CBSE Class–12 Mathematics

NCERT solution

Chapter - 2

Inverse Trigonometric Functions - Exercise 2.2

Prove the following:

1. $3\sin^{-1}x = \sin^{-1}\left(3x - 4x^3\right), x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Ans. We know that: $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

Putting $\sin\theta = x$

$\Rightarrow \theta = \sin^{-1}x$

$\therefore \sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\Rightarrow \sin 3\theta = 3x - 4x^3$

$\Rightarrow 3\theta = \sin^{-1}\left(3x - 4x^3\right)$

Putting $\theta = \sin^{-1}x$,

$3\sin^{-1}x = \sin^{-1}\left(3x - 4x^3\right)$

Proved.

2. $3\cos^{-1}x = \cos^{-1}\left(4x^3 - 3x\right), x \in \left[\frac{1}{2}, 1\right]$

Ans. We know that: $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

Putting $\cos\theta = x$

$\Rightarrow \theta = \cos^{-1}x$

$\therefore \cos 3\theta = 4x^3 - 3x$

$\Rightarrow 3\theta = \cos^{-1}\left(3x - 4x^3\right)$

Putting $\theta = \cos^{-1} x$,

$3\cos^{-1} x = \cos^{-1}\left(4x^3 - 3x\right)$ Proved.

---

3. $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

Ans. L.H.S. = $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24}$

= $\tan^{-1}\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}\times\frac{7}{24}}$ $\left[\because \tan^{-1} x + \tan^{-1} y = \tan^{-1}\frac{x+y}{1-xy}\right]$

= $\tan^{-1}\frac{48+77}{264-14}$

= $\tan^{-1}\frac{125}{250}$

= $\tan^{-1}\frac{1}{2}$

= R.H.S.

Proved.

---

4. $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

Ans. L.H.S. = $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}\dfrac{2\times\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}+\tan^{-1}\dfrac{1}{7}$

$\left[\because 2\tan^{-1}x=\tan^{-1}\dfrac{2x}{1-x^2}\right]$

$= \tan^{-1}\dfrac{4}{3}+\tan^{-1}\dfrac{1}{7}$

$= \tan^{-1}\dfrac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\times\frac{1}{7}}$

$\left[\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\dfrac{x+y}{1-xy}\right]$

$= \tan^{-1}\dfrac{28+3}{21-4}$

$= \tan^{-1}\dfrac{31}{17}$ = R.H.S.

Proved.

Write the following functions in the simplest form:

5. $\tan^{-1}\dfrac{\sqrt{1+x^2}-1}{x}$, $x\neq 0$

Ans. Putting $x=\tan\theta$ so that $\theta=\tan^{-1}x$

$\Rightarrow \tan^{-1}\dfrac{\sqrt{1+x^2}-1}{x}$

$= \tan^{-1} \frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}$

$= \tan^{-1} \frac{\sec\theta - 1}{\tan\theta}$

$= \tan^{-1} \left(\frac{\frac{1}{\cos\theta} - 1}{\frac{\sin\theta}{\cos\theta}} \right)$

$= \tan^{-1} \left(\frac{1-\cos\theta}{\sin\theta} \right)$

$= \tan^{-1} \left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} \right)$

$= \tan^{-1} \left(\tan\frac{\theta}{2} \right)$

$= \frac{\theta}{2} = \frac{1}{2}\tan^{-1} x$

---

6. $\tan^{-1} \frac{1}{\sqrt{x^2-1}}, |x| > 1$

Ans. Putting $x = \sec\theta$ so that $\theta = \sec^{-1} x$

$\Rightarrow \tan^{-1} \frac{1}{\sqrt{x^2-1}}$

$= \tan^{-1} \frac{1}{\sqrt{\sec^2\theta - 1}}$

$= \tan^{-1}\frac{1}{\sqrt{\tan^2\theta}}$

$= \tan^{-1}\left(\frac{1}{\tan\theta}\right)$

$= \tan^{-1}(\cot\theta)$

$= \tan^{-1}\tan\left(\frac{\pi}{2}-\theta\right)$

$= \frac{\pi}{2}-\theta = \frac{\pi}{2}-\sec^{-1}x$

---

7. $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}, x<\pi$

Ans. $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$

$= \tan^{-1}\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}$

$= \tan^{-1}\tan\frac{x}{2}$

$= \frac{x}{2}$

---

8. $\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right), 0<x<\pi$

Ans. $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$

Dividing the numerator and denominator by $\cos x$,

$$= \tan^{-1}\left(\frac{1-\tan x}{1+\tan x}\right)$$

$$= \tan^{-1}\left(\frac{\tan\frac{\pi}{4} - \tan x}{1+\tan\frac{\pi}{4}\tan x}\right)$$

$$= \tan^{-1}\tan\left(\frac{\pi}{4} - x\right) \qquad \left[\tan(x-y) = \frac{\tan x - \tan y}{1+\tan x . \tan y}\right]$$

$$= \frac{\pi}{4} - x$$

---

9. $\tan^{-1}\frac{x}{\sqrt{a^2 - x^2}}$, $|x| < a$

Ans. Putting $x = a\sin\theta$ so that $\theta = \sin^{-1}\frac{x}{a}$

$$\Rightarrow \tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2 - a^2\sin^2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2\left(1-\sin^2\theta\right)}}\right)$$

$$= \tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2\cos^2\theta}}\right)$$

$= \tan^{-1}\left(\frac{a\sin\theta}{a\cos\theta}\right)$

$= \tan^{-1}\tan\theta$

$= \theta = \sin^{-1}\frac{x}{a}$

10. $\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right), a>0, \left(-\frac{a}{\sqrt{3}}\le x\le\frac{a}{\sqrt{3}}\right)$

Ans. $\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)$

$= \tan^{-1}\left(\frac{3\left(\frac{x}{a}\right)-\left(\frac{x}{a}\right)^3}{1-3\left(\frac{x}{a}\right)^2}\right)$ [Dividing numerator and denominator by a^3]

Putting $\frac{x}{a}=\tan\theta$ so that $\theta=\tan^{-1}\frac{x}{a}$

$= \tan^{-1}\left(\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right)$

$= \tan^{-1}\tan 3\theta$

$= 3\theta = 3\tan^{-1}\frac{x}{a}$

Find the values of each of the following:

11. $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

Ans. $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

$= \tan^{-1}\left[2\cos\left(2\sin^{-1}\sin\frac{\pi}{6}\right)\right]$

$= \tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right]$

$= \tan^{-1}\left[2\cos\frac{\pi}{3}\right]$

$= \tan^{-1}\left[2\times\frac{1}{2}\right]$

$= \tan^{-1}1$

$= \tan^{-1}\tan\frac{\pi}{4} = \frac{\pi}{4}$

---

12. $\cot\left(\tan^{-1}a+\cot^{-1}a\right)$

Ans. $\cot\left(\tan^{-1}a+\cot^{-1}a\right)$

$= \cot\frac{\pi}{2} = 0 \left[\because \tan^{-1}x+\cot^{-1}x = \frac{\pi}{2}\right]$

---

13. $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right], |x|<1, y>0 \text{ and } xy<1$

Ans. Putting $x=\tan\theta$ and $y=\tan\phi$

$\therefore \tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right]$

$= \tan\frac{1}{2}\left[\sin^{-1}\frac{2\tan\theta}{1+\tan^2\theta}+\cos^{-1}\frac{1-\tan^2\phi}{1+\tan^2\phi}\right]$

$= \tan\frac{1}{2}\left[\sin^{-1}\sin 2\theta+\cos^{-1}\cos 2\phi\right]$

$= \tan\frac{1}{2}\left[2\theta+2\phi\right]$

$= \tan\left[\theta+\phi\right]$

$= \frac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}$

$= \frac{x+y}{1-xy}$

14. If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$**, then find the value of** x.

Ans. Given: $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1=\sin\frac{\pi}{2}$

$\Rightarrow \sin^{-1}\frac{1}{5}+\cos^{-1}x=\frac{\pi}{2}$

$\Rightarrow \cos^{-1}x=\frac{\pi}{2}-\sin^{-1}\frac{1}{5}$

$\Rightarrow \cos^{-1}x=\cos^{-1}\frac{1}{5}\quad\left[\because \sin^{-1}t+\cos^{-1}t=\frac{\pi}{2}\right]$

$\Rightarrow x=\frac{1}{5}$

15. If $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$, **then find the value of** x.

Ans. Given: $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}=\frac{\pi}{4}\left[\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}\right]$$

$$\Rightarrow \tan^{-1}\frac{(x-1)(x+2)+(x+1)(x-2)}{(x-2)(x+2)-(x-1)(x+1)}=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\frac{x^2+2x-x-2+x^2-2x+x-2}{x^2-4-\left(x^2-1\right)}=\frac{\pi}{4}$$

$$\Rightarrow \frac{2x^2-4}{x^2-4-x^2+1}=\tan\frac{\pi}{4}$$

$$\Rightarrow \frac{2x^2-4}{-3}=1$$

$$\Rightarrow 2x^2-4=-3$$

$$\Rightarrow 2x^2=1$$

$$\Rightarrow x^2=\frac{1}{2}$$

$$\Rightarrow x=\pm\frac{1}{\sqrt{2}}$$

Find the values of each of the expressions in Exercises 16 to 18.

16. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

Ans. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

$= \sin^{-1}\left(\sin\frac{3\pi-\pi}{3}\right)$

$= \sin^{-1}\left[\sin\left(\pi-\frac{\pi}{3}\right)\right]$

$= \sin^{-1}\sin\frac{\pi}{3}$

$= \frac{\pi}{3}$

---

17. $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

Ans. $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

$= \tan^{-1}\left(\tan\frac{4\pi-\pi}{4}\right)$

$= \tan^{-1}\left[\tan\left(\pi-\frac{\pi}{4}\right)\right]$

$= \tan^{-1}\left[-\tan\frac{\pi}{4}\right]$

$= \tan^{-1}\tan\left(-\frac{\pi}{4}\right)$

$= -\frac{\pi}{4}$

18. $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$

Ans. Putting $\sin^{-1}\frac{3}{5}=x$ and $\cot^{-1}\frac{3}{2}=y$ so that $\sin x=\frac{3}{5}$ and $\cot y=\frac{3}{2}$

Now, $\cos x=\sqrt{1-\sin^2 x}$ = $\sqrt{1-\frac{9}{25}}=\sqrt{\frac{16}{25}}=\frac{4}{5}$

And $\tan x=\frac{\sin x}{\cos x}=\frac{3}{4}$

and $\tan y=\frac{1}{\cot y}=\frac{2}{3}$

$\therefore \tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$

= $\tan(x+y)$

= $\frac{\tan x+\tan y}{1-\tan x\tan y}$ = $\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}}$

= $\frac{\frac{17}{12}}{\frac{1}{2}}$ = $\frac{17}{6}$

---

19. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ **is equal to:**

(A) $\frac{7\pi}{6}$

(B) $\frac{5\pi}{6}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{6}$

Ans. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

= $\cos^{-1}\left[\cos\left(2\pi-\frac{7\pi}{6}\right)\right]$ $\left[\because \cos(2\pi-\theta)=\cos\theta\right]$

= $2\pi-\frac{7\pi}{6}=\frac{12\pi-7\pi}{6}=\frac{5\pi}{6}$

Therefore, option (B) is correct.

---

20. $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$ **is equal to:**

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) 1

Ans. $\sin^{-1}\left(-\frac{1}{2}\right)=\sin^{-1}\left(-\sin\frac{\pi}{6}\right)=\sin^{-1}\left[\sin\left(-\frac{\pi}{6}\right)\right]$

= $-\frac{\pi}{6}$

$\therefore \sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$

$= \sin\left[\frac{\pi}{3}-\left(-\frac{\pi}{6}\right)\right]$

$= \sin\left[\frac{\pi}{3}+\frac{\pi}{6}\right]$

$= \sin\frac{3\pi}{6}=\sin\frac{\pi}{2}=1$

Therefore, option (D) is correct.

---

21. $\tan^{-1}\sqrt{3}-\cot^{-1}\left(-\sqrt{3}\right)$ **is equal to:**

(A) π

(B) $-\frac{\pi}{2}$

(C) 0

(D) $2\sqrt{3}$

Ans. $\tan^{-1}\sqrt{3}-\cot^{-1}\left(-\sqrt{3}\right)$

$= \tan^{-1}\tan\frac{\pi}{3}-\cot^{-1}\left(-\cot\frac{\pi}{6}\right)$

$= \frac{\pi}{3}-\cot^{-1}\left[\cot\left(\pi-\frac{\pi}{6}\right)\right]$

$= \frac{\pi}{3}-\left(\pi-\frac{\pi}{6}\right)$

$= \frac{\pi}{3}-\frac{5\pi}{6}$

$= \frac{2\pi-5\pi}{6}$

$= -\frac{3\pi}{6}=-\frac{\pi}{2}$

Therefore, option (B) is correct.