CBSE Class–12 Mathematics

NCERT solution

Chapter - 6

Application of Derivatives - Exercise 6.2

1. Show that the function given by $f(x) = 3x + 17$ is strictly increasing on R.

Ans. Given: $f(x) = 3x + 17$

$\therefore \quad f'(x) = 3(1) + 0 = 3 > 0$ i.e., positive for all $x \in$ R

Therefore, $f(x)$ is strictly increasing on R.

2. Show that the function given by $f(x) = e^{2x}$ is strictly increasing on R.

Ans. Given: $f(x) = e^{2x}$

$\therefore \quad f'(x) = e^{2x} \frac{d}{dx} 2x = e^{2x}(2) = 2e^{2x} > 0$ i.e., positive for all $x \in$ R

Therefore, $f(x)$ is strictly increasing on R.

3. Show that the function given by $f(x) = \sin x$ is (a) strictly increasing $\left(0, \frac{\pi}{2}\right)$, (b) strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$, (c) neither increasing nor decreasing in $(0, \pi)$.

Ans. Given: $f(x) = \sin x$

$\therefore \quad f'(x) = \cos x$

(a) Since, $f'(x) = \cos x > 0$, i.e., positive in first quadrant, i.e., in $\left(0, \frac{\pi}{2}\right)$.

Therefore, $f(x)$ is strictly increasing in $\left(0, \frac{\pi}{2}\right)$.

(b) Since, $f'(x) = \cos x$ < 0, i.e., negative in second quadrant, i.e., in $\left(\frac{\pi}{2}, \pi\right)$.

Therefore, $f(x)$ is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

(c) Since $f'(x) = \cos x$ > 0, i.e., positive in first quadrant, i.e., in $\left(0, \frac{\pi}{2}\right)$ and $f'(x) = \cos x$ < 0, i.e., negative in second quadrant, i.e., in $\left(\frac{\pi}{2}, \pi\right)$ and $f'\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0$.

$\therefore$ $f'(x)$ does not have the same sign in the interval $(0, \pi)$.

Therefore, $f(x)$ is neither increasing nor decreasing in $(0, \pi)$.

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4. Find the intervals in which the function f given by $f(x) = 2x^2 - 3x$ is (a) strictly increasing, (b) strictly decreasing.

Ans. Given: $f(x) = 2x^2 - 3x$

$\Rightarrow$ $f'(x) = 4x - 3$ ....(i)

Now $4x - 3 = 0$

$\Rightarrow$ $x = \frac{3}{4}$

Therefore, we have two disjoint sub intervals $\left(-\infty, \frac{3}{4}\right)$ and $\left(\frac{3}{4}, \infty\right)$.

(a) For interval $\left(\frac{3}{4}, \infty\right)$, taking $x = 1$, (say), then from eq. (i), $f'(x)$ > 0.

Therefore, f is strictly increasing in $\left(\frac{3}{4}, \infty\right)$.

(b) For interval $\left(-\infty, \frac{3}{4}\right)$, taking $x = 0.5$, (say), then from eq. (i), $f'(x)$ < 0.

Therefore, f is strictly decreasing in $\left(-\infty, \frac{3}{4}\right)$.

5. Find the intervals in which the function f given by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is (a) strictly increasing, (b) strictly decreasing.

Ans. (a) Given: $f(x) = 2x^3 - 3x^2 - 36x + 7$

$\Rightarrow \; f'(x) = 6x^2 - 6x - 36 = 6(x^2 - x - 6)$

$\Rightarrow \; f'(x) = 6(x+2)(x-3)$ ....(i)

Now $6(x+2)(x-3) = 0$

$\Rightarrow x + 2 = 0$ or $x - 3 = 0$

$\Rightarrow x = -2$ or $x = 3$

Therefore, we have three disjoint sub-intervals $(-\infty, -2), (-2, 3)$ and $(3, \infty)$.

For interval $(-\infty, -2)$, taking $x = -3$ (say), from eq. (i),

$f'(x) = (+)(-)(-) = (+)$ > 0

Therefore, f is strictly increasing in $(-\infty, -2)$.

For interval $(-2, 3)$, taking $x = 2$ (say), from eq. (i),

$f'(x) = (+)(+)(-) = (-)$ < 0

Therefore, f is strictly decreasing in $(-2, 3)$.

For interval $(3, \infty)$, taking $x = 4$ (say), from eq. (i),

$f'(x) = (+)(+)(+) = (+)$ > 0

Therefore, f is strictly increasing in $(3, \infty)$.

Hence, (a) f is strictly increasing in $(-\infty, -2)$ and $(3, \infty)$.

(b) f is strictly decreasing in $(-2, 3)$.

6. Find the intervals in which the following functions are strictly increasing or decreasing:

(a) x^2+2x-5

(b) $10-6x-2x^2$

(c) $-2x^3-9x^2-12x+1$

(d) $6-9x-x^2$

(e) $(x+1)^3(x-3)^3$

Ans. (a) Given: $f(x)=x^2+2x-5$

$\Rightarrow \ f'(x)=2x+2=2(x+1)$ ....(i)

Now $2(x+1)=0$

$\Rightarrow \ x=-1$

Therefore, we have two sub-intervals $(-\infty,-1)$ and $(-1,\infty)$.

For interval $(-\infty,-1)$ taking $x=-2$ (say), from eq. (i), $f'(x)=(-)<0$

Therefore, f is strictly decreasing.

For interval $(-1,\infty)$ taking $x=0$ (say), from eq. (i), $f'(x)=(+)>0$

Therefore, f is strictly increasing.

(b) Given: $f(x)=10-6x-2x^2$

$\Rightarrow \ f'(x)=-6-4x=-2(3+2x)$ ....(i)

Now $-2(3+2x)=0$

$\Rightarrow x = \frac{-3}{2}$

Therefore, we have two sub-intervals $\left(-\infty, \frac{-3}{2}\right)$ and $\left(\frac{-3}{2}, \infty\right)$.

For interval $\left(-\infty, \frac{-3}{2}\right)$ taking $x = -2$ (say), from eq. (i),

$f'(x) = (-)(-) = (+) > 0$

Therefore, f is strictly increasing.

For interval $\left(\frac{-3}{2}, \infty\right)$ taking $x = -1$ (say), from eq. (i),

$f'(x) = (-)(+) = (-) < 0$

Therefore, f is strictly decreasing.

(c) Given: $f(x) = -2x^3 - 9x^2 - 12x + 1$

$\Rightarrow f'(x) = -6x^2 - 18x - 12$

$\Rightarrow f'(x) = -6(x^2 + 3x + 2)$

$= -6(x+1)(x+2)$ ....(i)

Now $-6(x+1)(x+2) = 0$

$\Rightarrow x = -1$ or $x = -2$

Therefore, we have three disjoint intervals $(-\infty, -2)$, $(-2, -1)$ and $(-1, \infty)$.

For interval $(-\infty, -2)$, from eq. (i),

$f'(x) = (-)(-)(-) = (-) < 0$

Therefore, f is strictly decreasing.

For interval $(-2, -1)$, from eq. (i),

$f'(x) = (-)(-)(+) = (+) > 0$

Therefore, f is strictly increasing.

For interval $(-1, \infty)$, from eq. (i),

$f'(x) = (-)(+)(+) = (-) < 0$

Therefore, f is strictly decreasing.

(d) Given: $f(x) = 6 - 9x - x^2$

$\Rightarrow \; f'(x) = -9 - 2x$ ....(i)

Now $-9 - 2x = 0$

$\Rightarrow x = \frac{-9}{2}$

Therefore, we have two disjoint intervals $\left(-\infty, \frac{-9}{2}\right)$ and $\left(\frac{-9}{2}, \infty\right)$.

For interval $\left(-\infty, \frac{-9}{2}\right)$, taking x=-6 (say)

from eq. (i), $f'(x) = (+) > 0$

Therefore, f is strictly increasing.

For interval $\left(\frac{-9}{2}, \infty\right)$, taking x= 0 (say)

from eq. (i), $f'(x) = (-) < 0$

Therefore, f is strictly decreasing.

(e) Given: $f(x)=(x+1)^3(x-3)^3$

$\Rightarrow f'(x)=(x+1)^3.3(x-3)^2+(x-3)^3.3(x+1)^2$

$\Rightarrow f'(x)=3(x+1)^2(x-3)^2(x+1+x-3)$

$\Rightarrow f'(x)=3(x+1)^2(x-3)^2(2x-2)$

$\Rightarrow f'(x)=6(x+1)^2(x-3)^2(x-1)$

Here, factors $(x+1)^2$ and $(x-3)^2$ are non-negative for all x

Therefore, $f(x)$ is strictly increasing if $f'(x)>0$

$\Rightarrow x-1>0$

$\Rightarrow x>1$

And $f(x)$ is strictly decreasing if $f'(x)<0$

$\Rightarrow x-1<0$

$\Rightarrow x<1$

Hence, f is strictly increasing in $(1,\infty)$ and f is strictly decreasing in $(-\infty,1)$.

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7. Show that $y=\log(1+x)-\frac{2x}{2+x}, x>-1$ is an increasing function of x throughout its domain.

Ans. Given: $y=\log(1+x)-\frac{2x}{2+x}$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{1+x}\frac{d}{dx}(1+x) - \left[\frac{(2+x)\frac{d}{dx}(2x) - 2x\frac{d}{dx}(2+x)}{(2+x)^2}\right]$$

$$= \frac{1}{1+x} - \frac{(4+2x-2x)}{(2+x)^2}$$

$$= \frac{1}{1+x} - \frac{4}{(2+x)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(2+x)^2 - 4(1+x)}{(1+x)(2+x)^2}$$

$$= \frac{x^2}{(1+x)(2+x)^2} \quad ....(i)$$

Domain of the given function is given to be $x > -1$

$\Rightarrow x+1 > 0$

Also $(2+x)^2 > 0$ and $x^2 \geq 0$

$\therefore$ From eq. (i), $\frac{dy}{dx} \geq 0$ for all x in domain $x > -1$ and f is an increasing function.

8. Find the value of x for which $y = \{x(x-2)\}^2$ is an increasing function.

Ans. Given: $f(x) = y = (x(x-2))^2$

$$\Rightarrow \frac{dy}{dx} = 2x(x-2)\frac{d}{dx}[x(x-2)]$$

$\Rightarrow \frac{dy}{dx} = 2x(x-2)\left[x\frac{d}{dx}(x-2)+(x-2)\frac{d}{dx}x\right]$

[Applying Product Rule]

$\Rightarrow \frac{dy}{dx} = 2x(x-2)[x+x-2]$

= $2x(x-2)(2x-2)$

= $4x(x-2)(x-1)$ ....(i)

$\Rightarrow x=0, x=2, x=1$

Therefore, we have four disjoint subintervals $(-\infty,0),(0,1),(1,2),(2,\infty)$

For $(-\infty,0)$ taking $x=-1$ (say),

$\frac{dy}{dx} = (-)(-)(-) = (-) \leq 0$

$\therefore$ $f(x)$ is decreasing.

For $(0,1)$ taking $x=\frac{1}{2}$ (say),

$\frac{dy}{dx} = (+)(-)(-) = (+) \geq 0$

$\therefore$ $f(x)$ is increasing.

For $(1,2)$ taking $x=1.5$ (say),

$\frac{dy}{dx} = (+)(-)(+) = (-) \leq 0$

$\therefore$ $f(x)$ is decreasing.

For $(2,\infty)$ taking $x=3$ (say),

$$\frac{dy}{dx}=(+)(+)(+)=(+)\ \geq 0$$

$\therefore\ f(x)$ is increasing.

thus $f(x)$ is increasing in $(0,1)$ and $(2,\infty)$

$f(x)$ is decreasing in $(-\infty,0)$ and $(1,2)$

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9. Prove that $y=\dfrac{4\sin\theta}{(2+\cos\theta)}-\theta$ **is an increasing function of** θ **in** $\left[0,\dfrac{\pi}{2}\right]$.

Ans. Given: $y=\dfrac{4\sin\theta}{(2+\cos\theta)}-\theta$

$$\Rightarrow\ \frac{dy}{d\theta}=\frac{(2+\cos\theta).4\cos\theta-4\sin\theta(-\sin\theta)}{(2+\cos\theta)^2}-1$$

$$=\frac{8\cos\theta+4\cos^2\theta+4\sin^2\theta}{(2+\cos\theta)^2}-1$$

$$\Rightarrow\ \frac{dy}{d\theta}=\frac{8\cos\theta+4(\cos^2\theta+\sin^2\theta)-(2+\cos\theta)^2}{(2+\cos\theta)^2}$$

$$=\frac{8\cos\theta+4-(2+\cos\theta)^2}{(2+\cos\theta)^2}$$

$$\Rightarrow\ \frac{dy}{d\theta}=\frac{(8\cos\theta+4)-(4+4\cos\theta+\cos^2\theta)}{(2+\cos\theta)^2}$$

= $\dfrac{4\cos\theta - \cos^2\theta}{(2+\cos\theta)^2}$

= $\dfrac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2}$

Since $0 \le \theta \le \dfrac{\pi}{2}$ and we have $0 \le \cos\theta \le 1$, therefore $4-\cos\theta > 0$.

$\therefore \dfrac{dy}{d\theta} \ge 0$ for $0 \le \theta \le \dfrac{\pi}{2}$

Hence, y is an increasing function of θ in $\left[0, \dfrac{\pi}{2}\right]$.

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10. Prove that the logarithmic function is strictly increasing on $(0, \infty)$.

Ans. Given: $f(x) = \log x$

$\Rightarrow f'(x) = \dfrac{1}{x}$

for all x in $(0, \infty)$. $f'(x) > 0$

Therefore, $f(x)$ is strictly increasing on $(0, \infty)$.

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11. Prove that the function f given by $f(x) = x^2 - x + 1$ is neither strictly increasing nor strictly decreasing on $(-1, 1)$.

Ans. Given: $f(x) = x^2 - x + 1$

$\Rightarrow f'(x) = 2x - 1$

$f(x)$ is strictly increasing if $f'(x) > 0$

$\Rightarrow 2x-1>0$

$\Rightarrow x>\frac{1}{2}$

i.e., increasing on the interval $\left(\frac{1}{2},1\right)$

$f(x)$ is strictly decreasing if $f'(x)<0$

$\Rightarrow 2x-1<0$

$\Rightarrow x<\frac{1}{2}$

i.e., decreasing on the interval $\left(-1,\frac{1}{2}\right)$

hence, $f(x)$ is neither strictly increasing nor decreasing on the interval $(-1,1)$.

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12. Which of the following functions are strictly decreasing on $\left(0,\frac{\pi}{2}\right)$?

Ans. (A) $f(x)=\cos x$

$\Rightarrow f'(x)=-\sin x$

Since $0<x<\frac{\pi}{2}$

thus for x in $\left(0,\frac{\pi}{2}\right)$, sinx is positive in first quadrent

$\Rightarrow \sin x>0$

$\Rightarrow -\sin x<0$

So, $f'(x) < 0$

Therefore, $f(x)$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$.

(B) $f(x) = \cos 2x$

$\Rightarrow f'(x) = -2\sin 2x$

Since $0 < x < \frac{\pi}{2}$

$\therefore \ 0 < 2x < \pi$ therefore $\sin 2x > 0$

$\Rightarrow -2\sin 2x < 0$

So, $f'(x) < 0$

Therefore, $f(x)$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$.

(C) $f(x) = \cos 3x$

$\Rightarrow f'(x) = -3\sin 3x$

Since $0 < x < \frac{\pi}{2}$

$\therefore \ 0 < 3x < \frac{3\pi}{2}$

thus two cases $0 < 3x < \pi$ and $\pi < 3x < \frac{3\pi}{2}$

For $0 < 3x < \pi$

$\sin 3x > 0$

$\Rightarrow -3\sin 3x < 0$

So, $f'(x) < 0$

Therefore, $f(x)$ is strictly decreasing on $\left(0, \frac{\pi}{3}\right)$.

For $\pi < 3x < \frac{3\pi}{2}$

$\sin 3x < 0$

$\Rightarrow -3\sin 3x > 0$

So , $f'(x) > 0$

Therefore, $f(x)$ is strictly increasing on $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$.

Hence, $f(x)$ is neither strictly increasing not strictly decreasing on $\left(0, \frac{\pi}{2}\right)$.

(D) $f(x) = \tan x$

$\Rightarrow f'(x) = \sec^2 x > 0$

Therefore, $f(x)$ is strictly increasing on $\left(0, \frac{\pi}{2}\right)$.

13. On which of the following intervals is the function f given by $f(x) = x^{100} + \sin x - 1$ is strictly decreasing:

(A) (0, 1)

(B) $\left(\frac{\pi}{2}, \pi\right)$

(C) $\left(0, \frac{\pi}{2}\right)$

(D) None of these

Ans. Given: $f(x) = x^{100} + \sin x - 1$

$\Rightarrow f'(x) = 100x^{99} + \cos x$

(A) On (0, 1), $x > 0$ therefore $100x^{99} > 0$

And for $\cos x$

$\Rightarrow$ (0, 1 radian) = $(0, 57^{\circ} \text{ nearly})$ > 0

Therefore, $f(x)$ is strictly increasing on (0, 1).

(B) For $100x^{99}$ $x \in \left(\frac{\pi}{2}, \pi\right)$

= $\left(\frac{11}{7}, \frac{22}{7}\right)$ = (1.5, 3.1) > 1 and hence $100x^{99}$ > 100

For $\cos x$ $\left(\frac{\pi}{2}, \pi\right)$ is in second quadrant and hence $\cos x$ is negative and between -1 and 0.

Therefore, $f(x)$ is strictly increasing on $\left(\frac{\pi}{2}, \pi\right)$.

(C) On $\left(0, \frac{\pi}{2}\right)$ = (0, 1.5) both terms of given function are positive.

Therefore, $f(x)$ is strictly increasing on $\left(0, \frac{\pi}{2}\right)$.

(D) Option (D) is the correct answer.

14. Find the least value of a such that the function f given by $f(x)=x^2+ax+1$ strictly increasing on (1, 2).

Ans. $f(x)=x^2+ax+1$

$\Rightarrow \ f'(x)=2x+a$

Since $f(x)$ is strictly increasing on (1, 2), therefore $f'(x)=2x+a$ > 0 for all x in (1, 2)

$\therefore$ On (1, 2) $1<x<2$

$\Rightarrow 2<2x<4$

$\Rightarrow 2+a<2x+a<4+a$

$\therefore$ Minimum value of $f'(x)$ is $2+a$ and maximum value is $4+a$.

Since $f'(x)$ > 0 for all x in (1, 2)

$\therefore$ $2+a>0$ and $4+a>0$

$\Rightarrow a>-2$ and $a>-4$

Therefore least value of a is -2.

15. Let I be any interval disjoint from $[-1,1]$. Prove that the function f given by $f(x)=x+\frac{1}{x}$ is strictly increasing on I.

Ans. Given: $f(x)=x+\frac{1}{x}=x+x^{-1}$

$$\Rightarrow \ f'(x)=1+(-1)x^{-2}=1-\frac{1}{x^2}=\frac{x^2-1}{x^2}$$

$\Rightarrow f'(x) = \frac{(x-1)(x+1)}{x^2}$ ....(i)

Here for every x either $x < -1$ or $x > 1$

$\therefore$ for $x < -1$, $x = -2$ (say),

$f'(x) = \frac{(-)(-)}{(+)} = (+) > 0$

And for $x > 1$, $x = 2$ (say),

$f'(x) = \frac{(+)(+)}{(+)} = (+) > 0$

$\therefore$ $f'(x) > 0$ for all $x \in I$, hence $f(x)$ is strictly increasing on I

16. Prove that the function f given by $f(x) = \log \sin x$ is strictly increasing on $\left(0, \frac{\pi}{2}\right)$ and strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$.

Ans. Given: $f(x) = \log \sin x$

$\Rightarrow f'(x) = \frac{1}{\sin x}\frac{d}{dx}\sin x = \frac{1}{\sin x}\cos x = \cot x$

On the interval $\left(0, \frac{\pi}{2}\right)$ i.e., in first quadrant,

$f'(x) = \cot x > 0$

Therefore, $f(x)$ is strictly increasing on $\left(0, \frac{\pi}{2}\right)$.

On the interval $\left(\frac{\pi}{2}, \pi\right)$ i.e., in second quadrant,

$f'(x) = \cot x < 0$

Therefore, $f(x)$ is strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$.

17. Prove that the function f given by $f(x) = \log \cos x$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$ and strictly increasing on $\left(\frac{\pi}{2}, \pi\right)$.

Ans. Given: $f(x) = \log \cos x$

$$\Rightarrow \frac{1}{\cos x}\frac{d}{dx}\cos x = \frac{1}{\cos x}(-\sin x) = -\tan x$$

On the interval $\left(0, \frac{\pi}{2}\right)$ i.e., in first quadrant, $\tan x$ is positive, thus $f'(x) = -\tan x < 0$

Therefore, $f(x)$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$.

On the interval $\left(\frac{\pi}{2}, \pi\right)$ i.e., in second quadrant, $\tan x$ is negative thus $f'(x) = -\tan x > 0$

Therefore, $f(x)$ is strictly increasing on $\left(\frac{\pi}{2}, \pi\right)$.

18. Prove that the function given by $f(x) = x^3 - 3x^2 + 3x - 100$ is increasing in R.

Ans. Given: $f(x) = x^3 - 3x^2 + 3x - 100$

$$\Rightarrow f'(x) = 3x^2 - 6x + 3 = 3(x^2 - 2x + 1)$$

$\Rightarrow f'(x) = 3(x-1)^2 \geq 0$ for all x in R.

Therefore, $f(x)$ is increasing on R.

19. The interval in which $y = x^2 e^{-x}$ is increasing in:

(A) $(-\infty, \infty)$

(B) $(-2, 0)$

(C) $(2, \infty)$

(D) (0, 2)

Ans. Given: $f(x) = \left[y = x^2 e^{-x}\right]$

$$\Rightarrow \frac{dy}{dx} = x^2 \frac{d}{dx} e^{-x} + e^{-x} \frac{d}{dx} x^2$$

$$= x^2 e^{-x}(-1) + e^{-x}(2x)$$

$$\Rightarrow \frac{dy}{dx} = -x^2 e^{-x} + 2xe^{-x}$$

$$= xe^{-x}(-x+2)$$

$$\Rightarrow \frac{dy}{dx} = \frac{x(2-x)}{e^x}$$

In option (D), $\frac{dy}{dx} > 0$ for all x in the interval (0, 2).

Therefore, option (D) is correct.