CBSE Class–12 Mathematics

NCERT solution

Chapter - 1

Relations & Functions - Exercise 1.1

1. Determine whether each of the following relations are reflexive, symmetric and transitive:

(i) Relation R in the set A = {1, 2, 3, ……….. 13, 14} defined as R = $\{(x, y) : 3x - y = 0\}$ **.**

(ii) Relation R in the set N of natural numbers defined as R = {(x, y) : y = x + 5 and x < 4}

(iii) Relation R in the set A = {1, 2, 3, 4, 5, 6} as R = $\{(x, y) : y \text{ is divisible by } x\}$.

(iv) Relation R in the set Z of all integers defined as R = $\{(x, y) : x - y \text{ is an integer}\}$.

(v) Relation R in the set A of human beings in a town at a particular time given by:

(a) R = $\{(x, y) : x \text{ and } y \text{ work at the same place}\}$.

(b) R = $\{(x, y) : x \text{ and } y \text{ live in the same locality}\}$.

(c) R = $\{(x, y) : x \text{ is exactly 7 cm taller than } y\}$.

(d) R = $\{(x, y) : x \text{ is wife of } y\}$.

(e) R = $\{(x, y) : x \text{ is father of } y\}$.

Ans. (i) R = $\{(x, y) : 3x - y = 0\}$, in A = {1, 2, 3, 4, 5, 6, ……13, 14}

Clearly R = {(1, 3), (2, 6), (3, 9), (4, 12)}

Since, $(x, x) \notin$ R, $\therefore$ R is not reflexive.

Again $(x, y) \in$ R but $(y, x) \notin$ R $\therefore$ R is not symmetric.

Also (1, 3) $\in$ R and (3, 9) $\in$ R but (1, 9) $\notin$ R, $\therefore$ R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

(ii) R = $\{(x, y) : y = x + 5 \text{ and } x < 4\}$ in set N of natural numbers.

Clearly R = {(1, 6), (2, 7), (3, 8)}

Now $(x, x) \notin$ R, $\therefore$ R is not reflexive.

Again $(x, y) \in$ R but $(y, x) \notin$ R $\therefore$ R is not symmetric.

Also (1, 6) $\in$ R and (2, 7) $\in$ R but (1, 7) $\notin$ R, $\therefore$ R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

(iii) R = $\{(x, y) : y \text{ is divisible by } x\}$ in A = {1, 2, 3, 4, 5, 6}

Clearly R = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (5, 5), (6, 6)

Now (x, x) i.e., (1, 1), (2, 2) and (3, 3) $\in$ R $\therefore$ R is reflexive.

Again (x, y) i.e., (1, 2) $\in$ R but $(y, x) \notin$ R $\therefore$ R is not symmetric.

Also (1, 4) $\in$ R and (4, 4) $\in$ R and (1, 4) $\in$ R, $\therefore$ R is transitive.

Therefore, R is reflexive and transitive but not symmetric.

(iv) R = $\{(x, y) : x - y \text{ is an integer}\}$ in set Z of all integers.

Now (x, x) i.e., (1, 1) = 1 – 1 = 0 $\in$ Z $\therefore$ R is reflexive.

Again (x, y) $\in$ R and $(y, x) \in$ R, i.e., $x - y$ and $y - x$ are an integer $\therefore$ R is symmetric.

Also $(x_1, y_1) = x_1 - y_1$ $\in$ Z and $(y_1, z_1) = y_1 - z_1$ $\in$ Z and

(x_1, z_1) $\in$ R, $\therefore$ R is transitive.

Therefore, R is reflexive, symmetric and transitive.

(v) Relation R in the set A of human being in a town at a particular time.

(a) R = $\{(x, y): x \text{ and } y \text{ work at the same place}\}$

Since $(x, x) \in$ R, because x and x work at the same place. $\therefore$ R is reflexive.

Now, if $(x, y) \in$ R and $(x, y) \in$ R, since x and y work at the same place and y and x work at the same place. $\therefore$ R is symmetric.

Now, if $(x, y) \in$ R and $(y, z) \in$ R $\Rightarrow$ $(x, z) \in$ R. $\therefore$ R is transitive

Therefore, R is reflexive, symmetric and transitive.

(b) R = $\{(x, y): x \text{ and } y \text{ live in the same locality}\}$

Since $(x, x) \in$ R, because x and x live in the same locality. $\therefore$ R is reflexive.

Also $(x, y) \in$ R $\Rightarrow (y, x) \in$ R because x and y live in same locality and y and x also live in same locality. $\therefore$ R is symmetric.

Again $(x, y) \in$ R and $(y, z) \in$ R $\Rightarrow (x, z) \in$ R $\therefore$ R is transitive.

Therefore, R is reflexive, symmetric and transitive.

(c) R = $\{(x, y): x \text{ is exactly 7 cm taller than } y\}$

x is not exactly 7 cm taller than x, so $(x, x) \notin$ R $\therefore$ R is not reflexive.

Also x is exactly 7 cm taller than y but y is not 7 cm taller than x, so $(x, y) \in$ R but $(y, x) \notin$ R $\therefore$ R is not symmetric.

Now x is exactly 7 cm taller than y and y is exactly 7 cm taller than z then it does not imply that x is exactly 7 cm taller than z. $\therefore$ R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

(d) R = $\{(x, y) : x \text{ is wife of } y\}$

x is not wife of x, so $(x, x) \notin$ R $\therefore$ R is not reflexive.

Also x is wife of y but y is not wife of x, so $(x, y) \in$ R but $(y, x) \notin$ R $\therefore$ R is not symmetric.

Also $(x, y) \in$ R and $(y, z) \in$ R then $(x, z) \notin$ R $\therefore$ R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

(e) R = $\{(x, y) : x \text{ is father of } y\}$

x is not father of x, so $(x, x) \notin$ R $\therefore$ R is not reflexive.

Also x is father of y but y is not father of x,

so $(x, y) \in$ R but $(y, x) \notin$ R $\therefore$ R is not symmetric.

Also $(x, y) \in$ R and $(y, z) \in$ R then $(x, z) \notin$ R $\therefore$ R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

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2. Show that the relation R in the set R of real numbers defined as R = $\{(a, b) : a \leq b^2\}$ **is neither reflexive nor symmetric nor transitive.**

Ans. R = $\{(a, b) : a \leq b^2\}$, Relation R is defined as the set of real numbers.

(i) Whether $(a, a) \in$ R, then $a \leq a^2$ which is false. $\therefore$ R is not reflexive.

(ii) Whether $(a, b) = (b, a)$, then $a \leq b^2$ and $b \leq a^2$, it is false. $\therefore$ R is not symmetric.

(iii) Now $a \leq b^2$, $b \leq c^2 \Rightarrow a \leq c^4$, which is false. $\therefore$ R is not transitive

Therefore, R is neither reflexive, nor symmetric and nor transitive.

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3. Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as R = $\{(a, b) : b = a+1\}$ is reflexive, symmetric or transitive.

Ans. R = $\{(a, b) : b = a+1\}, a, b \in$ R

Now R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)} and $b = a+1$

(i) When $b = a \Rightarrow a = a+1$, which is false, so $(a, a) \notin$ R, $\therefore$ R is not reflexive.

(ii) Whether $(a, b) = (b, a)$, then $b = a+1$ and $a = b+1$, false $\therefore$ R is not symmetric.

(iii) Now if $(a, b) \in$ R, $(b, c) \in$ R $\Rightarrow (a, c) \in$ R

(iv) Then $b = a+1$ and $c = b+1 \Rightarrow c = a+2$ which is false. $\therefore$ R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

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4. Show that the relation R in R defined as R = $\{(a, b) : a \leq b\}$, is reflexive and transitive but not symmetric.

Ans. (i) $a \leq a$ which is true, so $(a, a) \in$ R, $\therefore$ R is reflexive.

(ii) $a \leq b$ but $b \leq a$ which is false. $\therefore$ R is not symmetric.

(iii) $a \leq b$ and $b \leq c \Rightarrow a \leq c$ which is true. $\therefore$ R is transitive.

Therefore, R is reflexive and transitive but not symmetric.

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5. Check whether the relation R in R defined by R = $\{(a, b) : a \leq b^3\}$ is reflexive, symmetric or transitive.

Ans. (i) For (a, a), $a < a^3$ which is false. $\therefore$ R is not reflexive.

(ii) For $(a, b), a < b^3$ and $(b, a), b < a^3$ which is false. $\therefore$ R is not symmetric.

(iii) For (a, b), (b, c) and (a, c), $a < a^9$ which is false. $\therefore$ R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

6. Show that the relation in the set A = {1, 2, 3} given by R = {(1, 2), (2, 1)} is symmetric but neither reflexive nor transitive.

Ans. R = {(1, 2), (2, 1)}, so for (a, a), (1, 1) $\notin$ R. $\therefore$ R is not reflexive.

Also if $(a, b) \in$ then $(b, a) \in$ R $\therefore$ R is symmetric.

Now $(a, b) \in$ R and $(b, c) \in$ then does not imply $(a, c) \notin$ R $\therefore$ R is not transitive..

Therefore, R is symmetric but neither reflexive nor transitive.

7. Show that the relation R in the set A of all the books in a library of a college, given by R = $\{(x, y) : x \text{ and } y \text{ have same number of pages}\}$ is an equivalence relation.

Ans. Books x and x have same number of pages $\Rightarrow$ $(x, x) \in$ R $\therefore$ R is reflexive.

If $(x, y) \in$ R $\Rightarrow$ $(y, x) \in$ R, so $(x, y) = (y, x)$ $\therefore$ R is symmetric.

Now if $(x, y) \in$ R, $(y, z) \in$ R $\Rightarrow$ $(x, z) \in$ R $\therefore$ R is transitive.

Since R is reflexive, symmetric and transistive, therefore, R is an equivalence relation.

8. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = $\{(a, b) : |a-b| \text{ is even}\}$, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}.

Ans. A = {1, 2, 3, 4, 5} and R = $\{(a, b) : |a-b| \text{ is even}\}$, then R = {(1, 3), (1, 5), (3, 5), (2, 4)}

(a) For (a, a), $|a-a| = 0$ which is even. $\therefore$ R is reflexive.

If $|a-b|$ is even, then $|b-a|$ is also even. $\therefore$ R is symmetric.

Now, if $|a-b|$ and $|b-c|$ is even then $|a-b+b-c| \Rightarrow |a-c|$ is also even. $\therefore$ R is transitive.

Therefore, R is an equivalence relation.

(b) Elements of {1, 3, 5} are related to each other.

Since $|1-3|=2, |3-5|=2, |1-5|=4,$ all are even numbers

$\Rightarrow$ Elements of {1, 3, 5} are related to each other.

Similarly elements of (2, 4) are related to each other.

Since $|2-4|=2$ an even number, then no element of the set {1, 3, 5} is related to any element of (2, 4).

Hence no element of {1, 3, 5} is related to any element of {2, 4}.

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9. Show that each of the relation R in the set A = $\{x \in Z : 0 \le x \le 12\}$, **given by:**

(i) R = $\{(a, b) : |a-b| \text{ is a multiple of } 4\}$

(ii) R = $\{(a, b) : a = b\}$ is an equivalence relation. Find the set of all elements related to 1 in each case.

Ans. (a) (i) A = $\{x \in Z : 0 \le x \le 12\}$ $\therefore$ A = {0, 1, 2, 3, ..., 12}

Now R = $\{(a, b) : |a-b| \text{ is a multiple of } 4\}$

$\therefore$ R = {(4, 0), (0, 4), (5, 1), (1, 5), (6, 2), (2, 6), ... (12, 9), (9, 12), ... (8, 0), (0, 8), ... (8, 4), (4, 8), ... (12, 12)}

Here, $|12-12| = |4-4| = |8-8| = 0$ is a multiple of 4.

$\therefore$ $(a, a) = |a-a| = 0$ is a multiple of 4. $\therefore$ R is reflexive.

Also we observe that $(a,b) = (b,a) = |a-b|$ $\therefore$ R is symmetric.

And $(a,b) = |a-b|, (b,c) = |b-c| \Rightarrow (a,c) = |a-c|$ is the multiple of 4. $\therefore$ R is transitive.

Hence R is an equivalence relation.

(ii) R = $\{(a,b): a=b\}$ and A = {0, 1, 2, 3, ………., 12}

$\therefore$ R = {(0, 0), (1, 1), (2, 2), …….. (12, 12)}

For (a,a), $a=a$ $\therefore$ R is reflexive.

As $a=b$, then $b=a$ $\therefore$ R is symmetric.

Also $a=b$, $b=c$ then $a=c$ $\therefore$ R is transitive.

Hence R is an equivalence relation.

(b) Now set of all elements related to 1 in each case.

(i) Required set = {1, 5, 9} (ii) Required set = {1}

10. Give an example of a relation, which is:

(i) Symmetric but neither reflexive nor transitive.

(ii) Transitive but neither reflexive nor symmetric.

(iii) Reflexive and symmetric but not transitive.

(iv) Reflexive and transitive but not symmetric.

(v) Symmetric and transitive but not reflexive.

Ans. (i) The relation "is perpendicular to" l_1 is not perpendicular to l_2.

If $l_1 \perp l_2$ then $l_2 \perp l_1$, however if $l_1 \perp l_2$ and $l_2 \perp l_3$ then l_1 is not perpendicular to l_3.

So it is clear that R "is perpendicular to" is a symmetric but neither reflexive nor transitive.

(ii) Relation R = $\left\{(x, y) : x > y\right\}$

We know that $x > x$ is false. Also $x > y$ but $y > x$ is false and if $x > y$, $y > z$ this implies $x > z$.

Therefore, R is transitive, but neither reflexive nor symmetric.

(iii) "is friend of" R = $\left\{(x, y) : x \text{ is a friend of } y\right\}$

It is clear that x is friend of x. $\therefore$ R is reflexive.

Also x is friend of y and y is friend of x. $\therefore$ R is symmetric.

Also if x is friend of y and y is friend of z then

x cannot be friend of z $\therefore$ R is not transitive.

Therefore, R is reflexive and symmetric but not transitive.

(iv) "is greater or equal to" R = $\left\{(x, y) : x \geq y\right\}$

It is clear that $x \geq x$ $\therefore$ R is reflexive.

And $x \geq y$ does not imply $y \geq x$ $\therefore$ R is not symmetric.

But $x \geq y$, $y \geq z \Rightarrow x \geq z$ $\therefore$ R is transitive.

Therefore, R is reflexive and transitive but not symmetric.

(v) "is brother of" R = $\left\{(x, y) : x \text{ is a brother of } y\right\}$

It is clear that x is not the brother of x. $\therefore$ R is not reflexive.

Also x is brother of y and y is brother of x. $\therefore$ R is symmetric.

Also if x is brother of y and y is brother of z then

x can be brother of z $\therefore$ R is transitive.

Therefore, R is symmetric and transitive but not reflexive.

11. Show that the relation R in the set A of points in a plane given by R = {(P. Q) : distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further, show that the set of all points related to a point P $\neq$ (0, 0) is the circle passing through P with origin as centre.

Ans. Part I: R = {(P, Q): distance of the point P from the origin is the same as the distance of the point Q from the origin}

Let P$\left(x_1, y_1\right)$ and Q$\left(x_2, y_2\right)$ and O (0, 0).

$\therefore$ OP = OQ $\Rightarrow \sqrt{x_1^2+y_1^2} = \sqrt{x_2^2+y_2^2} \Rightarrow x_1^2+y_1^2 = x_2^2+y_2^2$

Now, For (P, P), OP = OP $\therefore$ R is reflexive.

Also OP = OQ and OQ = OP $\Rightarrow$ (P, Q) = (Q, P) $\in$ R $\therefore$ R is symmetric.

Also OP = OQ and OQ = OR $\Rightarrow$ OP = OQ $\therefore$ R is transitive.

Therefore, R is an equivalent relation.

Part II: As $x_1^2+y_1^2 = x_2^2+y_2^2 = r^2$ (let) $\Rightarrow x^2+y^2=r^2$ which represents a circle with centre (0, 0) and radius r.

12. Show that the relation R defined in the set A of all triangles as R = {(T_1, T_2) : T_1 is similar to T_2}, is an equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

Ans. Part I: R = {(T_1, T_2): T_1 is similar to T_2} and T_1, T_2 are triangle.

We know that each triangle similar to itself and thus (T_1, T_2) $\in$ R $\therefore$ R is reflexive.

Also two triangles are similar, then $T_1 \cong T_2 \Rightarrow T_1 \cong T_2$ $\therefore$ R is symmetric.

Again, if then $T_1 \cong T_2$ and then $T_2 \cong T_3 \Rightarrow$ then $T_1 \cong T_3$ $\therefore$ R is transitive.

Therefore, R is an equivalent relation.

Part II: It is given that T_1, T_2 and T_3 are right angled triangles.

$\Rightarrow T_1$ with sides 3, 4, 5, T_2 with sides 5, 12, 13 and

T_3 with sides 6, 8, 10

Since, two triangles are similar if corresponding sides are proportional.

Therefore, $\frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{1}{2}$

Therefore, T_1 and T_3 are related.

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13. Show that the relation R defined in the set A of all polygons as R = {(P_1, P_2) : P_1 and P_2 have same number of sides}, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4, and 5?

Ans. Part I: R = {(P_1, P_2): P_1 and P_2 have same number of sides}

(i) Consider the element (P_1, P_2), it shows P_1 and P_2 have same number of sides. Therefore, R is reflexive.

(ii) If $(P_1, P_2) \in R$ then also $(P_2, P_1) \in R$

$\therefore$ $(P_1, P_2) = (P_2, P_1)$ as P_1 and P_2 have same number of sides, therefore, R is symmetric.

(iii) If $(P_1, P_2) \in R$ and $(P_2, P_3) \in R$ then also $(P_1, P_3) \in R$ as P_1, P_2 and P_3 have same number of sides, therefore, R is transitive.

Therefore, R is an equivalent relation.

Part II: we know that if 3, 4, 5 are the sides of a triangle, then the triangle is right angled triangle. Therefore, the set A is the set of right angled triangle.

14. Let L be the set of all lines in XY plane and R be the relation in L defined as R = {(L_1, L_2) : L_1 is parallel to L_2}. Show that R is an equivalence relation. Find the set of all lines related to the line $y=2x+4$.

Ans. Part I: R = {(L_1, L_2): L_1 is parallel to L_2}

(i) It is clear that $L_1 \parallel L_1$ i.e., $(L_1, L_1) \in R$ $\therefore$ R is reflexive.

(ii) If $L_1 \parallel L_2$ and $L_2 \parallel L_1$ then $(L_1, L_2) \in R$ $\therefore$ R is symmetric.

(iii) If $L_1 \parallel L_2$ and $L_2 \parallel L_3 \Rightarrow L_1 \parallel L_3$ $\therefore$ R is transitive.

Therefore, R is an equivalent relation.

Part II: All the lines related to the line $y=2x+4$ and $y=2x+k$ where k is a real number.

15. Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}. Choose the correct answer:

(A) R is reflexive and symmetric but not transitive.

(B) R is reflexive and transitive but not symmetric.

(C) R is symmetric and transitive but not reflexive.

(D) R is an equivalence relation.

Ans. Let R be the relation in the set {1, 2, 3, 4} is given by

R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}

(a) (1, 1), (2, 2), (3, 3), (4, 4) $\in$ R $\therefore$ R is reflexive.

(b) (1, 2) $\in$ R but (2, 1) $\notin$ R $\therefore$ R is not symmetric.

(c) If (1, 3) $\in$ R and (3, 2) $\in$ R then (1, 2) $\in$ R $\therefore$ R is transitive.

Therefore, option (B) is correct.

16. Let R be the relation in the set N given by R = $\{(a, b) : a = b - 2, b > 6\}$. **Choose the correct answer:**

(A) (2, 4) ∈ R

(B) (3, 8) ∈ R

(C) (6, 8) ∈ R

(D) (8, 7) ∈ R

Ans. Given: $a = b - 2, b > 6$

(A) $a = 2, b = 4$, Here $b > 6$ is not true, therefore, this option is incorrect.

(B) $a = 3, b = 8$ and $a = b - 2 \Rightarrow 3 = 8 - 2 \Rightarrow$ 3 = 6, which is false.

Therefore, this option is incorrect.

(C) $a = 6, b = 8$ and $a = b - 2 \Rightarrow 6 = 8 - 2 \Rightarrow$ 6 = 6, which is true.

Therefore, option (C) is correct.

(D) $a = 8, b = 7$ and $a = b - 2 \Rightarrow 8 = 7 - 2 \Rightarrow$ 8 = 5, which is false.