CBSE Class–12 Mathematics

NCERT solution

Chapter - 4

Determinants - Exercise 4.1

Evaluate the following determinants in Exercise 1 and 2.

1. $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$

Ans. $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$ = $2(-1)-4(-5)$ = $-2+20$ = 18

2. (i) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

(ii) $\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix}$

Ans. (i) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ = $\cos\theta.\cos\theta-(-\sin\theta).\sin\theta$ = $\cos^2\theta+\sin^2\theta$ = 1

(ii) $\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix}$ = $(x^2-x+1)(x+1)-(x+1)(x-1)$

= $(x^3+1)-(x^2-1)$ = x^3-x^2+2

3. If A = $\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, **then show that** $|2\text{A}|=4|\text{A}|$.

Ans. Given: A = $\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then 2A = $2\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$

= $\begin{bmatrix} 2\times1 & 2\times2 \\ 2\times4 & 2\times2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$

L.H.S. = $|2A| = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix} = 2\times4-4\times8$ = $8-32=-24$

R.H.S. = $4|A| = 4\begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix}$

= $4(1\times2-4\times8)$ = $4(-6)=-24$

Since L.H.S. = R.H.S.

Hence, proved.

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4. If A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, **then show that** $|3A| = 27|A|$.

Ans. Given: A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then 3A = $3\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$ = $\begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix}$

L.H.S. = $|3A| = \begin{vmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{vmatrix}$

= $3(36-0)$ = 3 x 36 = 108

R.H.S. = $27|A| = 27\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix}$

= 27 [1 (4 – 0)] = 27 x 4 = 108

Since L.H.S. = R.H.S.

Hence, proved.

5. Evaluate the determinants:

(i) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

(ii) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

(iii) $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$

(iv) $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

Ans. Evaluate the determinants:

(i) Given: $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

Expanding along first row, $3\begin{vmatrix} 0 & -1 \\ -5 & 0 \end{vmatrix} - (-1)\begin{vmatrix} 0 & -1 \\ 3 & 0 \end{vmatrix} + (-2)\begin{vmatrix} 0 & 0 \\ 3 & -5 \end{vmatrix}$

= $3(0-5)+1\{0-(-3)\}-2(0-0)$

= $-15+3-0$ = -12

(ii) Given: $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

Expanding along first row, $3\begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} - (-4)\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} + 5\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$

= $3(1+6)+4\{1-(-4)\}+5(3-2)$

= $3\times 7+4\times 5+5\times 1$

= 21 + 20 + 5 = 46

(iii) Given: $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$

Expanding along first row, $0\begin{vmatrix} 0 & -3 \\ 3 & 0 \end{vmatrix} - 1\begin{vmatrix} -1 & -3 \\ -2 & 0 \end{vmatrix} + 2\begin{vmatrix} -1 & 0 \\ -2 & 3 \end{vmatrix}$

= $0(0+9)-(0-6)+2(-3-0)$

= 0 + 6 – 6 = 0

(iv) Given: $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

Expanding along first row, $2\begin{vmatrix} 2 & -1 \\ -5 & 0 \end{vmatrix} - (-1)\begin{vmatrix} 0 & -1 \\ 3 & 0 \end{vmatrix} + (-2)\begin{vmatrix} 0 & 2 \\ 3 & -5 \end{vmatrix}$

= $2(0-5)+(0+3)-2(0-6)$

= $-10+3+12$ = 5

6. If A = $\begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$, **find** $|A|$.

Ans. Given: A = $\begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$,

$\Rightarrow |A| = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$

Expanding along first row, $1\begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} - 1\begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} + (-2)\begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix}$

= $\{-9-(-12)\}-\{-18-(-15)\}-2(8-5)$

= $-9+12-(-18+15)-2(3)$

= $3-(-3)-6$

= $3+3-6$ = 0

7. Find the value of x, **if:**

(i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

(ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

Ans. (i) Given: $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

$\Rightarrow\ 2-20=2x^2-24$

$\Rightarrow\ -18=2x^2-24$

$\Rightarrow\ 2x^2=-18+24$

$\Rightarrow\ 2x^2=6$

$\Rightarrow\ x^2=3$

$\Rightarrow\ x=\pm\sqrt{3}$

(ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}=\begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

$\Rightarrow\ 10-12=5x-6x$

$\Rightarrow\ -2=-x$

$\Rightarrow\ x=2$

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8. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix}=\begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, **then** x **is equal to:**

(A) 6

(B) ± 6

(C) -6

(D) 0

Ans. Given: $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix}=\begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

$\Rightarrow\ x^2-36=36-36$

$\Rightarrow\ x^2-36=0$

$\Rightarrow\ x^2=36$

$\Rightarrow\ x=\pm 6$

Therefore, option (B) is correct.