**CBSE Class–12 Mathematics**

**NCERT solution**

**Chapter - 5**

**Continuity & Differentiability - Exercise 5.6**

**If $x$ and $y$ are connected parametrically by the equations given in Exercise 1 to 5, without eliminating the parameter, find $\frac{dy}{dx}$.**

**1.** $x = 2at^2$, $y = at^4$

**Ans.** Given: $x = 2at^2$ and $y = at^4$

$\therefore \frac{dx}{dt} = \frac{d}{dt}\left(2at^2\right)$ and $\frac{dy}{dt} = \frac{d}{dt}\left(at^4\right)$

$\Rightarrow \frac{dx}{dt} = 2a\frac{d}{dt}\left(t^2\right) = 2a.2t = 4at$ and $\frac{dy}{dt} = a\frac{d}{dt}\left(t^4\right) = a.4t^3 = 4at^3$

Now $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4at^3}{4at} = t^2$

**2.** $x = a\cos\theta$, $y = b\cos\theta$

**Ans.** Given: $x = a\cos\theta$ and $y = b\cos\theta$

$\therefore \frac{dx}{d\theta} = \frac{d}{d\theta}\left(a\cos\theta\right)$ and $\frac{dy}{d\theta} = \frac{d}{d\theta}\left(b\cos\theta\right)$

$\Rightarrow \frac{dx}{d\theta} = a\frac{d}{d\theta}\left(\cos\theta\right)$ and $\frac{dy}{d\theta} = b\frac{d}{d\theta}\left(\cos\theta\right)$

$\Rightarrow \frac{dx}{d\theta} = -a\sin\theta$ and $\frac{dy}{d\theta} = -b\sin\theta$

Now $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-b\sin\theta}{-a\sin\theta} = \frac{b}{a}$

**3.** $x = \sin t$, $y = \cos 2t$

**Ans.** Given: $x = \sin t$ and $y = \cos 2t$

$\therefore \dfrac{dx}{dt} = \cos t$ and $\dfrac{dy}{dt} = -\sin 2t \dfrac{d}{dt}(2t) = -2\sin 2t$

Now $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{-2\sin 2t}{\cos t} = \dfrac{-2\times 2\sin t\cos t}{\cos t} = -4\sin t$

**4.** $x = 4t$, $y = \dfrac{4}{t}$

**Ans.** Given: $x = 4t$ and $y = \dfrac{4}{t}$

$\therefore \dfrac{dx}{dt} = \dfrac{d}{dt}(4t) = 4\dfrac{d}{dt}t = 4$

and $\dfrac{dy}{dt} = \dfrac{d}{dt}\left(\dfrac{4}{t}\right) = \dfrac{t\dfrac{d}{dt}4 - 4\dfrac{d}{dt}t}{t^2}$

$\Rightarrow \dfrac{dy}{dt} = \dfrac{t\times 0 - 4\times 1}{t^2} = -\dfrac{4}{t^2}$

Now $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{-\dfrac{4}{t^2}}{4} = \dfrac{-1}{t^2}$

**5.** $x = \cos\theta - \cos 2\theta$, $y = \sin\theta - \sin 2\theta$

**Ans.** Given: $x = \cos\theta - \cos 2\theta$ and $y = \sin\theta - \sin 2\theta$

$\therefore \dfrac{dx}{d\theta} = \dfrac{d}{d\theta}\cos\theta - \dfrac{d}{d\theta}\cos 2\theta$ and $\frac{dy}{d\theta} = \frac{d}{d\theta}\sin\theta - \frac{d}{d\theta}\sin 2\theta$

$\Rightarrow \frac{dx}{d\theta} = -\sin\theta - (-\sin 2\theta)\frac{d}{d\theta}2\theta$ and $\frac{dy}{d\theta} = \cos\theta - \cos 2\theta \frac{d}{d\theta}2\theta$

$\Rightarrow \frac{dx}{d\theta} = -\sin\theta + (\sin 2\theta)\,2$ and $\frac{dy}{d\theta} = \cos\theta - \cos 2\theta \times 2$

$\Rightarrow \frac{dx}{d\theta} = 2\sin 2\theta - \sin\theta$ and $\frac{dy}{d\theta} = \cos\theta - 2\cos 2\theta$

Now $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cos\theta - 2\cos 2\theta}{2\sin 2\theta - \sin\theta}$

**If $x$ and $y$ are connected parametrically by the equations given in Exercises 6 to 10, without eliminating the parameter, find $\frac{dy}{dx}$.**

**6.** $x = a(\theta - \sin\theta)$, $y = a(1+\cos\theta)$

**Ans.** Given: $x = a(\theta - \sin\theta)$ and $y = a(1+\cos\theta)$

$\therefore \frac{dx}{d\theta} = a\frac{d}{d\theta}(\theta - \sin\theta)$ and $\frac{dy}{d\theta} = a\frac{d}{d\theta}(1+\cos\theta)$

$\Rightarrow \frac{dx}{d\theta} = a\left[\frac{d}{d\theta}\theta - \frac{d}{d\theta}\sin\theta\right]$ and $\frac{dy}{d\theta} = a\left[\frac{d}{d\theta}(1) + \frac{d}{d\theta}\cos\theta\right]$

$\Rightarrow \frac{dx}{d\theta} = a(1-\cos\theta)$ and $\frac{dy}{d\theta} = a[0 - \sin\theta] = -a\sin\theta$

$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-a\sin\theta}{a(1-\cos\theta)} = \frac{-\sin\theta}{1-\cos\theta}$

$= -\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}$

= $-\dfrac{\cos\dfrac{\theta}{2}}{\sin\dfrac{\theta}{2}}$

= $-\cot\dfrac{\theta}{2}$

**7.** $x=\dfrac{\sin^3 t}{\sqrt{\cos 2t}}$, $y=\dfrac{\cos^3 t}{\sqrt{\cos 2t}}$

**Ans.** Given: $x=\dfrac{\sin^3 t}{\sqrt{\cos 2t}}$ and $y=\dfrac{\cos^3 t}{\sqrt{\cos 2t}}$

$\therefore\quad \dfrac{dx}{dt}=\dfrac{\sqrt{\cos 2t}.\dfrac{d}{dt}\left(\sin^3 t\right)-\sin^3 t.\dfrac{d}{dt}\left(\sqrt{\cos 2t}\right)}{\left(\sqrt{\cos 2t}\right)^2}$ [By quotient rule]

= $\dfrac{\sqrt{\cos 2t}.3\sin^2 t\dfrac{d}{dt}(\sin t)-\sin^3 t.\dfrac{1}{2}(\cos 2t)^{\frac{-1}{2}}\dfrac{d}{dt}(\cos 2t)}{\cos 2t}$

= $\dfrac{\sqrt{\cos 2t}.3\sin^2 t\cos t-\dfrac{\sin^3 t}{2\sqrt{\cos 2t}}(-2\sin 2t)}{\cos 2t}$

= $\dfrac{3\sin^2 t\cos t\cos 2t+\sin^3 t.\sin 2t}{(\cos 2t)^{\frac{3}{2}}}$

= $\dfrac{\sin^2 t\cos t\left(3\cos 2t+2\sin^2 t\right)}{(\cos 2t)^{\frac{3}{2}}}$

And $\frac{dy}{dt}=\frac{\sqrt{\cos 2t}.\frac{d}{dt}\left(\cos^3 t\right)-\cos^3 t.\frac{d}{dt}\left(\sqrt{\cos 2t}\right)}{\left(\sqrt{\cos 2t}\right)^2}$ [By quotient rule]

$$=\frac{\sqrt{\cos 2t}.3\cos^2 t\frac{d}{dt}\left(\cos t\right)-\cos^3 t.\frac{1}{2}\left(\cos 2t\right)^{\frac{-1}{2}}\frac{d}{dt}\left(\cos 2t\right)}{\cos 2t}$$

$$=\frac{\sqrt{\cos 2t}.3\cos^2 t\left(-\sin t\right)-\frac{\cos^3 t}{2\sqrt{\cos 2t}}\left(-2\sin 2t\right)}{\cos 2t}$$

$$=\frac{-3\cos^2 t\sin t\cos 2t+\cos^3 t.\sin 2t}{\left(\cos 2t\right)^{\frac{3}{2}}}$$

$$=\frac{-3\cos^2 t\sin t\cos 2t+\cos^3 t.2\sin t\cos t}{\left(\cos 2t\right)^{\frac{3}{2}}}$$

$$=\frac{\sin t\cos^2 t\left(2\cos^2 t-3\cos 2t\right)}{\left(\cos 2t\right)^{\frac{3}{2}}}$$

$$\therefore\ \frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{\frac{\sin t\cos^2 t\left(2\cos^2 t-3\cos 2t\right)}{\left(\cos 2t\right)^{\frac{3}{2}}}}{\frac{\sin^2 t\cos t\left(3\cos 2t+2\sin^2 t\right)}{\left(\cos 2t\right)^{\frac{3}{2}}}}$$

$$=\frac{\cos t\left[2\cos^2 t-3\left(2\cos^2 t-1\right)\right]}{\sin t\left[3\left(1-2\sin^2 t\right)+2\sin^2 t\right]}$$

$$=\frac{\cos t\left(3-4\cos^2 t\right)}{\sin t\left(3-4\sin^2 t\right)}$$

$= \dfrac{-\left(4\cos^3 t - 3\cos t\right)}{3\sin t - 4\sin^3 t}$

$= \dfrac{-\cos 3t}{\sin 3t} = -\cot 3t$

**8.** $x = a\left(\cos t + \log\tan\dfrac{t}{2}\right), y = a\sin t$

**Ans.** Given: $x = a\left(\cos t + \log\tan\dfrac{t}{2}\right)$ and $y = a\sin t$

$\therefore \dfrac{dx}{dt} = a\left[-\sin t + \dfrac{1}{\tan\frac{t}{2}} \cdot \dfrac{d}{dt}\left(\tan\dfrac{t}{2}\right)\right]$

$= a\left[-\sin t + \dfrac{1}{\tan\frac{t}{2}} \cdot \sec^2\dfrac{t}{2} \cdot \dfrac{1}{2}\right]$

$= a\left[-\sin t + \dfrac{\cos\frac{t}{2}}{\sin\frac{t}{2}} \cdot \dfrac{1}{\cos^2\frac{t}{2}} \cdot \dfrac{1}{2}\right]$

$= a\left[-\sin t + \dfrac{1}{2\sin\frac{t}{2}\cos\frac{t}{2}}\right]$

$= a\left[-\sin t + \dfrac{1}{\sin t}\right]$

$= a\left[\dfrac{1}{\sin t} - \sin t\right] = a\left(\dfrac{1-\sin^2 t}{\sin t}\right) = \dfrac{a\cos^2 t}{\sin t}$

And $\dfrac{dy}{dt} = a\cos t$

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{a\cos t}{\left(\frac{a\cos^2 t}{\sin t}\right)} = \frac{\sin t}{\cos t} = \tan t$

**9.** $x = a\sec\theta, y = b\tan\theta$

**Ans.** Given: $x = a\sec\theta$ and $y = b\tan\theta$

$\therefore \frac{dx}{d\theta} = a\sec\theta\tan\theta$ and $\frac{dy}{d\theta} = b\sec^2\theta$

$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{b\sec^2\theta}{a\sec\theta\tan\theta}$

$= \frac{b\sec\theta}{a\tan\theta}$

$= \frac{b.\frac{1}{\cos\theta}}{a.\frac{\sin\theta}{\cos\theta}}$

$= \frac{b}{a\sin\theta}$

$= \frac{b}{a}\cos ec\theta$

**10.** $x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$

**Ans.** Given: $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$

$\therefore \frac{dx}{d\theta} = a(-\sin\theta + \theta\cos\theta + \sin\theta.1)$

$= a\theta\cos\theta$

And $\frac{dy}{d\theta} = a\left[\cos\theta - \{\theta(-\sin\theta) + \cos\theta.1\}\right]$

$= a[\cos\theta + \theta\sin\theta - \cos\theta] = a\theta\sin\theta$

$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta$

**11. If** $x=\sqrt{a^{\sin^{-1}t}}$, $y=\sqrt{a^{\cos^{-1}t}}$, **show that** $\frac{dy}{dx}=\frac{-y}{x}$.

**Ans.** Given: $x=\sqrt{a^{\sin^{-1}t}}=\left(a^{\sin^{-1}t}\right)^{\frac{1}{2}}=a^{\frac{1}{2}\sin^{-1}t}$

and $y=\sqrt{a^{\cos^{-1}t}}=\left(a^{\cos^{-1}t}\right)^{\frac{1}{2}}=$

$= a^{\frac{1}{2}\cos^{-1}t}$

$\therefore \frac{dx}{dt}=a^{\frac{1}{2}\sin^{-1}t}\log a\frac{d}{dt}\left(\frac{1}{2}\sin^{-1}t\right)$

$= a^{\frac{1}{2}\sin^{-1}t}\log a\frac{1}{2}\frac{1}{\sqrt{1-t^2}}$

And $\frac{dy}{dt}=a^{\frac{1}{2}\cos^{-1}t}\log a\frac{d}{dt}\left(\frac{1}{2}\cos^{-1}t\right)$

$= a^{\frac{1}{2}\cos^{-1}t}\log a\frac{1}{2}.\frac{-1}{\sqrt{1-t^2}}$

$\therefore \frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{a^{\frac{1}{2}\cos^{-1}t}\log a\frac{1}{2}\frac{-1}{\sqrt{1-t^2}}}{a^{\frac{1}{2}\sin^{-1}t}\log a\frac{1}{2}\frac{1}{\sqrt{1-t^2}}}$

$=\frac{-a^{\frac{1}{2}\cos^{-1}t}}{a^{\frac{1}{2}\sin^{-1}t}}=\frac{-y}{x}$

Hence proved.