CBSE Class–12 Mathematics

NCERT solution

Chapter - 4

Determinants - Exercise 4.4

1. Write minors and cofactors of the elements of the following determinants:

(i) $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$

(ii) $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

Ans. (i) Let $\Delta = \begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$

M_{11} = Minor of $a_{11} = |3| = 3$ and A_{11} = $(-1)^{1+1} M_{11} = (-1)^2 (3) = 3$

M_{12} = Minor of $a_{12} = |0| = 0$ and A_{12} = $(-1)^{1+2} M_{12} = (-1)^3 (0) = 0$

M_{21} = Minor of $a_{21} = |-4| = -4$ and A_{21} = $(-1)^{2+1} M_{21} = (-1)^3 (-4) = 4$

M_{22} = Minor of $a_{22} = |2| = 2$ and A_{22} = $(-1)^{2+2} M_{22} = (-1)^4 (2) = 2$

(ii) Let $\Delta = \begin{vmatrix} a & c \\ b & d \end{vmatrix}$

M_{11} = Minor of $a_{11} = |d| = d$ and A_{11} = $(-1)^{1+1} M_{11} = (-1)^2 (d) = d$

M_{12} = Minor of $a_{12} = |b| = b$ and A_{12} = $(-1)^{1+2} M_{12} = (-1)^3 (b) = -b$

M_{21} = Minor of $a_{21} = |c| = c$ and A_{21} = $(-1)^{2+1} M_{21} = (-1)^3 (c) = -c$

M_{22} = Minor of $a_{22} = |a| = a$ and A_{22} = $(-1)^{2+2} M_{22} = (-1)^4 (a) = a$

2. Write minors and cofactors of the elements of the following determinants:

(i) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

(ii) $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -5 \\ 0 & 2 & 2 \end{vmatrix}$

Ans. (i) Let $\Delta = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

M_{11} = Minor of $a_{11} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$ and A_{11} = $(-1)^{1+1} M_{11} = (-1)^2 (1) = 1$

M_{12} = Minor of $a_{12} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 - 0 = 0$ and A_{12} = $(-1)^{1+2} M_{12} = (-1)^3 (0) = 0$

M_{13} = Minor of $a_{13} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$ and A_{13} = $(-1)^{1+3} M_{13} = (-1)^4 (0) = 0$

M_{21} = Minor of $a_{21} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 - 0 = 0$ and A_{21} = $(-1)^{2+1} M_{21} = (-1)^3 (0) = 0$

M_{22} = Minor of $a_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$ and A_{22} = $(-1)^{2+2} M_{22} = (-1)^4 (1) = 1$

M_{23} = Minor of $a_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$ and A_{23} = $(-1)^{2+3} M_{23} = (-1)^5 (0) = 0$

M_{31} = Minor of $a_{31} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0 - 0 = 0$ and A_{31} = $(-1)^{3+1} M_{31} = (-1)^4 (0) = 0$

M_{32} = Minor of $a_{32} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$ and A_{32} = $(-1)^{3+2} M_{32} = (-1)^5 (0) = 0$

M_{33} = Minor of $a_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$ and A_{31} = $(-1)^{3+3} M_{33} = (-1)^6 (1) = 1$

(ii) Let $\Delta = \begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -5 \\ 0 & 2 & 2 \end{vmatrix}$

M_{11} = Minor of $a_{11} = \begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = 10 - (-1) = 11$ and A_{11} = $(-1)^{1+1} M_{11} = (-1)^2 (11) = 11$

M_{12} = Minor of $a_{12} = \begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = 6 - 0 = 6$ and A_{12} = $(-1)^{1+2} M_{12} = (-1)^3 (6) = -6$

M_{13} = Minor of $a_{13} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$ and A_{13} = $(-1)^{1+3} M_{13} = (-1)^4 (3) = 3$

M_{21} = Minor of $a_{21} = \begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} = 0 - 4 = -4$ and A_{21} = $(-1)^{2+1} M_{21} = (-1)^3 (-4) = 4$

M_{22} = Minor of $a_{22} = \begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2$ and A_{22} = $(-1)^{2+2} M_{22} = (-1)^4 (2) = 2$

M_{23} = Minor of $a_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$ and A_{23} = $(-1)^{2+3} M_{23} = (-1)^5 (1) = -1$

M_{31} = Minor of $a_{31} = \begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix} = 0 - 20 = -20$ and A_{31} = $(-1)^{3+1} M_{31} = (-1)^4 (-20) = -20$

M_{32} = Minor of $a_{32} = \begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix} = -1-12 = -13$ and A_{32} = $(-1)^{3+2} M_{32} = (-1)^5 (-13) = 13$

M_{33} = Minor of $a_{33} = \begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 5-0 = 5$ and A_{31} = $(-1)^{3+3} M_{33} = (-1)^6 (5) = 5$

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3. Using cofactors of elements of second row, evaluate: $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$.

Ans. Elements of second row of Δ are $a_{21} = 2, a_{22} = 0, a_{23} = 1$

A_{21} = Cofactor of $a_{21} = (-1)^{2+1} \begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix}$ = $(-1)^3 (9-16) = -(-7) = 7$

A_{22} = Cofactor of $a_{22} = (-1)^{2+2} \begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix}$ = $(-1)^4 (15-8) = 7$

A_{23} = Cofactor of $a_{23} = (-1)^{2+3} \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix}$ = $(-1)^5 (10-3) = -7$

$\therefore\ \Delta = a_{21} + A_{21} + a_{22}A_{22} + a_{23}A_{23}$ = $2(7) + 0(7) + 1(-7) = 14 - 7 = 7$

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4. Using cofactors of elements of third column, evaluate: $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$.

Ans. Elements of third column of Δ are $a_{13} = yz, a_{23} = zx, a_{33} = xy$

A_{13} = Cofactor of $a_{13} = (-1)^{1+3} \begin{vmatrix} 1 & y \\ 1 & z \end{vmatrix}$ = $(-1)^4 (z-y) = z-y$

A_{23} = Cofactor of $a_{23} = (-1)^{2+3} \begin{vmatrix} 1 & x \\ 1 & z \end{vmatrix} = (-1)^5 (z-x) = x-z$

A_{33} = Cofactor of $a_{33} = (-1)^{3+3} \begin{vmatrix} 1 & x \\ 1 & y \end{vmatrix} = (-1)^6 (y-x) = y-x$

$\therefore \ \Delta = a_{13} + A_{13} + a_{23}A_{23} + a_{33}A_{33} = yz(z-y) + zx(x-z) + xy(y-x)$

$= yz^2 - y^2z + zx^2 - xz^2 + xy^2 - x^2y = (yz^2 - y^2z) + (xy^2 - xz^2) + (xz^2 - x^2y)$

$= yz(z-y) + x(y^2 - z^2) - x^2(y-z) = (y-z)\left[-yz + x(y+z) - x^2\right]$

$= (y-z)(-yz + xy + xz - x^2) = (y-z)\left[-y(z-x) + x(z-x)\right]$

$= (y-z)(z-x)(-y+x) = (x-y)(y-x)(z-x)$

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5. If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ **and** A_{ij} **is cofactor of** a_{ij}, **then value of** Δ **is given by:**

(A) $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$

(B) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$

(C) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$

(D) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

Ans. Option (D) is correct.