CBSE Class–12 Mathematics
NCERT solution
Chapter - 3
Matrices - Miscellaneous Exercise

1. Let A = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, **show that** $(aI+bA)^n = a^nI + na^{n-1}bA$ **where I is the identity matrix of order 2 and** $n \in$ **N.**

Ans. Using Mathematical Induction, we see the result is true for $n=1$, for

$$(aI+bA)^n = a^nI + na^{n-1}bA$$

Given: $p(k)$ is true, i.e. $(aI+bA)^k = a^kI + ka^{k-1}bA$

To prove: $(aI+bA)^{k+1} = a^{k+1}I + (k+1)a^kbA$

Proof: L.H.S. = $(aI+bA)^{k+1}$ = $(aI+bA)^k(aI+bA)$ = $(a^kI+ka^{k-1}bA)(aI+bA)$

= $a^{k+1}I\times I + ka^kbAI + a^kbAI + ka^{k-1}b^2A.A$

= $a^{k+1}I\times I + ka^kbAI + a^kbAI + ka^{k-1}b^2.0$

= $a^{k+1}I + (k+1)a^kbA$ = R.H.S.

Thus, $p(k+1)$ is true, therefore, $p(n)$ is true.

2. If A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, **prove that** A^n = $\begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$ $n \in$ **N.**

Ans. Given: A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ ....(i)

Let $p(n): A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$

$\Rightarrow \quad p(1): A = \begin{bmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{bmatrix}$ = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ = A

$\therefore \quad p(1)$ is true for $n = 1$.

Now $p(k): A^k = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$ ....(ii)

Multiplying eq. (ii) by eq. (i), $A^k A = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$$=> A^{k+1} = \begin{bmatrix} 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} \\ 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} \\ 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} \end{bmatrix}$$

$$=> A^{k+1} = \begin{bmatrix} 3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1} \\ 3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1} \\ 3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1} \end{bmatrix}$$

$$=> A^{k+1} = \begin{bmatrix} 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \\ 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \\ 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \end{bmatrix} = p(k+1)$$

Therefore, $p(n)$ is true for all natural numbers by P.M.I.

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3. If A = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then prove that A^n = $\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ where n is any positive integer.

Ans. Given: A" = $\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ $\quad \therefore \quad A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$

$\Rightarrow \quad A^1 = \begin{bmatrix} 1+2 & -4 \\ 1 & 1-2 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ which is true for $n=1$.

Now, $A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$ ....(i)

To Prove : $A^{k+1} = \begin{bmatrix} 1+2(k+1) & -4(k+1) \\ (k+1) & 1-2(k+1) \end{bmatrix}$ ....(ii)

Proof : Multiply A^k with A, We get

$$=> A^k.A = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

$$=> A^k.A = \begin{bmatrix} 3+6k-4k & -4-8k+4k \\ 3k+1-2k & -4k-1+2k \end{bmatrix}$$

$$=> A^k.A = \begin{bmatrix} 3+2k & -4-4k \\ k+1 & -2k-1 \end{bmatrix}$$

$$=> A^k . A = \begin{bmatrix} 1+2k+2 & -4-4k \\ k+1 & 1-2k-2 \end{bmatrix}$$

$$=> A^{k+1} = \begin{bmatrix} 1+2(k+1) & -4(k+1) \\ (k+1) & 1-2(k+1) \end{bmatrix}$$

Therefore, the result is true for $n = k+1$.

Hence, by the principal of mathematical induction, the result is true for all positive integers n.

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4. If A and B are symmetric matrices, prove that AB – BA is a skew symmetric matrix.

Ans. A and B are symmetric matrices. $\Rightarrow$ A’ = A and B’ = B ....(i)

Now, (AB – BA)’ = (AB)’ – (BA)’ $\Rightarrow$ (AB – BA)’ = B’A’ – A’B’ [Reversal law]

$\Rightarrow$ (AB – BA)’ = BA – AB [Using eq. (i)]

$\Rightarrow$ (AB – BA)’ = – (AB – BA)

Therefore, (AB – BA) is a skew symmetric.

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5. Show that the matrix B’AB is symmetric or skew symmetric according as A is symmetric or skew symmetric.

Ans. (B’AB)’ = [B’(AB]’ = (AB)’ (B’)’ [$\because$ (CD)’ = D’C’]

$\Rightarrow$ (B’AB)’ = B’A’B ....(i)

Case I: A is a symmetric matrix, then $\Rightarrow$ A’ = A

$\therefore$ From eq. (i) (B’AB)’ = B’AB

$\therefore$ B’AB is a symmetric matrix.

Case II: A is a skew symmetric matrix. $\Rightarrow$ A’ = – A

Putting A’ = – A in eq. (i), (B’AB)’ = B’(– A)B = – B’AB

$\therefore$ B'AB is a skew symmetric matrix.

6. Find the values of x, y, z if the matrix A = $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies the equation A'A = I.

Ans. Given: A = $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$

$\Rightarrow$ A' = $\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$

As, A'.A = I

$$=> \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0+x^2+x^2 & 0+xy-xy & 0-xz+xz \\ 0+xy-xy & 4y^2+y^2+y^2 & 2yz-yz-yz \\ 0-zx+zx & 2yz-yz-yz & z^2+z^2+z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equating corresponding entries, we have

$$2x^2 = 1 \Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm\frac{1}{\sqrt{2}}$$

And $6y^2 = 1 \Rightarrow y^2 = \frac{1}{6} \Rightarrow y = \pm\frac{1}{\sqrt{6}}$

And $3z^2 = 1 \Rightarrow z^2 = \frac{1}{3} \Rightarrow z = \pm\frac{1}{\sqrt{3}}$

$\therefore \quad x = \pm\frac{1}{\sqrt{2}}, \; y = \pm\frac{1}{\sqrt{6}}, \; z = \pm\frac{1}{\sqrt{3}}$

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7. For what value of x, $\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\ 2 & 0 & 1\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix} = 0$?

Ans. Given: $\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\ 2 & 0 & 1\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix} = 0$

$=> \begin{bmatrix}1+4+1 & 2+0+0 & 0+2+2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix} = 0$

$=> \begin{bmatrix}6 & 2 & 4\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix}0+4+4x\end{bmatrix}_{1\times1} = 0 = \begin{bmatrix}0\end{bmatrix}_{1\times1}$

Equating corresponding entries, we have

$0+4+4x = 0$

$\Rightarrow 4x = -4$

$\Rightarrow x = -1$

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8. If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$.

Ans. Given: A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$\therefore\ A^2 - 5A + 7I = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 8-15 & 5-5 \\ -5+5 & 3-10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} -7+7 & 0+7 \\ 0+7 & -7+7 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

=> 0 = R.H.S.   Proved.

9. Find x, if $\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$.

Ans. Given: $\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} x-0-2 & 0-10-0 & 2x-5-3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

$$\Rightarrow \begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \left[(x-2)x - 10(4) + (2x-8)1\right] = 0$$

$$\Rightarrow \left[x^2 - 2x - 40 + 2x - 8\right] = 0$$

$$\Rightarrow \left[x^2 - 48\right]_{1\times 1} = [0]_{1\times 1}$$

Equating corresponding entries, we have

$$x^2 - 48 = 0$$

$$\Rightarrow \ x^2 = 48$$

$$\Rightarrow \ x = \pm 4\sqrt{3}$$

10. A manufacturer produces three products, x, y, z which he sells in two markets. Annual sales are indicated below:

Market	**Products**		
I	**10000**	**2,000**	**18,000**
II	**6000**	**20,000**	**8,000**

(a) If unit sales prices of x, y and z are Rs 2.50, Rs 1.50 and Rs 1.00 respectively, find the total revenue in each market with the help of matrix algebra.

(b) If the unit costs of the above three commodities are Rs 2.00, Rs 1.00 and 50 paise respectively. Find the gross profit.

Ans. According to question, the matrix $A = \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 10,000 & 2,000 & 18,000 \\ 6,000 & 20,000 & 8,000 \end{bmatrix} \end{array}$

(a) Let B be the column matrix representing sale price of each unit of products $x, y, z.$

Then B = $\begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}_{3\times1}$

Now Revenue = Sale price * Number of items sold

$$\Rightarrow \begin{bmatrix} 10,000 & 2,000 & 18,000 \\ 6,000 & 20,000 & 8,000 \end{bmatrix}\begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix} = \begin{bmatrix} 25,000+3,000+18,000 \\ 15,000+30,000+8,000 \end{bmatrix} = \begin{bmatrix} 46,000 \\ 53,000 \end{bmatrix}$$

Therefore, the revenue collected by sale of all items in Market I = Rsb 46,000 and the revenue collected by sale of all items in Market II = Rs 53,000.

(b) Let C be the column matrix representing cost price of each unit of products $x, y, z.$

Then C = $\begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}_{3\times1}$

$\therefore$ Total cost = AC = $\begin{bmatrix} 10,000 & 2,000 & 18,000 \\ 6,000 & 20,000 & 8,000 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$

= $\begin{bmatrix} 20,000+2,000+9,000 \\ 12,000+20,000+4,000 \end{bmatrix} = \begin{bmatrix} 31,000 \\ 36,000 \end{bmatrix}$

$\therefore$ The profit collected in two markets is given in matrix form as

Profit matrix = Revenue matrix – Cost matrix

$$\Rightarrow \begin{bmatrix} 46,000 \\ 53,000 \end{bmatrix} - \begin{bmatrix} 31,000 \\ 36,000 \end{bmatrix} = \begin{bmatrix} 15,000 \\ 17,000 \end{bmatrix}$$

Therefore, the gross profit in both the markets =Rs 15000 + Rs 17000 = Rs 32,000.

11. Find the matrix X so that X $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$.

Ans. Given: X $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$ ....(i)

Putting X = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ in eq. (i), $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

Equating corresponding entries, we have

$a+4b=-7$ .....(ii)

$2a+5b=-8$ .....(iii)

$3a+6b=-9$ .....(iv)

$c+4d=2$ .....(v)

$2c+5d=4$ .....(vi)

$3c+6d=6$ .....(vi)

Solving eq. (ii) and (iii), we have $a=1$ and $b=-2$

Solving eq. (v) and (vi), we have $c=2$ and $d=0$

Putting these values in X = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, X = $\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

12. If A and B are square matrices of the same order such that AB = BA, then prove by induction that $AB^n = B^nA$. Further prove that $(AB)^n = A^nB^n$ for all $n \in$ N.

Ans. Given: AB = BA .....(i)

Let $p(n)$: $AB^n = B^nA$ ......(ii)

For $n=1$, $p(1)$: becomes AB = BA

$\therefore$ $p(1)$ is true for $n=1$.

For $n=k$, $p(k)$: $AB^k = B^kA$

Multiplying both sides by B, $AB^kB = B^kAB$ $\Rightarrow$ $AB^{k+1} = B^kAB$

$\Rightarrow$ $AB^{k+1} = B^{k+1}A$ [From eq. (i)]

$\therefore$ $p(k+1)$ is also true.

Therefore, $p(n)$ is true for all $n \in$ N by P.M.I.

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13. If A = $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that A^2 = I, then:

(A) $1+\alpha^2+\beta\gamma=0$

(B) $1-\alpha^2+\beta\gamma=0$

(C) $1-\alpha^2-\beta\gamma=0$

(D) $1+\alpha^2-\beta\gamma=0$

Ans. Given: A = $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ and A^2 = I

$$\Rightarrow \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha^2+\beta\gamma & \alpha\beta-\alpha\beta \\ \alpha\gamma-\gamma\alpha & \beta\gamma+\alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha^2+\beta\gamma & 0 \\ 0 & \beta\gamma+\alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Equating corresponding entries, we have

$$\alpha^2+\beta\gamma = 1$$

$$\Rightarrow 1-\alpha^2-\beta\gamma = 0$$

Therefore, option (C) is correct.

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14. If the matrix A is both symmetric and skew symmetric, then:

(A) A is a diagonal matrix

(B) A is a zero matrix

(C) A is a square matrix

(D) None of these

Ans. Since, A is symmetric, therefore, A’ = A ……..(i)

And A is skew-symmetric, therefore, A’ = – A

$\Rightarrow$ A = – A [From eq. (i)]

$\Rightarrow$ A + A = 0 $\Rightarrow$ 2A = 0 $\Rightarrow$ A = 0

Therefore, A is zero matrix.

Therefore, option (B) is correct.

15. If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to:

(A) A

(B) I – A

(C) I

(D) 3A

Ans. Given: $A^2 = A$ .....(i)

Multiplying both sides by A, $A^3 = A^2 = A$ [From eq. (i)] ......(ii)

Also given $(I + A)^3 - 7A = I^3 + A^3 + 3I^2A + 3IA^2 - 7A$

Putting $A^2 = A$ [from eq. (i)] and $A^3 = A$ [from eq. (ii)],

$= I + A + 3IA + 3IA - 7A = I + A + 3A + 3A - 7A$ [$\because$ $IA = A$]

$= I + 7A - 7A = I$

Therefore, option (C) is correct.