

CBSE Class-12 Mathematics
NCERT solution
Chapter - 6
Application of Derivatives - Exercise 6.1

1. Find the rate of change of the area of a circle with respect to its radius r when

(a) $r = 3$ cm

(b) $r = 4$ cm

Ans. Let x denote the area of the circle of variable radius r .

$$\therefore \text{Area of circle } (x) = \pi r^2$$

\therefore Rate of change of area x w.r.t. r

$$\frac{dx}{dr} = \pi(2r) = 2\pi r$$

(a) When $r = 3$ cm, then

$$\frac{dx}{dr} = 2\pi(3) = 6\pi \text{ cm}^2/\text{sec}$$

(b) When $r = 4$ cm, then

$$\frac{dx}{dr} = 2\pi(4) = 8\pi \text{ cm}^2/\text{sec}$$

2. The volume of a cube is increasing at the rate of $8 \text{ cm}^3/\text{sec}$. How fast is the surface area increasing when the length of an edge is 12 cm?

Ans. Let x cm be the edge of the cube and y be the surface area of the cube at any time t

Given: Rate of increase of volume of cube = $8 \text{ cm}^3/\text{sec}$

$$\Rightarrow \frac{d}{dt}(x \cdot x \cdot x) = \frac{d}{dt}(x^3) \text{ is positive} = 8$$

$$\Rightarrow 3x^2 \frac{dx}{dt} = 8$$

$$\Rightarrow \frac{dx}{dt} = \frac{8}{3x^2} \dots\dots\dots(i)$$

$$y = 6x^2$$

$$\begin{aligned} \therefore \text{Rate of change of surface area of the cube} &= \frac{dy}{dt} = 6 \frac{d}{dt} x^2 \\ &= 6 \left(2x \frac{dx}{dt} \right) \\ &= 12x \left(\frac{8}{3x^2} \right) \quad \text{(from (i))} \\ &= 4 \left(\frac{8}{x} \right) = \frac{32}{x} \text{ cm}^2/\text{sec} \end{aligned}$$

Putting $x = 12$ cm (given),

$$\frac{dy}{dt} = \frac{32}{12} = \frac{8}{3} \text{ cm}^2/\text{sec}$$

Since $\frac{dy}{dt}$ is positive, therefore surface area is increasing at the rate of $\frac{8}{3} \text{ cm}^2/\text{sec}$.

3. The radius of the circle is increasing uniformly at the rate of 3 cm per second. Find the rate at which the area of the circle is increasing when the radius is 10 cm.

Ans. Let r cm be the radius and A be the area of the circle at any time t .

Rate of increase of radius of circle = 3 cm/sec

$$\Rightarrow \frac{dr}{dt} \text{ is positive and } = 3 \text{ cm/sec}$$

$$\frac{dr}{dt} = 3 \dots\dots (1) \dots\dots(i)$$

$$\text{Now, } A = \pi r^2$$

$$\therefore \text{Rate of change of area of circle} = \frac{dA}{dt} = \pi 2r \frac{dr}{dt}$$

$$\Rightarrow \pi 2r(3) = 6\pi r \text{ cm}^2/\text{sec} \quad (\text{from (i)})$$

putting $r = 10 \text{ cm}$ (given),

$$\frac{dA}{dt} = 6\pi(10) = 60\pi \text{ cm}^2/\text{sec}$$

Since $\frac{dA}{dt}$ is positive, therefore surface area is increasing at the rate of $60\pi \text{ cm}^2/\text{sec}$.

4. An edge of a variable cube is increasing at the rate of 3 cm per second. How fast is the volume of the cube increasing when the edge is 10 cm long?

Ans. Let $x \text{ cm}$ be the edge and y be the volume of the variable cube at any time t .

Rate of increase of edge = 3 cm/sec

$$\Rightarrow \frac{dx}{dt} \text{ is positive and } = 3 \text{ cm/sec}$$

$$\Rightarrow \frac{dx}{dt} = 3 \text{ cm/sec} \dots\dots(i)$$

$$\Rightarrow y = x^3$$

$$\therefore \text{Rate of change of volume of cube} = \frac{dy}{dt} = \frac{d}{dt} x^3$$

$$= 3x^2 \frac{dx}{dt} = 3x^2(3) \quad (\text{from (i)})$$

$$= 9x^2 \text{ cm}^3/\text{sec}$$

Putting $x = 10$ cm (given),

$$\frac{dy}{dt} = 9(10)^2 = 900 \text{ cm}^3/\text{sec}$$

Since $\frac{dy}{dt}$ is positive, therefore volume of cube is increasing at the rate of $900 \text{ cm}^3/\text{sec}$.

5. A stone is dropped into a quite lake and waves move in circles at the rate of 5 cm/sec. At the instant when radius of the circular wave is 8 cm, how fast is the enclosed area increasing?

Ans. Let x cm be the radius and y be the enclosed area of the circular wave at any time t .

Rate of increase of radius of circular wave = 5 cm/sec

$$\Rightarrow \frac{dx}{dt} \text{ is positive and } = 5 \text{ cm/sec}$$

$$\Rightarrow \frac{dx}{dt} = 5 \text{ cm / sec} \dots\dots\dots(i)$$

$$y = \pi x^2$$

$$\begin{aligned} \therefore \text{Rate of change of area} &= \frac{dy}{dt} = \pi \frac{d}{dt} x^2 \\ &= \pi \cdot 2x \frac{dx}{dt} = 2\pi x(5) && \text{(from (i))} \\ &= 10\pi x \text{ cm}^2/\text{sec} \end{aligned}$$

Putting $x = 8$ cm (given),

$$\frac{dy}{dt} = 10\pi(8) = 80\pi \text{ cm}^2/\text{sec}$$

Since $\frac{dy}{dt}$ is positive, therefore area of circular wave is increasing at the rate of 80π cm²/sec.

6. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference?

Ans. Let x cm be the radius and y be the circumference of the circle at any time t .

Rate of increase of radius of circle = 0.7 cm/sec

$$\Rightarrow \frac{dx}{dt} \text{ is positive and } = 0.7 \text{ cm/sec}$$

$$\Rightarrow \frac{dx}{dt} = 0.7 \text{ cm / sec} \dots\dots\dots(i)$$

$$\Rightarrow y = 2\pi x$$

$$\therefore \text{Rate of change of circumference of circle} = \frac{dy}{dt}$$

$$= 2\pi \frac{d}{dt} x = 2\pi(0.7) \quad (\text{from (i)})$$

$$= 1.4\pi \text{ cm/sec}$$

7. The length x of a rectangle is decreasing at the rate of 5 cm/minute and width y is increasing at the rate of 4 cm/minute. When $x = 8$ cm and $y = 6$ cm, find the rate of change of (a) the perimeter and (b) the area of the rectangle.

Ans. Given: Rate of decrease of length x of rectangle is 5 cm/minute.

$$\Rightarrow \frac{dx}{dt} \text{ is negative } = -5 \text{ cm/minute}$$

$$\Rightarrow \frac{dx}{dt} = -5 \text{ cm/minute} \dots\dots\dots(i)$$

Also, Rate of increase of width y of rectangle is 4 cm/minute

$$\Rightarrow \frac{dy}{dt} \text{ is positive} = 4 \text{ cm/minute}$$

$$\Rightarrow \frac{dy}{dt} = 4 \text{ cm/minute} \dots\dots\dots(\text{ii})$$

let z denotes the perimeter of rectangle at any time t

$$\therefore z = 2x + 2y$$

$$\Rightarrow \frac{dz}{dt} = 2 \frac{dx}{dt} + 2 \frac{dy}{dt} \quad (\text{from (i) and (ii)})$$

$$= 2(-5) + 2(4) = -2 \quad \text{which is negative.}$$

\therefore Perimeter of the rectangle is decreasing at the rate of 2 cm/min.

(b) Let z denotes the area of rectangle at any time t

$$\Rightarrow z = xy$$

$$\Rightarrow \frac{dz}{dt} = x \frac{dy}{dt} + y \frac{dx}{dt}$$

$$= 8(4) + 6(-5) = 2 \quad \text{which is positive.} \quad (\text{from (i) and (ii)})$$

\therefore Area of the rectangle is increasing at the rate of 2 cm²/min

8. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.

Ans. Let x cm be the radius of the spherical balloon at time t .

According to the question,
$$\frac{d}{dt} \left(\frac{4}{3} \pi x^3 \right) = 900$$

$$\Rightarrow \frac{4\pi}{3} \frac{d}{dt} x^3 = 900$$

$$\Rightarrow \frac{4\pi}{3} \cdot 3x^2 \frac{dx}{dt} = 900$$

$$\Rightarrow 4\pi x^2 = \frac{dx}{dt} = 900$$

$$\Rightarrow \frac{dx}{dt} = \frac{900}{4\pi x^2}$$

$$\Rightarrow \frac{dx}{dt} = \frac{900}{4\pi(15)^2}$$

$$\Rightarrow \frac{dx}{dt} = \frac{900}{4\pi(225)}$$

$$\Rightarrow \frac{dx}{dt} = \frac{900}{900\pi} = \frac{1}{\pi} \text{ cm sec}$$

\therefore Radius of balloon is increasing at the rate of $\frac{1}{\pi}$ cm sec.

9. A balloon, which always remains spherical has a variable radius. Find the rate at which its volume is increasing with the radius when the radius is 10 cm.

Ans. Since, $V = \frac{4}{3} \pi x^3$

$$\therefore \frac{dV}{dx} = \frac{d}{dx} \left(\frac{4}{3} \pi x^3 \right)$$

$$= \frac{4}{3} \pi \cdot 3x^2 = 4\pi x^2$$

as $x = 10$ cm

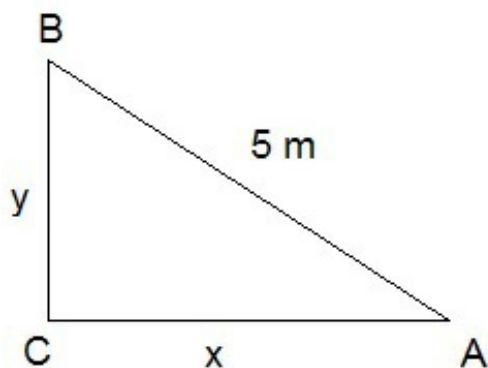
$$\Rightarrow \frac{dV}{dx} = 4\pi(10)^2 = 400\pi$$

Therefore, the volume is increasing at the rate of 400π cm³/sec.

10. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?

Ans. Let AB be the ladder and C is the junction of wall and ground, AB = 5 m

Let CA = x meters, CB = y meters



It is given that the bottom of the ladder is pulled along the ground away from the wall at the rate of 2 cm/sec

So x increases and y decreases

and $\frac{dx}{dt} = 2 \text{ cm/s}$

In $\triangle ABC$, $AC^2 + BC^2 = AB^2$ [Using Pythagoras theorem]

$$\Rightarrow x^2 + y^2 = 25 \quad \text{.....(i)}$$

$$\Rightarrow 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$\Rightarrow 2x(2) + 2y \frac{dy}{dt} = 0 \quad \left(\text{ as } \frac{dx}{dt} = 2 \text{ cm/s } \right)$$

$$\Rightarrow 2y \frac{dy}{dt} = -4x$$

$$\Rightarrow \frac{dy}{dt} = \frac{-2x}{y} \dots\dots\dots(ii)$$

When $x=4$, $16+y^2=25$ [From eq. (i)]

$$\Rightarrow y=3$$

$$\therefore \text{From eq. (ii), } \frac{dy}{dt} = \frac{-2 \times 4}{3} = \frac{-8}{3} \text{ cm/s}$$

-ve sign indicates that it is decreasing.

11. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.

Ans. Given: Equation of the curve $6y = x^3 + 2 \dots\dots\dots(i)$

Let (x, y) be the required point on curve (i)

since y-coordinate is changing 8 times as fast as the x-coordinate

$$\frac{dy}{dt} = 3x^2 \frac{dx}{dt} \dots\dots\dots(ii)$$

\therefore From eq. (i), $6y = x^3 + 2$

$$6 \frac{dy}{dt} = 3x^2 \frac{dx}{dt}$$

now from (ii) we substitute the value of $\frac{dy}{dt}$

$$\Rightarrow 6\left(8 \frac{dx}{dt}\right) = 3x^2 \frac{dx}{dt}$$

$$\Rightarrow 6 \times 8 = 3x^2 \quad \left(\text{we cancel } \frac{dx}{dt} \text{ from both side}\right)$$

$$\Rightarrow x^2 = \frac{6 \times 8}{3}$$

$$\Rightarrow x = \pm 4$$

Taking $x = 4$, $6y = 64 + 2$

$$\Rightarrow y = 11$$

\therefore Required point is (4, 11).

Taking $x = -4$, $6y = -64 + 2$

$$\Rightarrow y = \frac{-31}{3}$$

\therefore Required point is $\left(-4, \frac{-31}{3}\right)$.

so the required points are $\left(-4, \frac{-31}{3}\right)$ and (4, 11).

12. The radius of an air bubble is increasing at the rate of $\frac{1}{2}$ cm/s. At what rate is the volume of the bubble increasing when the radius is 1 cm?

Ans. Let x cm be the radius of the air bubble at time t .

According to question, $\frac{dx}{dt}$ is positive = $\frac{1}{2}$ cm/sec(i)

Volume of air bubble $(z) = \frac{4\pi}{3} x^3$

$$\Rightarrow \frac{dz}{dt} = \frac{4\pi}{3} \frac{d}{dt} x^3$$

$$= \frac{4\pi}{3} \cdot 3x^2 \frac{dx}{dt}$$

$$= 4\pi x^2 \left(\frac{1}{2}\right)$$

$$\Rightarrow \frac{dz}{dt} = 2\pi x^2$$

$$= 2\pi(1)^2 = 2\pi$$

Therefore, required rate of increase of volume of air bubble is 2π cm³/sec.

13. A balloon which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$. Find the rate of change of its volume with respect to x .

Ans. Given: Diameter of the balloon = $\frac{3}{2}(2x+1)$

$$\therefore \text{Radius of the balloon} = \frac{3}{4}(2x+1)$$

$$\therefore \text{Volume of the balloon} = \frac{4}{3}\pi\left(\frac{3}{4}(2x+1)\right)^3$$

$$= \frac{9\pi}{16}(2x+1)^3 \text{ cu. units}$$

$$\therefore \text{Rate of change of volume w.r.t. } x = \frac{dV}{dx}$$

$$= \frac{9\pi}{16} \cdot 3(2x+1)^2 \cdot \frac{d}{dx}(2x+1)$$

$$= \frac{27\pi}{16}(2x+1)^2 \cdot 2$$

$$= \frac{27\pi}{8}(2x+1)^2$$

14. Sand is pouring from a pipe at the rate of 12 cm³/s. The falling sand forms a cone on

the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4cm?

Ans. Let the height and radius of the sand-cone formed at time t second be y cm and x cm respectively.

According to question, $y = \frac{1}{6}x$

$$\Rightarrow x = 6y$$

$$\text{Volume of cone (V)} = \frac{1}{3}\pi x^2 y$$

$$= \frac{1}{3}\pi(6y)^2 y$$

$$= 12\pi y^3$$

$$\Rightarrow \frac{dV}{dt} = 36\pi y^2 \frac{dy}{dt}$$

$$\text{Now, since } \frac{dV}{dt} = 12$$

$$\Rightarrow 36\pi y^2 \times \frac{dy}{dt} = 12$$

$$\Rightarrow \frac{dy}{dt} = \frac{1}{3\pi y^2}$$

$$\Rightarrow \frac{dy}{dt} = \frac{1}{3\pi 4^2} = \frac{1}{48\pi} \text{ cm/sec}$$

15. The total cost $C(x)$ in rupees associated with the production of x units of an item given by $C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000$. Find the marginal cost when 17 units are produced.

Ans. Marginal cost = $\frac{dC}{dx}$

$$= \frac{d}{dx}(0.007x^3 - 0.003x^2 + 15x + 4000)$$
$$= 0.021x^2 - 0.006x + 15$$

Now, when $x = 17$, MC

$$= 0.021(17)^2 - 0.006 \times 17 + 15$$
$$= 6.069 - 0.102 + 15 = 20.967$$

Therefore, required Marginal cost is Rs 20.97

16. The total revenue in rupees received from the sale of x units of a product is given by

$R(x) = 13x^2 + 26x + 15$. Find the marginal revenue when $x = 7$.

Ans. Marginal Revenue (MR) = $\frac{dR}{dx}$

$$= \frac{d}{dx}(13x^2 + 26x + 15)$$
$$= 26x + 26$$

Now, when $x = 7$, $MR = 26 \times 7 + 26 = 208$

Therefore, the required marginal revenue is Rs 208

Choose the correct answer in Exercises 17 and 18.

17. The rate of change of the area of a circle with respect to its radius r at $r = 6$ cm is:

(A) 10π

(B) 12π

(C) 8π

(D) 11π

Ans. Area of circle (A) = πr^2

$$\Rightarrow \frac{dA}{dr} = 2\pi r$$

$$= 2\pi \times 6 = 12\pi$$

Therefore, option (B) is correct.

18. The total revenue in Rupees received from the sale of x units of a product is given by

$R(x) = 3x^2 + 36x + 5$. The marginal revenue, when $x = 15$ is:

(A) 116

(B) 96

(C) 90

(D) 126

Ans. Total revenue $R(x) = 3x^2 + 36x + 5$

$$\therefore \text{Marginal revenue} = \frac{d}{dx} R(x) = 6x + 36$$

$$= 6 \times 15 + 36 = 126$$

Therefore, option (D) is correct.
