

CBSE Class-12 Mathematics

NCERT solution

Chapter - 6

Application of Derivatives - Exercise 6.4

1. Using differentials, find the approximate value of each of the following up to 3 places of decimal:

(i) $\sqrt{25.3}$

(ii) $\sqrt{49.5}$

(iii) $\sqrt{0.6}$

(iv) $(0.009)^{\frac{1}{3}}$

(v) $(0.999)^{\frac{1}{10}}$

(vi) $(15)^{\frac{1}{4}}$

(vii) $(26)^{\frac{1}{3}}$

(viii) $(255)^{\frac{1}{4}}$

(ix) $(82)^{\frac{1}{4}}$

(x) $(401)^{\frac{1}{2}}$

(xi) $(0.0037)^{\frac{1}{2}}$

(xii) $(26.57)^{\frac{1}{3}}$

(xiii) $(81.5)^{\frac{1}{4}}$

(xiv) $(3.968)^{\frac{3}{4}}$

(xv) $(32.15)^{\frac{1}{5}}$

Ans. (i) $\sqrt{25.3}$

Let $y = \sqrt{x}$ (i)

$$\therefore \frac{dy}{dx} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$\Rightarrow dy = \frac{dx}{2\sqrt{x}} \text{(ii)}$$

Now, from eq. (i), $y + \Delta y = \sqrt{x + \Delta x}$

$$= \sqrt{25.3} = \sqrt{25 + 0.3}$$

Here, $x = 25$ and $\Delta x = 0.3$, then $\Delta y = \sqrt{x + \Delta x} - \sqrt{x}$

$$= \sqrt{25.3} - \sqrt{25} = \sqrt{25.3} - 5$$

$$\Rightarrow \sqrt{25.3} = \Delta y + 5$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

\therefore From eq. (ii),

$$dy = \frac{0.3}{2\sqrt{25}} = 0.03$$

Therefore, approximately value of $\sqrt{25.3}$ is $5 + 0.03 = 5.03$.

(ii) $\sqrt{49.5}$

Let $y = \sqrt{x}$ (i)

$$\therefore \frac{dy}{dx} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$\Rightarrow dy = \frac{dx}{2\sqrt{x}} \text{(ii)}$$

Now, from eq. (i), $y + \Delta y = \sqrt{x + \Delta x}$

$$= \sqrt{49.5} = \sqrt{49 + 0.5}$$

Here, $x = 49$ and $\Delta x = 0.5$, then

$$\Delta y = \sqrt{x + \Delta x} - \sqrt{x}$$

$$= \sqrt{49.5} - \sqrt{49} = \sqrt{49.5} - 7$$

$$\Rightarrow \sqrt{49.5} = \Delta y + 7$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

$$\therefore \text{From eq. (ii), } dy = \frac{0.5}{2\sqrt{49}}$$

$$= 0.0357$$

Therefore, approximately value of $\sqrt{49.5}$ is $7 + 0.0357 = 7.0357$.

(iii) $\sqrt{0.6}$

Let $y = \sqrt{x}$ (i)

$$\therefore \frac{dy}{dx} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$\Rightarrow dy = \frac{dx}{2\sqrt{x}} \text{(ii)}$$

Now, from eq. (i), $y + \Delta y = \sqrt{x + \Delta x}$

$$= \sqrt{0.6} = \sqrt{0.64 - 0.04}$$

Here, $x = 0.64$ and $\Delta x = -0.04$, then

$$\Delta y = \sqrt{x + \Delta x} - \sqrt{x}$$

$$= \sqrt{0.6} - \sqrt{0.64} = \sqrt{0.6} - 0.8$$

$$\Rightarrow \sqrt{0.6} = \Delta y + 0.8$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

$$\therefore \text{From eq. (ii), } dy = \frac{-0.04}{2\sqrt{0.64}} = -0.025$$

Therefore, approximately value of $\sqrt{0.6}$ is $0.8 - 0.025 = 0.775$.

(iv) $(0.009)^{\frac{1}{3}}$

Let $y = \sqrt[3]{x}$ (i)

$$\therefore \frac{dy}{dx} = \frac{1}{3} x^{-\frac{2}{3}} = \frac{1}{3x^{\frac{2}{3}}}$$

$$\Rightarrow dy = \frac{dx}{3x^{\frac{2}{3}}} = \frac{dx}{3\left(x^{\frac{1}{3}}\right)^2} \text{(ii)}$$

Now, from eq. (i), $y + \Delta y = \sqrt[3]{x + \Delta x}$

$$= (0.009)^{\frac{1}{3}} = (0.008 + 0.001)^{\frac{1}{3}}$$

Here, $x = 0.008$ and $\Delta x = 0.001$,

$$\text{then } \Delta y = \sqrt[3]{x + \Delta x} - \sqrt[3]{x}$$

$$= (0.009)^{\frac{1}{3}} - (0.008)^{\frac{1}{3}} = (0.009)^{\frac{1}{3}} - 0.2$$

$$\Rightarrow (0.009)^{\frac{1}{3}} = \Delta y + 0.2$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

$$\therefore \text{From eq. (ii), } dy = \frac{0.001}{3 \left((0.008)^{\frac{1}{3}} \right)^2}$$

$$= \frac{0.001}{3 \times 0.04} = 0.0083$$

Therefore, approximately value of $(0.009)^{\frac{1}{3}}$ is $0.2 + 0.0083 = 0.2083$.

(v) $(0.999)^{\frac{1}{10}}$

Let $y = x^{\frac{1}{10}}$ (i)

$$\therefore \frac{dy}{dx} = \frac{1}{10} x^{\frac{-9}{10}} = \frac{1}{10x^{\frac{9}{10}}}$$

$$\Rightarrow dy = \frac{dx}{10 \left(x^{\frac{1}{10}} \right)^9} \text{(ii)}$$

Now, from eq. (i), $y + \Delta y = \sqrt[10]{x + \Delta x}$

$$= (0.999)^{\frac{1}{10}} = (1 - 0.001)^{\frac{1}{10}} \text{(iii)}$$

Here $x = 1$ and $\Delta x = -0.001$

Then $\Delta y = (x + \Delta x)^{\frac{1}{10}} - x^{\frac{1}{10}}$

$$= (0.999)^{\frac{1}{10}} - 1$$

$$\Rightarrow (0.999)^{\frac{1}{10}} = 1 + \Delta y$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

\therefore From eq. (ii),

$$dy = \frac{-0.001}{10 \left(1^{\frac{1}{10}} \right)^9} = -0.0001$$

Therefore, approximate value of $(0.999)^{\frac{1}{10}}$ is $1 - 0.0001 = 0.9999$.

(vi) $(15)^{\frac{1}{4}}$

Let $y = x^{\frac{1}{4}}$ (i)

$$\therefore \frac{dy}{dx} = \frac{1}{4} x^{\frac{-3}{4}} = \frac{1}{4x^{\frac{3}{4}}}$$

$$\Rightarrow dy = \frac{dx}{4 \left(x^{\frac{1}{4}} \right)^3} \text{(ii)}$$

Now, from eq. (i), $y + \Delta y = \sqrt[4]{x + \Delta x}$

$$= (15)^{\frac{1}{4}} = (16 - 1)^{\frac{1}{4}} \text{(iii)}$$

Here $x = 16$ and $\Delta x = -1$

Then $\Delta y = (x + \Delta x)^{\frac{1}{4}} - x^{\frac{1}{4}}$

$$= (15)^{\frac{1}{4}} - 16^{\frac{1}{4}} = (15)^{\frac{1}{4}} - 2$$

$$\Rightarrow (15)^{\frac{1}{4}} = 2 + \Delta y$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

$$\therefore \text{From eq. (ii), } dy = \frac{-1}{4 \left(16^{\frac{1}{4}} \right)^3} = \frac{-1}{32}$$

Therefore, approximate value of $(15)^{\frac{1}{4}}$ is $2 - \frac{1}{32} = \frac{63}{32} = 1.96875$.

(vii) $(26)^{\frac{1}{3}}$

Let $y = \sqrt[3]{x}$ (i)

$$\therefore \frac{dy}{dx} = \frac{1}{3} x^{\frac{-2}{3}} = \frac{1}{3x^{\frac{2}{3}}}$$

$$\Rightarrow dy = \frac{dx}{3x^{\frac{2}{3}}} = \frac{dx}{3 \left(x^{\frac{1}{3}} \right)^2} \text{(ii)}$$

Now, from eq. (i), $y + \Delta y = \sqrt{x + \Delta x}$

$$= (26)^{\frac{1}{3}} = (27 - 1)^{\frac{1}{3}}$$

Here, $x = 27$ and $\Delta x = -1$,

$$\text{then } \Delta y = \sqrt{x + \Delta x} - \sqrt{x} = (26)^{\frac{1}{3}} - (27)^{\frac{1}{3}} = (26)^{\frac{1}{3}} - 3$$

$$\Rightarrow (26)^{\frac{1}{3}} = \Delta y + 3$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

$$\therefore \text{From eq. (ii), } dy = \frac{-1}{3\left((27)^{\frac{1}{3}}\right)^2} = \frac{-1}{27}$$

Therefore, approximately value of $(26)^{\frac{1}{3}}$ is $3 - \frac{1}{27} = \frac{80}{27} = 2.9629$.

(viii) $(255)^{\frac{1}{4}}$

$$\text{Let } y = x^{\frac{1}{4}} \dots\dots\dots\text{(i)}$$

$$\therefore \frac{dy}{dx} = \frac{1}{4} x^{\frac{-3}{4}} = \frac{1}{4x^{\frac{3}{4}}}$$

$$\Rightarrow dy = \frac{dx}{4\left(x^{\frac{1}{4}}\right)^3} \dots\dots\dots\text{(ii)}$$

Now, from eq. (i), $y + \Delta y = \sqrt[4]{x + \Delta x}$

$$= (255)^{1/4} = (256 - 1)^{\frac{1}{4}} \dots\dots\dots\text{(iii)}$$

Here $x = 256$ and $\Delta x = -1$

$$\text{Then } \Delta y = (x + \Delta x)^{\frac{1}{4}} - x^{\frac{1}{4}}$$

$$= (255)^{\frac{1}{4}} - (256)^{\frac{1}{4}} = (255)^{\frac{1}{4}} - 4$$

$$\Rightarrow (255)^{\frac{1}{4}} = 4 + \Delta y$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

∴ From eq. (ii),
$$dy = \frac{-1}{4 \left(256^{\frac{1}{4}} \right)^3} = \frac{-1}{256}$$

Therefore, approximate value of $(255)^{\frac{1}{4}}$ is $4 - \frac{1}{256} = \frac{1023}{256} = 3.9961$.

(ix) $(82)^{\frac{1}{4}}$

Let $y = x^{\frac{1}{4}}$ (i)

∴ $\frac{dy}{dx} = \frac{1}{4} x^{\frac{-3}{4}}$

$= \frac{1}{4x^{\frac{3}{4}}}$

$\Rightarrow dy = \frac{dx}{4 \left(x^{\frac{1}{4}} \right)^3}$ (ii)

Now, from eq. (i), $y + \Delta y = \sqrt[4]{x + \Delta x} = (82)^{\frac{1}{4}} = (81 + 1)^{\frac{1}{4}}$ (iii)

Here $x = 81$ and $\Delta x = 1$

Then $\Delta y = (x + \Delta x)^{\frac{1}{4}} - x^{\frac{1}{4}}$

$= (82)^{\frac{1}{4}} - (81)^{\frac{1}{4}} = (82)^{\frac{1}{4}} - 3$

$\Rightarrow (82)^{\frac{1}{4}} = 3 + \Delta y$

Since, Δx and Δy is approximately equal to dx and dy respectively.

∴ From eq. (ii),
$$dy = \frac{1}{4 \left(81^{\frac{1}{4}} \right)^3} = \frac{1}{108}$$

Therefore, approximate value of $(82)^{\frac{1}{4}}$ is $3 + \frac{1}{108} = \frac{325}{108} = 3.0092$.

(x) $\sqrt{401}$

Let $y = \sqrt{x}$ (i)

∴ $\frac{dy}{dx} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$\Rightarrow dy = \frac{dx}{2\sqrt{x}}$ (ii)

Now, from eq. (i), $y + \Delta y = \sqrt{x + \Delta x}$

$= \sqrt{401} = \sqrt{400 + 1}$

Here, $x = 400$ and $\Delta x = 1$, then $\Delta y = \sqrt{x + \Delta x} - \sqrt{x}$

$= \sqrt{401} - \sqrt{400} = \sqrt{401} - 20$

$\Rightarrow \sqrt{401} = \Delta y + 20$

Since, Δx and Δy is approximately equal to dx and dy respectively.

∴ From eq. (ii), $dy = \frac{1}{2\sqrt{400}} = \frac{1}{40}$

Therefore, approximately value of $\sqrt{401}$ is $20 + \frac{1}{40} = \frac{801}{40} = 20.025$.

(xi) $\sqrt{0.0037}$

Let $y = \sqrt{x}$ (i)

$$\therefore \frac{dy}{dx} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$\Rightarrow dy = \frac{dx}{2\sqrt{x}} \text{(ii)}$$

Now, from eq. (i), $y + \Delta y = \sqrt{x + \Delta x}$

$$= \sqrt{0.0037} = \sqrt{0.0036 + 0.0001}$$

Here, $x = 0.0036$ and $\Delta x = 0.0001$, then

$$\Delta y = \sqrt{x + \Delta x} - \sqrt{x}$$

$$= \sqrt{0.0037} - \sqrt{0.0036} = \sqrt{0.0037} - 0.06$$

$$\Rightarrow \sqrt{0.0037} = \Delta y + 0.06$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

$$\therefore \text{From eq. (ii), } dy = \frac{0.0001}{2\sqrt{0.0036}}$$

$$= \frac{0.0001}{0.12}$$

Therefore, approximately value of $\sqrt{0.0037}$ is $0.06 + \frac{0.0001}{0.12} = 0.060833$.

$$\text{(xi) } (26.57)^{\frac{1}{3}}$$

Let $y = \sqrt[3]{x}$ (i)

$$\therefore \frac{dy}{dx} = \frac{1}{3} x^{-\frac{2}{3}} = \frac{1}{3x^{\frac{2}{3}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{dx}{3x^{\frac{2}{3}}} = \frac{dx}{3\left(x^{\frac{1}{3}}\right)^2} \dots\dots\dots(ii)$$

Now, from eq. (i), $y + \Delta y = \sqrt{x + \Delta x}$

$$= (26.57)^{\frac{1}{3}} = (27 - 0.43)^{\frac{1}{3}}$$

Here, $x = 27$ and $\Delta x = -0.43$,

$$\text{then } \Delta y = \sqrt{x + \Delta x} - \sqrt{x}$$

$$= (26.57)^{\frac{1}{3}} - (27)^{\frac{1}{3}} = (26.57)^{\frac{1}{3}} - 3$$

$$\Rightarrow (26.57)^{\frac{1}{3}} = \Delta y + 3$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

$$\therefore \text{From eq. (ii), } dy = \frac{-0.43}{3\left((27)^{\frac{1}{3}}\right)^2}$$

$$= \frac{-0.43}{27} = 0.0159$$

Therefore, approximately value of $(26.57)^{\frac{1}{3}}$ is $3 - 0.0159 = 2.9841$.

(xii) $(81.5)^{\frac{1}{4}}$

$$\text{Let } y = x^{\frac{1}{4}} \dots\dots\dots(i)$$

$$\therefore \frac{dy}{dx} = \frac{1}{4} x^{\frac{-3}{4}} = \frac{1}{4x^{\frac{3}{4}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{dx}{4 \left(x^{\frac{1}{4}} \right)^3} \dots\dots\dots(ii)$$

Now, from eq. (i), $y + \Delta y = \sqrt{x + \Delta x}$

$$= (81.5)^{\frac{1}{4}} = (81 + 0.5)^{\frac{1}{4}} \dots\dots\dots(iii)$$

Here $x = 81$ and $\Delta x = 0.5$

$$\text{Then } \Delta y = (x + \Delta x)^{\frac{1}{4}} - x^{\frac{1}{4}}$$

$$= (81.5)^{\frac{1}{4}} - (81)^{\frac{1}{4}} = (81.5)^{\frac{1}{4}} - 3$$

$$\Rightarrow (81.5)^{\frac{1}{4}} = 3 + \Delta y$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

$$\therefore \text{ From eq. (ii), } \frac{dy}{dx} = \frac{0.5}{4 \left(81^{\frac{1}{4}} \right)^3} = \frac{0.5}{108}$$

$$= 0.00462$$

Therefore, approximate value of $(82)^{\frac{1}{4}}$ is $3 + 0.00462 = 3.00462$.

$$\text{(xiv) } (3.968)^{\frac{3}{2}}$$

$$\text{Let } y = x^{\frac{3}{2}} = x^{1+\frac{1}{2}} = x\sqrt{x} \dots\dots\dots(i)$$

$$\therefore \frac{dy}{dx} = \frac{3}{2} x^{\frac{1}{2}}$$

$$\Rightarrow dy = \frac{3\sqrt{x}}{2} dx \dots\dots\dots(ii)$$

Now, from eq. (i), $y + \Delta y = \sqrt{x + \Delta x}$

$$= (3.968)^{\frac{3}{2}} = (4 - 0.032)^{\frac{3}{2}}$$

Here, $x = 4$ and $\Delta x = 0.032$, then $\Delta y = \sqrt{x + \Delta x} - \sqrt{x}$

$$= (3.968)^{\frac{3}{2}} - (4)^{\frac{3}{2}} = (3.968)^{\frac{3}{2}} - 8$$

$$\Rightarrow (3.968)^{\frac{3}{2}} = \Delta y + 8$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

\therefore From eq. (ii),

$$dy = \frac{3}{2} \sqrt{4} (-0.032) = -0.096$$

Therefore, approximately value of $(3.968)^{\frac{3}{2}}$ is $8 - 0.096 = 7.904$.

(xv) $(32.15)^{\frac{1}{5}}$

Let $y = x^{\frac{1}{5}} \dots\dots\dots(i)$

$$\therefore \frac{dy}{dx} = \frac{1}{5} x^{\frac{-4}{5}} = \frac{1}{5x^{\frac{4}{5}}}$$

$$\Rightarrow dy = \frac{dx}{5 \left(x^{\frac{1}{5}} \right)^4} \dots\dots\dots(ii)$$

Now, from eq. (i), $y + \Delta y = \sqrt{x + \Delta x}$

$$= (32.15)^{\frac{1}{5}} = (32 + 0.15)^{\frac{1}{5}} \dots\dots\dots(iii)$$

Here $x = 32$ and $\Delta x = 0.15$

$$\text{Then } \Delta y = (x + \Delta x)^{\frac{1}{5}} - x^{\frac{1}{5}}$$

$$= (32.15)^{\frac{1}{5}} - (32)^{\frac{1}{5}} = (32.15)^{\frac{1}{5}} - 2$$

$$\Rightarrow (32.15)^{\frac{1}{5}} = 2 + \Delta y$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

\therefore From eq. (ii),

$$dy = \frac{0.15}{5 \left(32^{\frac{1}{5}} \right)^4} = \frac{0.15}{80} = 0.001875$$

Therefore, approximate value of $(32.15)^{\frac{1}{5}}$ is $2 + 0.001875 = 2.001875$.

2. Find the approximate value of $f(2.01)$ where $f(x) = 4x^2 + 5x + 2$.

Ans. Let $f(x) = y = 4x^2 + 5x + 2 \dots\dots\dots(i)$

$$\therefore f'(x) = \frac{dy}{dx} = 8x + 5$$

$$\Rightarrow dy = (8x + 5) dx$$

$$= (8x + 5) \Delta x \dots\dots\dots(ii)$$

Changing x to $x + \Delta x$ and y to $y + \Delta y$ in eq. (i),

$$y + \Delta y = f(x + \Delta x)$$

$$= f(2.01) = f(2 + 0.01) \dots\dots\dots(iii)$$

Here, $x = 2$ and $\Delta x = 0.01$

$$\therefore \text{From eq. (iii), } f(2.01) = y + \Delta y$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

$$\therefore \text{From eq. (i) and (ii), } f(2.01) = (4x^2 + 5x + 2) + (8x + 5)\Delta x$$

$$\Rightarrow f(2.01) = 4(4) + 5(2) + 2 + (8 \times 2 + 5)(0.01) = 28.21$$

Therefore, approximate value of $f(2.01)$ is 28.21.

3. Find the approximate value of $f(5.001)$ where $f(x) = x^3 - 7x^2 + 15$.

Ans. Let $f(x) = y = x^3 - 7x^2 + 15 \dots\dots\dots(i)$

$$\therefore f'(x) = \frac{dy}{dx} = 3x^2 - 14x$$

$$\Rightarrow dy = (3x^2 - 14x) dx = (3x^2 - 14x)\Delta x \dots\dots\dots(ii)$$

Changing x to $x + \Delta x$ and y to $y + \Delta y$ in eq. (i),

$$y + \Delta y = f(x + \Delta x)$$

$$= f(5.001) = f(5 + 0.001) \dots\dots\dots(iii)$$

Here, $x = 5$ and $\Delta x = 0.001$

$$\therefore \text{From eq. (iii), } f(5.001) = y + \Delta y$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

∴ From eq. (i) and (ii),

$$f(5.001) = (x^3 - 7x^2 + 15) + (3x^2 - 14x)\Delta x$$

$$\Rightarrow f(5.001) = 125 - 175 + 15 + (75 - 70)(0.001)$$

$$= -35 + 0.005 = -34.995$$

Therefore, approximate value of $f(5.001)$ is -34.995 .

4. Find the approximate change in the volume of a cube of side x meters caused by increasing the side by 1%.

Ans. Since Volume (V) = x^3 (i)

$$\therefore \frac{dV}{dx} = 3x^2 \text{(ii)}$$

It is given that increase in side = 1% of $x = \frac{x}{100}$

$$\Rightarrow \Delta x = \frac{x}{100} \text{(iii)}$$

Since approximate change in volume V of cube = $\Delta V \sim dV = \frac{dV}{dx} dx$

$$= \frac{dV}{dx} \Delta x \sim 3x^2 \left(\frac{x}{100} \right) \sim \frac{3}{100} x^3 \sim 0.03x^3 \text{ cubic meters}$$

5. Find the approximate change in the surface area of a cube of side x meters caused by decreasing the side by 1%.

Ans. Since Surface area (S) = $6x^2$

$$\therefore \frac{dS}{dx} = 12x$$

It is given that decrease in side = -1% of $x = -0.01x$

$$\Rightarrow \Delta x = -0.01x$$

Since approximate change in surface area S of cube = $\Delta S \sim dS = \frac{dS}{dx} dx$

$$= \frac{dS}{dx} \Delta x \sim 12x(-0.01x) \sim -0.12x^2 \text{ square meters (decreasing)}$$

6. If the radius of a sphere is measured as 7 m with an error of 0.02 m, then find the approximate error in calculating its volume.

Ans. Let r be the radius of the sphere and Δr be the error in measuring the radius.

Then, according to the question, $r = 7$ m and $\Delta r = 0.02$ m

$$\therefore \text{Volume of sphere (V)} = \frac{4}{3} \pi r^3$$

$$\therefore \frac{dV}{dr} = \frac{4}{3} \pi \cdot 3r^2$$

\therefore Approximate error in calculating the volume = Approximate value of ΔV

$$= dV = \frac{dV}{dr} (dr)$$

$$= \left(\frac{4}{3} \pi \cdot 3r^2 \right) dr$$

$$= 4\pi(7)^2(0.02)$$

$$= 3.92 \times \frac{22}{7} = 12.32 \text{ m}^3$$

Therefore, the approximate error in calculating volume is 12.32 m^3 .

7. If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating its surface area.

Ans. Let r be the radius of the sphere.

$$\therefore \text{Surface area of the sphere (S)} = 4\pi r^2$$

$$\therefore \frac{dS}{dr} = 8\pi r$$

$$\Rightarrow dS = 8\pi r \, dr$$

$$\Rightarrow dS = 8\pi r \, \Delta r$$

According to question $r=9$ m and $\Delta r=.03$ m

$$ds = 8\pi(9)(.03)$$

$$= 2.16\pi \text{ square meters}$$

8. If $f(x) = 3x^2 + 15x + 5$, then the approximate value of $f(3.02)$ is:

(A) 47.66

(B) 57.66

(C) 67.66

(D) 77.66

Ans. Let $f(x) = y = 3x^2 + 15x + 5$ (i)

$$\therefore f'(x) = \frac{dy}{dx} = 6x + 15$$

$$\Rightarrow dy = (6x + 15) \, dx = (6x + 15) \, \Delta x \text{(ii)}$$

Changing x to $x + \Delta x$ and y to $y + \Delta y$ in eq. (i),

$$y + \Delta y = f(x + \Delta x)$$

$$= f(3.02) = f(3 + 0.02) \dots\dots\dots(iii)$$

Here, $x = 3$ and $\Delta x = 0.02$

$$\therefore \text{From eq. (iii), } f(3.02) = y + \Delta y$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

\therefore From eq. (i) and (ii),

$$f(3.02) = (3x^2 + 15x + 5) + (6x + 15)\Delta x$$

$$\Rightarrow f(3.02) = 3(9) + 15(3) + 5 + (6 \times 3 + 15)(0.02)$$

$$= 77 + 0.66 = 77.66$$

Therefore, option (D) is correct.

9. The approximate change in the volume of a cube of side x meters caused by increasing the side by 3% is:

(A) $0.06 x^3 \text{ m}^3$

(B) $0.6 x^3 \text{ m}^3$

(C) $0.09 x^3 \text{ m}^3$

(D) $0.9 x^3 \text{ m}^3$

Ans. Since Volume (V) = x^3 (i)

$$\therefore \frac{dV}{dx} = 3x^2 \dots\dots\dots(ii)$$

It is given that increase in side = 3% of $x = \frac{3x}{100}$

$$\Rightarrow \Delta x = \frac{3x}{100} \dots\dots\dots(iii)$$

Since approximate change in volume V of cube = $\Delta V \sim dV = \frac{dV}{dx} dx$

$$= \frac{dV}{dx} \Delta x \sim 3x^2 \left(\frac{3x}{100} \right) \sim \frac{9}{100} x^3 \sim 0.09x^3 \text{ cubic meters}$$

Therefore, option (C) is correct.