CBSE Class–12 Mathematics

NCERT solution

Chapter - 3

Matrices - Exercise 3.1

1. In the matrix A = $\begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$**, write:**

(i) The order of the matrix.

(ii) The number of elements.

(iii) Write the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$

Ans. (i) There are 3 horizontal lines (rows) and 4 vertical lines (columns) in the given matrix A.

Therefore, Order of the matrix is 3 x 4.

(ii) The number of elements in the matrix A is 3 x 4 = 12.

(iii) $a_{13} \to$ Element in first row and third column = 19

$a_{21} \to$ Element in second row and first column = 35

$a_{33} \to$ Element in third row and third column = -5

$a_{24} \to$ Element in second row and fourth column = 12

$a_{23} \to$ Element in second row and third column = $\frac{5}{2}$

2. If a matrix has 24 elements, what are possible orders it can order? What, if it has 13 elements?

Ans. Since, a matrix having mn element is of order $m \times n$.

(i) Therefore, there are 8 possible matrices having 24 elements of orders 1 x 24, 2 x 12, 3 x 8, 4 x 6, 24 x 1, 12 x 2, 8 x 3, 6 x 4.

(ii) Prime number 13 = 1 x 13 and 13 x 1

Therefore, there are 2 possible matrices of order 1 x 13 (Row matrix) and 13 x 1 (Column matrix).

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3. If a matrix has 18 elements, what are the possible orders it can have? What if has 5 elements?

Ans. Since, a matrix having mn element is of order $m \times n$.

(i) Therefore, there are 6 possible matrices having 18 elements of orders 1 x 18, 2 x 9, 3 x 6, 18 x 1, 9 x 2, 6 x 3.

(ii) Prime number 5 = 1 x 5 and 5 x 1

Therefore, there are 2 possible matrices of order 1 x 5 (Row matrix) and 5 x 1 (Column matrix).

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4. Construct a 2 x 2 matrix A = $\left[a_{ij}\right]$ whose elements are given by:

(i) $a_{ij} = \frac{(i+j)^2}{2}$

(ii) $a_{ij} = \frac{i}{j}$

(iii) $a_{ij} = \frac{(i+2j)^2}{2}$

Ans. (i) Given: $a_{ij} = \frac{(i+j)^2}{2}$ ....(i)

Putting $i=1, j=1$ in eq. (i) $a_{11}=\frac{(1+1)^2}{2}=\frac{(2)^2}{2}=\frac{4}{2}=2$

Putting $i=1, j=2$ in eq. (i) $a_{12}=\frac{(1+2)^2}{2}=\frac{(3)^2}{2}=\frac{9}{2}$

Putting $i=2, j=1$ in eq. (i) $a_{11}=\frac{(2+1)^2}{2}=\frac{(3)^2}{2}=\frac{9}{2}$

Putting $i=2, j=2$ in eq. (i) $a_{11}=\frac{(2+2)^2}{2}=\frac{(4)^2}{2}=\frac{16}{2}=8$

$\therefore$ $A_{2 \times 2}$ = $\left[a_{ij}\right]=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}=\begin{bmatrix} 2 & 9/2 \\ 9/2 & 8 \end{bmatrix}$

(ii) Given: $a_{ij}=\frac{i}{j}$ ....(i)

Putting $i=1, j=1$ in eq. (i) $a_{11}=\frac{1}{1}=1$

Putting $i=1, j=2$ in eq. (i) $a_{12}=\frac{1}{2}$

Putting $i=2, j=1$ in eq. (i) $a_{11}=\frac{2}{1}=2$

Putting $i=2, j=2$ in eq. (i) $a_{11}=\frac{2}{2}=1$

$\therefore$ $A_{2 \times 2}$ = $\left[a_{ij}\right]=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}=\begin{bmatrix} 1 & 1/2 \\ 2 & 1 \end{bmatrix}$

(iii) Given: $a_{ij} = \dfrac{(i+2j)^2}{2}$ ....(i)

Putting $i=1, j=1$ in eq. (i) $a_{11} = \dfrac{(1+2)^2}{2} = \dfrac{(3)^2}{2} = \dfrac{9}{2}$

Putting $i=1, j=2$ in eq. (i) $a_{12} = \dfrac{(1+4)^2}{2} = \dfrac{(5)^2}{2} = \dfrac{25}{2}$

Putting $i=2, j=1$ in eq. (i) $a_{11} = \dfrac{(2+2)^2}{2} = \dfrac{(4)^2}{2} = \dfrac{16}{2} = 8$

Putting $i=2, j=2$ in eq. (i) $a_{11} = \dfrac{(2+4)^2}{2} = \dfrac{(6)^2}{2} = \dfrac{36}{2} = 18$

$\therefore$ $A_{2 \times 2} = \left[a_{ij}\right] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 9/2 & 25/2 \\ 8 & 18 \end{bmatrix}$

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5. Construct a 3 x 4 matrix, whose elements are given by:

(i) $a_{ij} = \dfrac{1}{2}\left|-3i + j\right|$

(ii) $a_{ij} = 2i - j$

Ans. (i) Given: $a_{ij} = \dfrac{1}{2}\left|-3i + j\right|$ ....(i)

Putting $i=1, j=1$ in eq. (i) $a_{11} = \dfrac{1}{2}\left|-3+1\right| = \dfrac{1}{2}\left|-2\right| = \dfrac{1}{2}(2) = 1$

Putting $i=1, j=2$ in eq. (i) $a_{12} = \dfrac{1}{2}\left|-3+2\right| = \dfrac{1}{2}\left|-1\right| = \dfrac{1}{2}(1) = \dfrac{1}{2}$

Putting $i=1, j=3$ in eq. (i) $a_{13}=\frac{1}{2}|-3+3|=\frac{1}{2}|0|=\frac{1}{2}(0)=0$

Putting $i=1, j=4$ in eq. (i) $a_{14}=\frac{1}{2}|-3+4|=\frac{1}{2}|1|=\frac{1}{2}(1)=\frac{1}{2}$

Putting $i=2, j=1$ in eq. (i) $a_{21}=\frac{1}{2}|-6+1|=\frac{1}{2}|-5|=\frac{1}{2}(5)=\frac{5}{2}$

Putting $i=2, j=2$ in eq. (i) $a_{22}=\frac{1}{2}|-6+2|=\frac{1}{2}|-4|=\frac{1}{2}(4)=2$

Putting $i=2, j=3$ in eq. (i) $a_{23}=\frac{1}{2}|-6+3|=\frac{1}{2}|-3|=\frac{1}{2}(3)=\frac{3}{2}$

Putting $i=2, j=4$ in eq. (i) $a_{24}=\frac{1}{2}|-6+4|=\frac{1}{2}|-2|=\frac{1}{2}(2)=1$

Putting $i=3, j=1$ in eq. (i) $a_{31}=\frac{1}{2}|-9+1|=\frac{1}{2}|-8|=\frac{1}{2}(8)=4$

Putting $i=3, j=2$ in eq. (i) $a_{32}=\frac{1}{2}|-9+2|=\frac{1}{2}|-7|=\frac{1}{2}(7)=\frac{7}{2}$

Putting $i=3, j=3$ in eq. (i) $a_{33}=\frac{1}{2}|-9+3|=\frac{1}{2}|-6|=\frac{1}{2}(6)=3$

Putting $i=3, j=4$ in eq. (i) $a_{34}=\frac{1}{2}|-9+4|=\frac{1}{2}|-5|=\frac{1}{2}(5)=\frac{5}{2}$

$$\therefore \; A_{3\times 4}=\left[a_{34}\right]=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}=\begin{bmatrix} 1 & 1/2 & 0 & 1/2 \\ 5/2 & 2 & 3/2 & 1 \\ 4 & 7/2 & 3 & 5/2 \end{bmatrix}$$

(ii) Given: $a_{ij}=2i-j$ ....(i)

Putting $i=1, j=1$ in eq. (i) $a_{11}=2-1=1$

Putting $i=1, j=2$ in eq. (i) $a_{12}=2-2=0$

Putting $i=1, j=3$ in eq. (i) $a_{13}=2-3=-1$

Putting $i=1, j=4$ in eq. (i) $a_{14}=2-4=-2$

Putting $i=2, j=1$ in eq. (i) $a_{21}=4-3=3$

Putting $i=2, j=2$ in eq. (i) $a_{22}=4-2=2$

Putting $i=2, j=3$ in eq. (i) $a_{23}=4-3=1$

Putting $i=2, j=4$ in eq. (i) $a_{24}=4-4=0$

Putting $i=3, j=1$ in eq. (i) $a_{31}=6-1=5$

Putting $i=3, j=2$ in eq. (i) $a_{32}=6-2=4$

Putting $i=3, j=3$ in eq. (i) $a_{33}=6-3=3$

Putting $i=3, j=4$ in eq. (i) $a_{34}=6-4=2$

$\therefore$ $A_{3 \times 4} = [a_{34}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$

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6. Find the values of x, y and z from the following equations:

(i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

(ii) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

(iii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Ans. (i) Given: $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

By definition of Equal matrices, $x = 1, y = 4, z = 3$

(ii) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

Equating corresponding entries, $x + y = 6$ ....(i)

$5 + z = 5 \Rightarrow z = 5 - 5 \Rightarrow z = 0$ ....(ii)

And $xy = 8 \Rightarrow x(6 - x) = 8$ [From eq. (i), $y = 6 - x$]

$\Rightarrow 6x - x^2 = 8$

$\Rightarrow x^2 - 6x + 8 = 0$

$\Rightarrow (x-4)(x-2) = 0$

$\Rightarrow x = 4$ or $x = 2$

Putting these values of x in eq. (i), we have $y = 2$ and $y = 4$

$\therefore x = 2, y = 4, z = 0$ or x = 4, y=2, z=0

(iii) Given: $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Equating corresponding entries, $x + y + z = 9$ ....(i)

$x + z = 5$ .... (ii)

And $y+z=7$ ....(iii)

Eq. (i) – Eq. (ii) = $y=$ 9 – 5 = 4

Eq. (i) – Eq. (iii) = $x=$ 9 – 7 = 2

Putting values of x and y in eq. (i),

$2+4+z=9 \Rightarrow z=3$

$\therefore \quad x=2, y=4, z=3$

7. Find the values of a, b, c and d from the equation $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$.

Ans. Equating corresponding entries,

$a-b=-1$ ....(i)

$2a-b=0$ ....(ii)

$2a+c=5$ ....(iii)

$3c+d=13$ ....(iv)

Eq. (i) – Eq. (ii) = $-a=-1$

$\Rightarrow \quad a=1$

Putting $a=1$ in eq. (i), $1-b=-1$

$\Rightarrow \quad -b=-2 \Rightarrow b=2$

Putting $a=1$ in eq. (iii), $2+c=5$

$\Rightarrow \quad c=5-2 \Rightarrow c=3$

Putting $c=3$ in eq. (iv), $9+d=13$

$\Rightarrow \quad d=13-9 \Rightarrow d=4$

$\therefore \quad a=1, b=2, c=3, d=4$

8. A = $\left[a_{ij}\right]_{m\times n}$ is a square matrix if:

(A) $m<n$ **(B)** $m>n$ **(C)** $m=n$ **(D) None of these**

Ans. By definition of square matrix $m=n$, option (C) is correct.

9. Which of the given values of x and y make the following pairs of matrices equal:

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

(A) $x=\frac{-1}{3}, y=7$

(B) Not possible to find

(C) $y=7, x=\frac{-2}{3}$

(D) $x=\frac{-1}{3}, y=\frac{-2}{3}$

Ans. Equating corresponding sides,

$3x+7=0 \Rightarrow x=\frac{-7}{3}$

And $5=y-2 \Rightarrow y=7$

Also $y+1=8 \Rightarrow y=7$

And $2-3x=4 \Rightarrow x=\frac{-2}{3}$

Since, values of x are not equal, therefore, no values of x and y exist to make the two matrices equal.

Therefore, option (B) is correct.

10. The number of all possible matrices of order 3 x 3 with each entry 0 or 1 is:

(A) 27

(B) 18

(C) 81

(D) 512

Ans. Since, general matrix of order 3 x 3 is $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$.

This matrix has 9 elements.

The number of choices for a_{11} is 2 (as 0 or 1 can be used)

Similarly, the number of choices for each other element is 2.

Therefore, total possible arrangements (matrices) = $2 \times 2 \times 2 \times \ldots 9$ times = $2^9 = 512$

Therefore, option (D) is correct.