CBSE Class–12 Mathematics

NCERT solution

Chapter - 1

Relations & Functions - Exercise 1.2

1. Show that the function $f: \mathrm{R}_* \to \mathrm{R}_*$ defined by $f(x)=\frac{1}{x}$ is one-one and onto, where R_* is the set of all non-zero real numbers. Is the result true, if the domain R_* is replaced by N with co-domain being same as R_*?

Ans. $f(x)=\frac{1}{x}, f: \mathrm{R}_* \to \mathrm{R}_*$

Part I: $f(x_1)=\frac{1}{x_1}$ and $f(x_2)=\frac{1}{x_2}$

If $f(x_1)=f(x_2)$ then $\frac{1}{x_1}=\frac{1}{x_2}$

$\Rightarrow x_1=x_2$

$\therefore$ f is one-one.

$f(x)=\frac{1}{x}$

$\Rightarrow y=\frac{1}{x}$

$\Rightarrow x=\frac{1}{y}$

$\Rightarrow f\left(\frac{1}{y}\right)=y$ $\therefore$ f is onto.

Part II: When domain R is replaced by N, co-domain R remaining the same, then,

$f = \mathrm{N} \to \mathrm{R}$

If $f(x_1) = f(x_2)$

$\Rightarrow \frac{1}{n_1} = \frac{1}{n_2}$

$\Rightarrow n_1 = n_2$ where $n_1, n_2 \in \mathrm{N}$

$\therefore$ f is one-one.

But every real number belonging to co-domain may not have a pre-image in N.

e.g. $\frac{1}{3} = \frac{3}{2} \neq \mathrm{N}$ $\therefore$ f is not onto.

2. Check the injectivity and surjectivity of the following functions:

(i) $f: \mathrm{N} \to \mathrm{N}$ **given by** $f(x) = x^2$

(ii) $f: \mathrm{Z} \to \mathrm{Z}$ **given by** $f(x) = x^2$

(iii) $f: \mathrm{R} \to \mathrm{R}$ **given by** $f(x) = x^2$

(iv) $f: \mathrm{N} \to \mathrm{N}$ **given by** $f(x) = x^3$

(v) $f: \mathrm{Z} \to \mathrm{Z}$ **given by** $f(x) = x^3$

Ans. (i) $f: \mathrm{N} \to \mathrm{N}$ given by $f(x) = x^2$

If $f(x_1) = f(x_2)$ then $x_1^2 = x_2^2$

$\Rightarrow x_1 = x_2$

$\therefore$ f is injective.

There are such numbers of co-domain which have no image in domain N.

e.g. 3 $\in$ co-domain N, but there is no pre-image in domain of f.

therefore f is not onto. $\therefore$ f is not surjective.

(ii) $f: Z \rightarrow Z$ given by $f(x) = x^2$

Since, Z = $\{0, \pm1, \pm2, \pm3, \ldots\ldots\}$ therefore, $f(-1) = f(1) = 1$

$\Rightarrow$ -1 and 1 have same image. $\therefore$ f is not injective.

There are such numbers of co-domain which have no image in domain Z.

e.g. 3 $\in$ co-domain, but $\sqrt{3} \notin$ domain of f. $\therefore$ f is not surjective.

(iii) $f: R \rightarrow R$ given by $f(x) = x^2$

As $f(-1) = f(1) = 1$

$\Rightarrow$ -1 and 1 have same image. $\therefore$ f is not injective.

e.g. $-2 \in$ co-domain, but $\sqrt{-2} \notin$ domain R of f. $\therefore$ f is not surjective.

(iv) $f: N \rightarrow N$ given by $f(x) = x^3$

If $f(x_1) = f(x_2)$ then $x_1^3 = x_2^3$

$\Rightarrow$ $x_1 = x_2$

i.e., for every $x \in$ N, has a unique image in its co-domain. $\therefore$ f is injective.

There are many such members of co-domain of f which do not have pre-image in its domain e.g., 2, 3, etc.

Therefore f is not onto. $\therefore$ f is not surjective.

(v) $f: Z \rightarrow Z$ given by $f(x) = x^3$

If $f(x_1) = f(x_2)$ then $x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2$

i.e., for every $x \in$ Z, has a unique image in its co-domain. $\therefore f$ is injective.

There are many such members of co-domain of f which do not have pre-image in its domain.

Therefore f is not onto. f is not surjective.

3. Prove that the Greatest integer Function $f: \mathbb{R} \to \mathbb{R}$, given by $f(x) = [x]$, is neither one-one nor onto, where $[x]$ denotes the greatest integer less than or equal to x.

Ans. Function $f: \mathbb{R} \to \mathbb{R}$, given by $f(x) = [x]$

$\because 1 \le x \le 2, \quad f(x) = 1$

$\therefore f(1) = 1$ and $f(1.1) = 1$

$\therefore f$ is not one-one.

All the images of $x \notin$ R belong to its domain have integers as the images in co-domain. But no fraction proper or improper belonging to co-domain of f has any pre-image in its domain.

Therefore, f is not onto.

4. Show that the Modulus Function $f: \mathbb{R} \to \mathbb{R}$, given by $f(x) = |x|$, is neither one-one nor onto, where is x, if x is positive or 0 and $|x|$ is -1, if x is negative.

Ans. Modulus Function $f: \mathbb{R} \to \mathbb{R}$, given by $f(x) = |x|$

Now $|x|=\begin{cases}-x, & \text{if } x<0\\ 0, & \text{if } x=0\\ x, & \text{if } x\geq 0\end{cases}$

$\Rightarrow f$ contains $(-1,1),(1,1),(-2,2),(2,2)$

Thus negative integers are not images of any element. $\therefore f$ is not one-one.

Also second set R contains some negative numbers which are not images of any real number.

$\therefore f$ is not onto.

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5. Show that the Signum Function $f:\mathrm{R}\rightarrow\mathrm{R}$, given by $f(x)=\begin{cases}1, \text{ if } x>0\\ 0, \text{ if } x=0\\ -1, \text{ if } x<0\end{cases}$ is neither one-one nor onto.

Ans. Signum Function $f:\mathrm{R}\rightarrow\mathrm{R}$, given by $f(x)=\begin{cases}1, \text{ if } x>0\\ 0, \text{ if } x=0\\ -1, \text{ if } x<0\end{cases}$

$f(1)=f(2)=1$

$\Rightarrow f(x_1)=f(x_2)=1$ for $n>0$

$\Rightarrow x_1\neq x_2 \quad \therefore f$ is not one-one.

Except $-1,0,1$ no other members of co-domain of f has any pre-image its domain.

$\therefore f$ is not onto.

Therefore, f is neither one-one nor onto.

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6. Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. Show that f is one-one.

Ans. A = {1, 2, 3}, B = {4, 5, 6, 7} and f = {(1, 4), (2, 5), (3, 6)}

Here, $f(1)=4$, $f(2)=5$ and $f(3)=6$

Here, also distinct elements of A have distinct images in B.

Therefore, f is a one-one function.

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7. In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.

(i) $f:\mathrm{R}\rightarrow\mathrm{R}$ **defined by** $f(x)=3-4x$

(ii) $f:\mathrm{R}\rightarrow\mathrm{R}$ **defined by** $f(x)=1+x^2$

Ans. (i) $f:\mathrm{R}\rightarrow\mathrm{R}$ defined by $f(x)=3-4x$

Now, if $x_1, x_2 \in$ R, then $f(x_1)=3-4x_1$ and $f(x_2)=3-4x_2$

And if $f(x_1)=f(x_2)$, then $x_1=x_2$ $\therefore$ f is one-one.

Again, if every element of Y (– R) is image of some element of X (R) under f, i.e., for every $y \in$ Y, there exists an element x in X such that $f(x)=y$.

Now $y=3-4x$

$$\Rightarrow x=\frac{3-y}{4}$$

$$\therefore f\left(\frac{3-y}{4}\right)=3-4\left(\frac{3-y}{4}\right)$$

$$\Rightarrow f(x)=3-3+y=y$$

$\therefore$ f is onto or bijective function.

(ii) $f: \mathrm{R} \to \mathrm{R}$ defined by $f(x) = 1 + x^2$

Now, if $x_1, x_2 \in$ R, then $f(x_1) = 1 + x_1^2$ and $f(x_2) = 1 + x_2^2$

And if $f(x_1) = f(x_2)$, then $x_1^2 = x_2^2$

$\Rightarrow x_1 = \pm x_2 \therefore f$ is not one-one.

Again, if every element of Y (– R) is image of some element of X (R) under f, i.e., for every $y \in$ Y, there exists an element x in X such that $f(x) = y$.

Now, $y = 1 + x^2 \Rightarrow x = \pm\sqrt{y-1}$

$\therefore f\left(\sqrt{y-1}\right) = 1 + y - 1 = -y \neq y$

$\therefore f$ is not onto.

Therefore, f is not bijective.

8. Let A and B be sets. Show that f : A x B $\to$ B x A such that $f(a, b) = (b, a)$ is a bijective function.

Ans. Injectivity: Let (a_1, b_1) and $(a_2, b_2) \in$ A x B such that $f(a_1, b_1) = f(a_2, b_2)$

$\Rightarrow (b_1, a_1) = (b_2, a_2)$

$\Rightarrow b_1 = b_2$ and $a_1 = a_2$

$\Rightarrow (a_1, b_1) = (a_2, b_2)$

$\Rightarrow (a_1, b_1) = (a_2, b_2)$ for all $(a_1, b_1), (a_2, b_2) \in$ A x B

So, f is injective.

Surjectivity: Let (b, a) be an arbitrary element of B x A. Then $b \in$ B and $a \in$ A.

$\Rightarrow (a,b) \in$ A x B

Thus, for all $(b,a) \in$ B x A, their exists $(a,b) \in$ A x B such that $f(a,b) = (b,a)$

So, f : A x B $\rightarrow$ B x A is an onto function, therefore f is bijective.

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9. Let $f: \mathrm{N} \rightarrow \mathrm{N}$ be defined by $f(n) = \begin{cases} \frac{n+1}{2}, \text{ if } n \text{ is odd} \\ \frac{n}{2}, \text{ if } n \text{ is even} \end{cases}$ for all $n \in$ N.

State whether the function f is bijective. Justify your answer.

Ans. $f: \mathrm{N} \rightarrow \mathrm{N}$ be defined by $f(n) = \begin{cases} \frac{n+1}{2}, \text{ if } n \text{ is odd} \\ \frac{n}{2}, \text{ if } n \text{ is even} \end{cases}$

(a) $f(1) = \frac{n+1}{2} = \frac{1+1}{2} = \frac{2}{2} = 1$ and $f(2) = \frac{n}{2} = \frac{2}{2} = 1$

The elements 1, 2, belonging to domain of f have the same image 1 in its co-domain.

So, f is not one-one, therefore, f is not injective.

(b) Every number of co-domain has pre-image in its domain e.g., 1 has two pre-images 1 and 2.

So, f is onto, therefore, f is not bijective.

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10. Let A = R – {3} and B = R – {1}. Consider the function f : A $\rightarrow$ B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

Ans. A = R – {3} and B = R – {1} and $f(x) = \frac{x-2}{x-3}$

Let $x_1, x_2 \in$ A, then $f(x_1) = \frac{x_1 - 2}{x_1 - 3}$ and $f(x_2) = \frac{x_2 - 2}{x_2 - 3}$

Now, for $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$

$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow -3x_1 - 2x_2 = -2x_1 - 3x_2$

$\Rightarrow x_1 = x_2 \therefore f$ is one-one function.

Now $y = \frac{x-2}{x-3}$

$\Rightarrow y(x-3) = x-2$

$\Rightarrow xy - 3y = x - 2$

$\Rightarrow x(y-1) = 3y - 2$

$\Rightarrow x = \frac{3y-2}{y-1}$

$\therefore f\left(\frac{3y-2}{y-1}\right) = \frac{\frac{3y-2}{y-1} - 2}{\frac{3y-2}{y-1} - 3} = \frac{3y - 2 - 2y + 2}{3y - 2 - 3y + 3} = y$

$\Rightarrow f(x) = y$

Therefore, f is an onto function.

11. Let $f:\ \mathbb{R}\to\mathbb{R}$ be defined as $f(x)=x^4$. Choose the correct answer:

(A) f is one-one onto

(B) f is many-one onto

(C) f is one-one but not onto

(D) f is neither one-one nor onto

Ans. $f(x)=x^4$ and $\underline{\mathrm{R}}\to\underline{\mathrm{R}}$

Let $x_1, x_2 \in \underline{\mathrm{R}}$, then $f(x_1)=x_1^4$ and $f(x_2)=x_2^4$

$\therefore\ x_1^4 = x_2^4$

$\Rightarrow\ \pm x_1 = \pm x_2$

Therefore, f is not one-one function.

Now, $y=x^4$

$\Rightarrow\ x=\pm y^{\frac{1}{4}}$

$\therefore\ f\left(y^{\frac{1}{4}}\right)=y^{\frac{1}{4}}=y$ and $f\left(-y^{\frac{1}{4}}\right)=-y^{\frac{1}{4}}=y$

Therefore, f is not onto function.

Therefore, option (D) is correct.

12. Let $f:\ \mathbb{R}\to\mathbb{R}$ be defined as $f(x)=3x$. Choose the correct answer:

(A) f is one-one onto

(B) f is many-one onto

(C) f **is one-one but not onto**

(D) f **is neither one-one nor onto**

Ans. Let $x_1, x_2 \rightarrow$ R such that $f(x_1) = f(x_2)$

$\Rightarrow 3x_1 = 3x_2$

$\Rightarrow x_1 = x_2$

Therefore, f is one-one function.

Now, consider $y \in$ R (co-domain of f) certainly $x = \frac{y}{3} \in$ R (domain of f)

Thus for all $y \in$ R (co-domain of f) there exists $x = \frac{y}{3} \in$ R (domain of f) such that

$$f(x) = f\left(\frac{y}{3}\right) = 3.\frac{y}{3} = y$$

Therefore, f is onto function.

Therefore, option (A) is correct.