CBSE Class–12 Mathematics

NCERT solution

Chapter - 1

Relations & Functions - Miscellaneous Exercise

1. Let $f: \mathrm{R} \rightarrow \mathrm{R}$ be defined as $f(x) = 10x + 7$. Find the function $g: \mathrm{R} \rightarrow \mathrm{R}$ such that $gof = fog = \mathrm{I}_{\mathrm{R}}$.

Ans. Given: $f(x) = 10x + 7$

Now $gof = g[f(x)]$ and $fog = f[g(x)] = 10g(x) + 7$

$\Rightarrow 10g(x) + 7 = \mathrm{I}_{\mathrm{R}}(x) = x$

$\Rightarrow g(x) = \dfrac{x-7}{10}$

2. Let $f: \mathrm{W} \rightarrow \mathrm{W}$ be defined as $f(n) = n - 1$, if n is odd and $f(n) = n + 1$, if n is even. Show that f is invertible. Find the inverse of f. Here, W is the set of all whole numbers.

Ans. Given: $f: \mathrm{W} \rightarrow \mathrm{W}$ defined as $f(n) = \begin{cases} n-1, & \text{if } n \text{ is odd} \\ n+1, & \text{if } n \text{ is even} \end{cases}$

Injectivity: Let n, m be any two odd real numbers, then $f(n) = f(m)$

$\Rightarrow n - 1 = m - 1$

$\Rightarrow n = m$

Again, let n, m be any two even whole numbers, then $f(n) = f(m)$

$\Rightarrow n + 1 = m + 1$

$\Rightarrow n = m$

Is n is even and m is odd, then $n \neq m$

Also, if $f(n)$ odd and $f(m)$ is even, then $f(n) \neq f(m)$

Hence, $n \neq m$

$\Rightarrow f(n) \neq f(m)$

$\therefore$ f is an injective mapping.

Surjectivity: Let n be an arbitrary whole number.

If n is an odd number, then there exists an even whole number $(n+1)$ such that

$f(n+1) = n+1-1 = n$

If n is an even number, then there exists an odd whole number $(n-1)$ such that

$f(n-1) = n-1+1 = n$

Therefore, every $n \in$ W has its pre-image in W.

So, $f:$ W $\rightarrow$ W is a surjective. Thus f is invertible and f^{-1} exists.

For f^{-1}: $y = n-1$

$\Rightarrow n = y+1$ and $y = n+1 \Rightarrow n = y-1$

$$\therefore f^{-1}(n) = \begin{cases} n-1, & \text{if } n \text{ is odd} \\ n+1, & \text{if } n \text{ is even} \end{cases}$$

Hence, $f^{-1}(y) = y$

3. If $f:$ R $\rightarrow$ R is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$.

Ans. Given: $f(x) = x^2 - 3x + 2$

$\Rightarrow f[f(x)] = f(x^2 - 3x + 2)$

$\Rightarrow (x^2 - 3x + 2)^2 - 3(x^2 - 3x + 2) + 2$

= $x^4 + 9x^2 + 4 - 6x^3 - 12x + 4x^2 - 3x^2 + 9x - 6 + 2$

$\Rightarrow x^4 - 6x^3 + 10x^2 - 3x$

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4. Show that the function $f : \mathrm{R} \rightarrow \{x \in \mathrm{R} : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in$ R is one-one and onto function.

Ans. f is one-one: For any $x, y \in$ R – {+1}, we have $f(x) = f(y)$

$\Rightarrow \frac{x}{1+|x|} = \frac{y}{|y|+1}$

$\Rightarrow xy + x = xy + y$

$\Rightarrow x = y$

Therefore, f is one-one function.

If f is one-one, let $y =$ R – {1}, then $f(x) = y$

$\Rightarrow \frac{x}{x+1} = y$

$\Rightarrow x = \frac{y}{1-y}$

It is cleat that $x \in$ R for all $y =$ R – {1}, also $x \neq = -1$

Because $x = -1$

$\Rightarrow \dfrac{y}{1-y} = -1$

$\Rightarrow y = -1 + y$ which is not possible.

Thus for each R – {1} there exists $x = \dfrac{y}{1-y} \in$ R – {1} such that

$$f(x) = \frac{x}{x+1} = \frac{\frac{y}{1-y}}{\frac{y}{1-y}+1} = y$$

Therefore f is onto function.

5. Show that the function f: R → R given by $f(x) = x^3$ is injective.

Ans. Let $x_1, x_2 \in$ R be such that $f(x_1) = f(x_2)$

$\Rightarrow x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2$

Therefore, f is one-one function, hence $f(x) = x^3$ is injective.

6. Give examples of two functions f : N → Z and g : Z → Z such that gof is injective but g is not injective.

(Hint: Consider $f(x) = x$ and $g(x) = |x|$)

Ans. Given: two functions f : N → Z and g : Z → Z

Let $f(x) = x$ and $g(x) = x \therefore (gof)(x) = f[f(x)] = g(x)$

Therefore, gof is injective but g is not injective.

7. Give examples of two functions $f: \mathbb{N} \rightarrow \mathbb{N}$ and $g: \mathbb{N} \rightarrow \mathbb{N}$ such that gof is onto but f is not onto.

(Hint: Consider $f(x) = x+1$ and $g(x) = \begin{cases} x-1 \text{ if } x > 1 \\ 1 \text{ if } x = 1 \end{cases}$)

Ans. Let $f(x) = x+1$

$$\therefore \; g(x) = \begin{cases} x-1 & \text{if } x > 1 \\ 1 & \text{if } x = 1 \end{cases}$$

These are two examples in which gof is onto but f is not onto.

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8. Given a non empty set X, consider P (X) which is the set of all subsets of X.

Define the relation R in P (X) as follows:

For subsets A, B in P (X), ARB if and only if A $\subset$ B. Is R an equivalence relation on P (X)? Justify your answer.

Ans. (i) A $\subset$ A $\therefore$ R is reflexive.

(ii) A $\subset$ B $\neq$ B $\subset$ A $\therefore$ R is not commutative.

(iii) If A $\subset$ B, B $\subset$ C, then A $\subset$ C $\therefore$ R is transitive.

Therefore, R is not equivalent relation.

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9. Given a non-empty set X, consider the binary operation * : P (X) x P (X) $\rightarrow$ P (X) given by A * B = A $\cap$ B $\forall$ A, B in P (X), where P (X) is the power set of X. Show that X is the identity element for this operation and X is the only invertible element in P (X) with respect to the operation *.

Ans. Let S be a non-empty set and P(S) be its power set. Let any two subsets A and B of S.

$\Rightarrow$ A $\cup$ B $\subset$ S

$\Rightarrow$ A $\cup$ B $\in$ P(S)

Therefore, '$\cup$ ' is an binary operation on P(S).

Similarly, if A, B $\in$ P(S) and A – B $\in$ P(S), then the intersection of sets $\cap$ and difference of sets are also binary operation on P(S) and A $\cap$ S = A = S $\cap$ A for every subset A of sets

$\Rightarrow$ A $\cap$ S = A = S $\cap$ A for all A $\in$ P(S)

$\Rightarrow$ S is the identity element for intersection $(\cap)$ on P(S).

10. Find the number of all onto functions from the set {1, 2, 3, ……., n} to itself.

Ans. The number of onto functions that can be defined from a finite set A containing n elements onto a finite set B containing n elements = $2^n - n$.

11. Let S = $\{a, b, c\}$ and T = {1, 2, 3}. Find F^{-1} of the following functions F from S to T, if it exists.

(i) F = $\{(a,3),(b,2),(c,1)\}$

(ii) F = $\{(a,2),(b,1),(c,1)\}$

Ans. S = $\{a, b, c\}$ and T = {1, 2, 3}

(i) F = $\{(a,3),(b,2),(c,1)\}$

$\Rightarrow F(a) = 3, F(b) = 2, F(c) = 1$

$\Rightarrow F^{-1}(3) = a, F^{-1}(2) = b, F^{-1}(1) = c$

$\therefore\ F^{-1} = \{(3,a),(2,b),(1,c)\}$

(ii) $\{(a,2),(b,1),(c,1)\}$

F is not one-one function, since element b and c have the same image 1.

Therefore, F is not one-one function.

12. Consider the binary operation * : R x R $\to$ R and o = R x R $\to$ R defined as $a * b = |a-b|$ and $aob = a, \forall a, b \in$ R. Show that * is commutative but not associative, o is associative but not commutative. Further, show that $\forall, a, b, c \in$ R, $a * (boc) = (a * b) o (a * b)$. [If it is so, we say that the operation * distributes over the operation o]. Does o distribute over *? Justify your answer.

Ans. Part I: $a * b = |a-b|$ also $b * a = |b-a| = (a-b)$ $\therefore$ operation * is commutative.

Now, $a * (b * c) = a * |b-c| = |a-(b-c)| = |a-b+c|$

And $(a * b) * c = |a-b| * c = |a-b-c|$

Here, $a * (b * c) \neq (a * b) * c$ $\therefore$ operation * is not associative.

Part II: $aob = a \forall a, b \in$ R

And, $boa = b$

$\Rightarrow aob \neq boa$ $\therefore$ operation o is not commutative.

Now $ao(boc) = aob = a$ and $(aob)oc = aoc = a$

Here $ao(boc) = (aob)oc$ $\therefore$ operation o is associative.

Part III: L.H.S. $a * (boc)$ = $a * b = |a-b|$

R.H.S. $(a * b) o (a * b)$ = $(a-b) o (a-c) = |a-b|$ = L.H.S. Proved.

Now, another distribution law: $ao(b * c) = (aob) * (aob)$

L.H.S. $ao(b * c) = ao(|b-c|) = a$

R.H.S. $(aob) * (aob) = a * a = |a-a| = 0$

As L.H.S. $\neq$ R.H.S.

Therefore, the operation O does not distribute over.

13. Given a non-empty set X, let * : P (X) x P (X) $\rightarrow$ P (X) be defined as A * B = (A – B) $\cup$ (B – A), $\forall$ A, B $\in$ P (X). Show that the empty set ϕ is the identity for the operation * and all the elements A of P (X) are invertible with A^{-1} = A. (Hint: $(A-\phi)\cup(\phi-A)=A$ and $(A-A)\cup(A-A)=A*A=\phi$)

Ans. For every A $\in$ P(X), we have

$\phi * A = (\phi - A)\cup(A-\phi) = \phi \cup A = A$

And $A * \phi = (A-\phi)\cup(\phi-A) = A\cup\phi = A$

$\Rightarrow$ ϕ is the identity element for the operation * on P(X).

Also A * A = (A – A) $\cup$ (A – A) = $\phi\cup\phi=\phi$

$\Rightarrow$ Every element A of P(X) is invertible with A^{-1} = A.

14. Define binary operation * on the set {0, 1, 2, 3, 4, 5} as $a * b = \begin{cases} a+b, \text{if} a+b<6 \\ a+b-6 \text{if} a+b \geq 6 \end{cases}$

Show that zero is the identity for this operation and each element a of the set is invertible with $6-a$ being the inverse of a.

Ans. A binary operation (or composition) * on a (non-empty) set is a function * : A x A $\rightarrow$A. We denote $*(a,b)$ by $a*b$ for every ordered pair $(a,b)\in$ A x A.

$\Rightarrow$ A binary operation on a no-empty set A is a rule that associates with every ordered pair of elements a,b (distinct or equal) of A some unique element $a*b$ of A.

*	0	1	2	3	4	5
0	0	1	2	3	4	5

1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

For all $a \notin$ A, we have $0 * a$ (mod 6) = 0

And $a * 1 = a(\text{mod } 6) = a$ and $a * 1 = a(\text{mod } 6) = a = 0$

$\Rightarrow$ 0 is the identity element for the operation.

Also on 0 = 0 – 0 = 0 *

2 * 1 = 3 = 1 * 2

$0^{-1} = 0 \quad 0^{-1} = 5$

15. Let A = {–1, 0, 1, 2}, B = {–4, –2, 0, 2} and $f, g: A \to B$ be the functions defined by $f(x) = x^2 - x, x \in$ A and $g(x) = 2\left|x - \frac{1}{2}\right| - 1, x \in$ A. Are f and g equal? Justify your answer.

(Hint: One may note that two functions $f: A \to B$ and $g: A \to B$ such that $f(a) = g(a)\ \forall\ a \in$ A, are called equal functions).

Ans. When $x = -1$ then $f(x) = 1^2 + 1 = 2$ and $g(x) = 2\left|-1 - \frac{1}{2}\right| - 1 = 2$

At $x = 0$, $f(0) = 0$ and $g(0) = 2\left|-\frac{1}{2}\right| - 1 = 2 \times \frac{1}{2} - 1 = 0$

At $x = 1$, $f(1) = 1^2 - 1 = 0$ and $g(1) = 2\left|1 - \frac{1}{2}\right| - 1 = 2 \times \frac{1}{2} - 1 = 0$

At $x=2$, $f(2)=2^2-2=2$ and $g(2)=2\left|2-\frac{1}{2}\right|-1=3-1=2$

Thus for each $a\in$ A, $f(a)=g(a)$

Therefore, f and g are equal function.

16. Let A = {1, 2, 3}. Then number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is:

(A) 1

(B) 2

(C) 3

(D) 4

Ans. It is clear that 1 is reflexive and symmetric but not transitive.

Therefore, option (A) is correct.

17. Let A = {1, 2, 3}. Then number of equivalence relations containing (1, 2) is:

(A) 1
(B) 2
(C) 3
(D) 4

Ans. 2
Therefore, option (B) is correct.

18. Let f: R $\rightarrow$ R be the Signum Function defined as $\begin{cases}1, x>0\\0, x=0\\-1, x<0\end{cases}$ **and g: R $\rightarrow$ R be the Greatest Function given by $g(x)=[x]$, where $[x]$ is greatest integer less than**

or equal to x. Then, does fog and gof coincide in (0, 1)?

Ans. It is clear that $gof: \mathbb{R} \to \mathbb{R}$ and $fog: \mathbb{R} \to \mathbb{R}$

Consider $x = \frac{1}{2}$ which lie on (0, # 1)

Now, $(gof)\left(\frac{1}{2}\right) = g\left\{f\left(\frac{1}{2}\right)\right\} = g(1) = [1] = 1$

And $(fog)\left(\frac{1}{2}\right) = f\left\{g\left(\frac{1}{2}\right)\right\} = f\left(\left[\frac{1}{2}\right]\right) = f(0) = 0$

$\Rightarrow gof \neq fog$ in (0, 1]

No, fog and gof don't coincide in (0, 1].

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19. Number of binary operation on the set $\{a, b\}$ are:

(A) 10

(b) 16

(C) 20

(D) 8

Ans. A = $\{a, b\}$

A x A = $\{(a, a), (a, b), (b, b)(b, a)\}$

$\therefore n(A \times A)$ = 4

Number of subsets = 2^4 = 16

Hence number of binary operation is 16.

Therefore, option (B) is correct.