi}{2} = \frac{2b}{a}$

And Slope of the normal at $\theta = \frac{\pi}{2}$

= $\frac{-1}{m} = \frac{-1}{2b/a}$

= $\frac{-a}{2b}$

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7. Find the point at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to the $x-$ axis.

Ans. Given: Equation of the curve $y = x^3 - 3x^2 - 9x + 7$ ....(i)

$\therefore \quad \frac{dy}{dx} = 3x^2 - 6x - 9$

Since, the tangent is parallel to the $x-$ axis, i.e., $\frac{dy}{dx} = 0$

$\therefore \ 3x^2 - 6x - 9 = 0$

$\Rightarrow \ x^2 - 2x - 3 = 0$

$\Rightarrow \ (x-3)(x+1) = 0$

$\Rightarrow x=3, x=-1$

From eq. (i), when $x=3$,

$y=27-27-27+7=-20$

when $x=-1$, $y=-1-3+9+7=12$

Therefore, the required points are $(3,-20)$ and $(-1,12)$.

8. Find the point on the curve $y=(x-2)^2$ at which the tangent is parallel to the chord joining the points (2, 0) and (4, 4).

Ans. Let the given points are A (2, 0) and B (4, 4).

Slope of the chord AB = $\frac{4-0}{4-2}=2\left[\because m=\frac{y_2-y_1}{x_2-x_1}\right]$

Equation of the curve is $y=(x-2)^2$

$\therefore$ Slope of the tangent at (x,y)

= $\frac{dy}{dx}=2(x-2)$

If the tangent is parallel to the chord AB, then Slope of tangent = Slope of chord

$\Rightarrow 2(x-2)=2$

$\Rightarrow x=3$

$\therefore y=(3-2)^2=1$

Therefore, the required point is (3, 1).

9. Find the point on the curve $y=x^3-11x+5$ at which the tangent is $y=x-11$.

Ans. Given: Equation of the curve $y = x^3 - 11x + 5$ ....(i)

Equation of the tangent $y = x - 11$ ....(ii)

$\Rightarrow \ x - y - 11 = 0$

From eq. (i), $\frac{dy}{dx} = 3x^2 - 11$

= Slope of the tangent at (x, y)

But from eq. (ii), the slope of tangent = $\frac{-a}{b} = \frac{-1}{-1} = 1$

$\therefore \ 3x^2 - 11 = 1$

$\Rightarrow \ x^2 = 4$

$\Rightarrow \ x = \pm 2$

From eq. (i), when $x = 2$, $y = 8 - 22 + 5 = -9$

And when $x = -2$, $y = -8 + 22 + 5 = 19$

Since $(-2, 19)$ does not satisfy eq. (ii), therefore the required point is $(2, -9)$.

10. Find the equation of all lines having slope -1 that are tangents to the curve $y = \frac{1}{x-1}, x \neq 1$.

Ans. Given: Equation of the curve $y = \frac{1}{x-1} = (x-1)^{-1}$ ....(i)

$\therefore \ \frac{dy}{dx} = (-1)(x-1)^{-2} \frac{d}{dx}(x-1)$

= $\frac{-1}{(x-1)^2}$ = Slope of the tangent at (x, y)

But according to question, slope = -1

$\therefore$ $\frac{-1}{(x-1)^2}$ = -1

$\Rightarrow (x-1)^2 = 1$

$\Rightarrow x-1 = \pm 1$

$\Rightarrow x = 1+1 = 2$ or $x = 1-1 = 0$

From eq. (i), when $x = 2$, $y = \frac{1}{2-1} = 1$

And when $x = 0$, $y = \frac{1}{0-1} = -1$

$\therefore$ Points of contact are (2, 1) and $(0, -1)$.

And Equation of two tangents are $y-1 = -1(x-2)$

= $x+y-3 = 0$ and

$y-(-1) = -1(x-0)$ = $x+y+1 = 0$

11. Find the equations of all lines having slope 2 which are tangents to the curve $y = \frac{1}{x-3}, x \neq 3$.

Ans. Given: Equation of the curve $y = \frac{1}{x-3} = (x-3)^{-1}$

$\therefore$ $\frac{dy}{dx} = (-1)(x-3)^{-2}$

= $\frac{-1}{(x-3)^2}$ = Slope of the tangent at (x, y)

But according to question, slope = 2

$\therefore \dfrac{-1}{(x-3)^2} = 2$

$\Rightarrow (x-3)^2 = \dfrac{-1}{2}$ which is not possible.

Hence, there is no tangent to the given curve having slope 2.

12. Find the equations of all lines having slope 0 which are tangents to the curve $y = \dfrac{1}{x^2 - 2x + 3}$.

Ans. Given: Equation of the curve $y = \dfrac{1}{x^2 - 2x + 3}$ ....(i)

$\therefore \dfrac{dy}{dx} = \dfrac{d}{dx}\left[\left(x^2 - 2x + 3\right)^{-1}\right]$

$= -\left(x^2 - 2x + 3\right)^{-2} .\left(2x - 2\right)$

$= \dfrac{-2(x-1)}{\left(x^2 - 2x + 3\right)^2}$

But according to question, slope = 0

$\therefore \dfrac{-2(x-1)}{\left(x^2 - 2x + 3\right)^2} = 0$

$\Rightarrow -2(x-1) = 0$

$\Rightarrow x = 1$

From eq. (i), $y = \dfrac{1}{1-2+3} = \dfrac{1}{2}$

Therefore, the point on the curve which tangent has slope 0 is $\left(1, \frac{1}{2}\right)$.

$\therefore$ Equation of the tangent is $y - \frac{1}{2} = 0(x-1)$

$\Rightarrow y - \frac{1}{2} = 0$

$\Rightarrow y = \frac{1}{2}$

13. Find the points on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are:

(i) parallel to $x-$ axis

(ii) parallel to $y-$ axis

Ans. Given: Equation of the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ ....(i)

$\Rightarrow \frac{2x}{9} + \frac{2y}{16}\frac{dy}{dx} = 0$

$\Rightarrow \frac{2y}{16}\frac{dy}{dx} = -\frac{2x}{9}$

$\Rightarrow 18y\frac{dy}{dx} = -32x$

$\Rightarrow \frac{dy}{dx} = \frac{-32x}{18y} = \frac{-16x}{9y}$ ....(ii)

(i) If tangent is parallel to $x-$ axis, then Slope of tangent = 0

$\therefore \frac{dy}{dx} = 0$

$\Rightarrow \dfrac{-16x}{9y} = 0$

$\Rightarrow x = 0$

$\therefore$ From eq. (i), $\dfrac{y^2}{16} = 1$

$\Rightarrow y^2 = 16$

$\Rightarrow y = \pm 4$

Therefore, the points on curve (i) where tangents are parallel to $x-$ axis are $(0, \pm 4)$.

(ii) If the tangent parallel to $y-$axis.

$\Rightarrow$ Slope of the tangent = $\pm\infty$

$\Rightarrow \dfrac{dy}{dx} = \pm\infty$

$\Rightarrow \frac{dx}{dy} = 0$

$\therefore$ From eq. (ii), $\dfrac{9y}{-16x} = 0$

$\Rightarrow y = 0$

$\therefore$ From eq. (i), $\dfrac{x^2}{9} = 1$

$\Rightarrow x^2 = 9$

$\Rightarrow x = \pm 3$

Therefore, the points on curve (i) where tangents are parallel to $y-$axis are $(\pm 3, 0)$.

14. Find the equations of the tangent and normal to the given curves at the indicated

points:

(i) $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ **at (0, 5)**

(ii) $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ **at (1, 3)**

(iii) $y = x^3$ **at (1, 1)**

(iv) $y = x^2$ **at (0, 0)**

(v) $x = \cos t, y = \sin t$ **at** $t = \frac{\pi}{4}$

Ans. (i) Equation of the curve $y = x^4 - 6x^3 + 13x^2 - 10x + 5$

$\therefore \frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$

slope of tangent = $\frac{dy}{dx}$ at (0, 5)

$= 4(0)^3 - 18(0)^2 + 26(0) - 10 = -10 = m$ (say)

$\therefore$ Slope of the normal at (0, 5) is $\frac{-1}{m} = \frac{-1}{-10} = \frac{1}{10}$

$\therefore$ Equation of the tangent at (0, 5) is $y - 5 = -10(x - 0)$

$\Rightarrow y - 5 = -10x$

$\Rightarrow 10x + y = 5$

And Equation of the normal at (0, 5) is $y - 5 = \frac{1}{10}(x - 0)$

$\Rightarrow 10y - 50 = x$

$\Rightarrow x - 10y + 50 = 0$

(ii) Equation of the curve $y=x^4-6x^3+13x^2-10x+5$

$\therefore \dfrac{dy}{dx}=4x^3-18x^2+26x-10$

slope of tangent = $\dfrac{dy}{dx}$ at (1, 3)

$=4(1)^3-18(1)^2+26(1)-10=4-18+26-10=2=m$ (say)

$\therefore$ Slope of the normal at (1, 3) is $\dfrac{-1}{m}=\dfrac{-1}{2}$

$\therefore$ Equation of the tangent at (1, 3) is $y-3=2(x-1)$

$\Rightarrow y-3=2x-2$

$\Rightarrow y=2x+1$

And Equation of the normal at (1, 3) is $y-3=\dfrac{-1}{2}(x-1)$

$\Rightarrow 2y-6=-x+1$

$\Rightarrow x+2y-7=0$

(iii) Equation of the curve $y=x^3$ ....(i)

$\therefore \dfrac{dy}{dx}=3x^2$

slope of tangent = $\dfrac{dy}{dx}$ at (1, 1)

= $3(1)^2=3$ = m (say)

$\therefore$ Slope of the normal at (1, 1) is $\frac{-1}{m} = \frac{-1}{3}$

$\therefore$ Equation of the tangent at (1, 1) is $y-1=3(x-1)$

$\Rightarrow \ y-1=3x-3$

$\Rightarrow \ y=3x-2$

And Equation of the normal at (1, 1) is $y-1=\frac{-1}{3}(x-1)$

$\Rightarrow 3y-3=-x+1$

$\Rightarrow x+3y-4=0$

(iv) Equation of the curve $y=x^2$ ....(i)

$\therefore \ \frac{dy}{dx}=2x$

slope of tangent = $\frac{dy}{dx}$ at (0, 0)

= $2\times 0=0$ = m (say)

$\therefore$ Equation of the tangent at (0, 0) is $y-0=0(x-0)$

$\Rightarrow \ y=0$ which is equaion of x-axis

thus normal at (0, 0) is $y-$axis.

$\therefore$ equation of normal is

x=0

(v) Equation of the curves are $x=\cos t, y=\sin t$

$\therefore \ \frac{dx}{dt}=-\sin t$ and $\frac{dy}{dt}=\cos t$

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos t}{-\sin t} = -\cot t$

Slope of the tangent at $t = \frac{\pi}{4}$ = $-\cot\frac{\pi}{4} = -1 = m$ (say)

$\therefore$ Slope of the normal at $t = \frac{\pi}{4}$ is $\frac{-1}{m} = \frac{-1}{-1} = 1$

$\therefore$ Point (x, y) = $(\cos t, \sin t)$

= $\left(\cos\frac{\pi}{4}, \sin\frac{\pi}{4}\right)$

= $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\therefore$ Equation of the tangent is $y - \frac{1}{\sqrt{2}} = -1\left(x - \frac{1}{\sqrt{2}}\right)$

$\Rightarrow x + y = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$\Rightarrow x + y = \sqrt{2}$

And Equation of the normal is $y - \frac{1}{\sqrt{2}} = 1\left(x - \frac{1}{\sqrt{2}}\right)$

$\Rightarrow y - \frac{1}{\sqrt{2}} = x - \frac{1}{\sqrt{2}}$

$\Rightarrow y = x$

15. Find the equation of the tangent line to curve $y = x^2 - 2x + 7$ which is:

(a) parallel to the line $2x - y + 9 = 0$

(b) perpendicular to the line $5y - 15x = 13$

Ans. Given: Equation of the curve $y = x^2 - 2x + 7$ ....(i)

$\therefore$ Slope of tangent = $\frac{dy}{dx} = 2x - 2$ ....(ii)

(a) Slope of the line $2x - y + 9 = 0$ is $\frac{-a}{b} = \frac{-2}{-1} = 2$

$\therefore$ Slope of tangent parallel to this line is also = 2

$\therefore$ From eq. (ii), $2x - 2 = 2$

$\Rightarrow x = 2$

$\therefore$ From eq. (i), $y = 4 - 4 + 7 = 7$

Therefore, point of contact is (2, 7).

$\therefore$ Equation of the tangent at (2, 7) is $y - 7 = 2(x - 2)$

$\Rightarrow y - 7 = 2x - 4$

$\Rightarrow y - 2x - 3 = 0$

(b) Slope of the line $-15x + 5y = 13$ is $\frac{-a}{b} = \frac{-(-15)}{5} = 3$ = m

$\therefore$ Slope of the required tangent perpendicular to this line = $\frac{-1}{m} = \frac{-1}{3}$

$\therefore$ From eq. (ii), $2x - 2 = \frac{-1}{3}$

$\Rightarrow 6x - 6 = -1$

$\Rightarrow x = \frac{5}{6}$

$\therefore$ From eq. (i), $y = \frac{25}{36} - \frac{5}{3} + 7$

= $\frac{25 - 60 + 252}{36}$ = $\frac{217}{36}$

Therefore, point of contact is $\left(\frac{5}{6}, \frac{217}{36}\right)$.

$\therefore$ Equation of the required tangent is $y - \frac{217}{36} = \frac{-1}{3}\left(x - \frac{5}{6}\right)$

$\Rightarrow 3y - \frac{217}{12} = -x + \frac{5}{6}$

$\Rightarrow x + 3y = \frac{217}{12} + \frac{5}{6}$

$\Rightarrow x + 3y = \frac{217 + 10}{12} = \frac{227}{12}$

$\Rightarrow 12x + 36y = 227$

16. Show that the tangents to the curve $y = 7x^3 + 11$ at the points where $x = 2$ and $x = -2$ are parallel.

Ans. Given: Equation of the curve $y = 7x^3 + 11$

$\therefore$ Slope of tangent at (x, y) = $\frac{dy}{dx} = 21x^2$

At the point $x = 2$, Slope of the tangent = $21(2)^2 = 21 \times 4 = 84$

At the point $x = -2$, Slope of the tangent = $21(-2)^2 = 21 \times 4 = 84$

Since, the slopes of the two tangents are equal.

Therefore, tangents at $x=2$ and $x=-2$ are parallel.

17. Find the points on the curve $y=x^3$ at which the slope of the tangent is equal to the $y-$coordinate of the point.

Ans. Given: Equation of the curve $y=x^3$ ....(i)

$\therefore$ Slope of tangent at (x, y)

= $\frac{dy}{dx}=3x^2$ ....(ii)

According to question, Slope of the tangent = $y-$coordinate of the point

$\therefore\ 3x^2=x^3$

$\Rightarrow\ 3x^2-x^3=0$

$\Rightarrow\ x^2(3-x)=0$

$\Rightarrow\ x^2=0$ or $3-x=0$

$\Rightarrow\ x=0$ or $x=3$

$\therefore$ From eq. (i), at $x=0$, $y=0$ The point is (0, 0).

And From eq. (i), at $x=3$, $y=27$ The point is (3, 27).

Therefore, the required points are (0, 0) and (3, 27).

18. For the curve $y=4x^3-2x^5$, find all points at which the tangent passes through the origin.

Ans. Given: Equation of the curve $y=4x^3-2x^5$ ....(i)

$\therefore$ Slope of the tangent at (x, y) passing through origin (0, 0)

= $\frac{dy}{dx} = 12x^2 - 10x^4$

= $\frac{y-0}{x-0}$

$\Rightarrow$ $\frac{y}{x} = 12x^2 - 10x^4$

$\Rightarrow$ $y = 12x^3 - 10x^5$

Substituting this value of y in eq. (i), we get,

$12x^3 - 10x^5 = 4x^3 - 2x^5$

$\Rightarrow$ $8x^3 - 8x^5 = 0$

$\Rightarrow$ $8x^3\left(1 - x^2\right) = 0$

$\Rightarrow$ $8x^3 = 0$ or $1 - x^2 = 0$

$\Rightarrow$ $x = 0$ or $x = \pm 1$

$\therefore$ From eq. (i), at $x = 0$, $y = 0$

From eq. (i), at $x = 1$, $y = 4 - 2 = 2$

From eq. (i), at $x = -1$, $y = -4 + 2 = -2$

Therefore, the required points are (0, 0), (1, 2) and $(-1, -2)$.

19. Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangents are parallel to $x-$ axis.

Ans. Equation of the curve $x^2 + y^2 - 2x - 3 = 0$ ....(i)

$\therefore$ $2x + 2y\frac{dy}{dx} - 2 = 0$

$\Rightarrow 2y\frac{dy}{dx}=2-2x$

$\Rightarrow \frac{dy}{dx}=\frac{2(1-x)}{2y}=\frac{1-x}{y}$

$\because \frac{dy}{dx}=0$ [tangent is parallel to $x-$ axis]

$\therefore \frac{1-x}{y}=0$

$\Rightarrow x=1$

$\therefore$ From eq. (i), $1+y^2-2-3=0$

$\Rightarrow y^2=4$

$\Rightarrow y=\pm 2$

Therefore, the required points are (1, 2) and $(1,-2)$.

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20. Find the equation of the normal at the point (am^2, am^3) for the curve $ay^2=x^3$.

Ans. Given: Equation of the curve $ay^2=x^3$ ....(i)

$\therefore a\frac{d}{dx}y^2=\frac{d}{dx}x^3$

$\Rightarrow a.2y\frac{dy}{dx}=3x^2$

$\Rightarrow \frac{dy}{dx}=\frac{3x^2}{2ay}$

$\therefore$ Slope of the tangent at the point (am^2, am^3)

= $\frac{3\left(am^2\right)^2}{2a.am^3}$ = $\frac{3m}{2}$

$\therefore$ Slope of the normal at the point $\left(am^2, am^3\right)$ = $\frac{-2}{3m}$

Equation of the normal at $\left(am^2, am^3\right)$

= $y - am^3 = \frac{-2}{3m}\left(x - am^2\right)$

$\Rightarrow 3my - 3am^4 = -2x + 2am^2$

$\Rightarrow 2x + 3my - 2am^2 - 3am^4 = 0$

$\Rightarrow 2x + 3my - am^2\left(2 + 3m^2\right) = 0$

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21. Find the equations of the normal to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$.

Ans. Given: Equation of the curve $y = x^3 + 2x + 6$ ....(i)

$\therefore$ Slope of the tangent at (x, y)

= $\frac{dy}{dx} = 3x^2 + 2$

$\therefore$ Slope of the normal to the curve at (x, y)

= $\frac{-1}{3x^2 + 2}$ ....(ii)

But Slope of the normal (given) = $\frac{-1}{14}$

$\therefore \dfrac{-1}{3x^2+2} = \dfrac{-1}{14}$

$\Rightarrow 3x^2+2=14$

$\Rightarrow 3x^2=12$

$\Rightarrow x^2=4$

$\Rightarrow x=\pm 2$

$\therefore$ From eq. (i), at $x=2$, $y=8+4+6=18$

at $x=-2$, $y=-8-4+6=-6$

Therefore, the points of contact are (2, 18) and $(-2,-6)$.

$\therefore$ Equation of the normal at (2, 18) is $y-18=\dfrac{-1}{14}(x-2)$

$\Rightarrow 14y-252=-x+2$

$\Rightarrow x+14y-254=0$

And Equation of the normal at $(-2,-6)$ is $y+6=\dfrac{-1}{14}(x+2)$

$\Rightarrow 14y+84=-x-2$

$\Rightarrow x+14y+86=0$

22. Find the equation of the tangent and normal to the parabola $y^2=4ax$ at the point $(at^2, 2at)$.

Ans. Given: Equation of the parabola $y^2=4ax$ ....(i)

Differentiating eq. (i) w.r.t x

$\Rightarrow 2y\frac{dy}{dx} = 4a$

$\Rightarrow \frac{dy}{dx} = \frac{4a}{2y} = \frac{2a}{y}$

$\therefore$ Slope of the tangent at the point $(at^2, 2at)$ = $\frac{2a}{2at} = \frac{1}{t}$

$\therefore$ Slope of the normal = $-t$

$\therefore$ Equation of the tangent at the point $(at^2, 2at)$

= $y - 2at = \frac{1}{t}(x - at^2)$

$\Rightarrow ty - 2at^2 = x - at^2$

$\Rightarrow ty = x + at^2$

And Equation of the normal at the point $(at^2, 2at)$

= $y - 2at = -t(x - at^2)$

$\Rightarrow tx + y = 2at + at^3$

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23. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$.

Ans. Given: Equations of the curves are $x = y^2$ .....(i) and $xy = k$ ....(ii)

Substituting the value of x in eq. (ii), we get $y^2.y = k$

$\Rightarrow y = k^{1/3}$

Putting the value of y in eq. (i), we get $x = \left(k^{1/3}\right)^2 = k^{2/3}$

Therefore, the point of intersection (x, y) is = $\left(k^{2/3}, k^{1/3}\right)$……….(iii)

Differentiating eq. (i) w.r.t x $1 = 2y\dfrac{dy}{dx}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2y} = m_1$ ……….(iv)

Differentiating eq. (ii) w.r.t x $x\dfrac{dy}{dx} + y = 0$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{-y}{x} = m_2$……….(v)

According to the question, $m_1 m_2 = -1$

$\Rightarrow \dfrac{1}{2y}\left(\dfrac{-y}{x}\right) = -1$

$\Rightarrow \dfrac{1}{2x} = 1$

$\Rightarrow 2x = 1$

$\Rightarrow 2k^{2/3} = 1$ [From eq. (iii)]

$\Rightarrow 8k^2 = 1$ [Cubing both sides]

24. Find the equations of the tangent and normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_0, y_0).

Ans. Given: Equation of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$……….(i)

$\therefore \dfrac{2x}{a^2} - \dfrac{2y}{b^2}.\dfrac{dy}{dx} = 0$

$\Rightarrow \frac{-2y}{b^2} . \frac{dy}{dx} = \frac{-2x}{a^2}$

$\Rightarrow \frac{dy}{dx} = \frac{b^2 x}{a^2 y}$ ....(ii)

$\therefore$ Slope of tangent at (x_0, y_0) is $\frac{b^2 x_0}{a^2 y_0}$

$\therefore$ Equation of the tangent at (x_0, y_0) is $y - y_0 = \frac{b^2 x_0}{a^2 y_0}(x - x_0)$

$\Rightarrow yy_0 - y_0^2 = \frac{b^2}{a^2}(xx_0 - x_0^2)$

$\Rightarrow \frac{yy_0}{b^2} - \frac{y_0^2}{b^2} = \frac{xx_0}{a^2} - \frac{x_0^2}{a^2}$

$\Rightarrow \frac{xx_0}{a^2} - \frac{yy_0}{b^2} = \frac{x_0^2}{a^2} - \frac{y_0^2}{b^2}$ ....(iii)

Since (x_0, y_0) lies on the hyperbola (i), therefore, $\frac{x_0^2}{a^2} - \frac{y_0^2}{b^2} = 1$

$\therefore$ From eq. (iii), $\frac{xx_0}{a^2} - \frac{yy_0}{b^2} = 1$

Now, Slope of normal at (x_0, y_0) = $\frac{-a^2 y_0}{b^2 x_0}$

$\therefore$ Equation of the normal at (x_0, y_0) is $y - y_0 = \frac{-a^2 y_0}{b^2 x_0}(x - x_0)$

$\Rightarrow b^2 x_0 y - b^2 x_0 y_0 = -a^2 y_0 x + a^2 x_0 y_0$

$\Rightarrow b^2 x_0 (y - y_0) = -a^2 y_0 (x - x_0)$

Dividing both sides by $a^2b^2x_0y_0$

$$\frac{y-y_0}{a^2y_0}=-\frac{(x-x_0)}{b^2x_0}$$

$$\Rightarrow \frac{(x-x_0)}{b^2x_0}+\frac{y-y_0}{a^2y_0}=0$$

25. Find the equation of the tangent to the curve $y=\sqrt{3x-2}$ which is parallel to the line $4x-2y+5=0$.

Ans. Given: Equation of the curve $y=\sqrt{3x-2}$ ....(i)

$\therefore$ Slope of the tangent at point (x,y) is $\frac{dy}{dx}=\frac{d}{dx}(3x-2)^{\frac{1}{2}}$

$\Rightarrow \frac{dy}{dx}=\frac{1}{2}(3x-2)^{-1/2}\frac{d(3x-2)}{dx}$

$=\frac{1}{2\sqrt{3x-2}}.3$..........(ii)

Again slope of the line $4x-2y+5=0$ is $\frac{-a}{b}=\frac{-4}{-2}=2$..........(iii)

According to the question, $\frac{3}{2\sqrt{3x-2}}=2$ [Parallel lines have same slope]

$\Rightarrow 4\sqrt{3x-2}=3$

$\Rightarrow 16(3x-2)=9$

$\Rightarrow 48x-32=9$

$\Rightarrow 48x=32+9$

$\Rightarrow 48x=41$

$\Rightarrow x = \frac{41}{48}$

$\therefore$ From eq. (i), $y = \sqrt{3\left(\frac{41}{48}\right) - 2}$

= $\sqrt{\frac{41}{16} - 2}$

= $\sqrt{\frac{41-32}{16}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$

Therefore, point of contact is $\left(\frac{41}{48}, \frac{3}{4}\right)$.

$\therefore$ Equation of the required tangent is $y - \frac{3}{4} = 2\left(x - \frac{41}{48}\right)$

$\Rightarrow y - \frac{3}{4} = 2x - \frac{41}{24}$

$\Rightarrow y = 2x + \frac{3}{4} - \frac{41}{24}$

$\Rightarrow y = 2x + \frac{18-41}{24}$

$\Rightarrow 24y = 48x - 23$

$\Rightarrow 48x - 24y = 23$

Choose the correct answer in Exercises 26 and 27.

26. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is:

(A) 3

(B) $\frac{1}{3}$

(C) -3

(D) $\frac{-1}{3}$

Ans. Given: Equation of the curve $y = 2x^2 + 3\sin x$ ....(i)

$\therefore$ Slope of the tangent at point (x, y) is $\frac{dy}{dx} = 4x + 3\cos x$

$\therefore$ Slope of the tangent at $x = 0$, $4(0) + 3\cos 0 = 3 = m$ (say)

$\therefore$ Slope of the normal = $\frac{-1}{m} = \frac{-1}{3}$

Therefore, option (D) if correct.

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27. The line $y = x + 1$ is a tangent to the curve $y^2 = 4x$ at the point:

(A) (1, 2)

(B) (2, 1)

(C) $(1, -2)$

(D) $(-1, 2)$

Ans. Given: Equation of the curve $y^2 = 4x$ ....(i)

$\therefore$ Slope of the tangent at point (x, y) is $2y\frac{dy}{dx} = 4$

$\Rightarrow \frac{dy}{dx} = \frac{2}{y}$ ....(ii)

$\therefore$ Slope of the line $y = x + 1$

$\Rightarrow x-y+1=0$ is $\frac{-a}{b}=\frac{-1}{-1}=1$..........(iii)

From eq. (ii) and (iii), $\frac{2}{y}=1$

$\Rightarrow y=2$

$\therefore$ From eq. (i), $4=4x$

$\Rightarrow x=1$

Therefore, required point is (1, 2).

Therefore, option (A) is correct.