

CBSE Class-12 Mathematics
NCERT solution
Chapter - 4
Determinants - Exercise 4.5

Find adjoint of each of the matrices in Exercise 1 and 2.

1. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Ans. Here $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$\Rightarrow |A| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

$$\therefore A_{11} = \text{Cofactor of } a_{11} = (-1)^2 (4) = 4$$

$$A_{12} = \text{Cofactor of } a_{12} = (-1)^3 (3) = -3$$

$$A_{21} = \text{Cofactor of } a_{21} = (-1)^3 (2) = -2$$

$$A_{22} = \text{Cofactor of } a_{22} = (-1)^4 (1) = 1$$

$$\therefore \text{adj. } A = \begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix}'$$

$$= \begin{vmatrix} 4 & -3 \\ -2 & 1 \end{vmatrix}'$$

$$= \begin{vmatrix} 4 & -2 \\ -3 & 1 \end{vmatrix}$$

2. $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$

Ans. Here $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$

$$\Rightarrow |A| = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{vmatrix}$$

$$A_{11} = + \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3$$

$$A_{12} = - \begin{vmatrix} 2 & 5 \\ -2 & 1 \end{vmatrix} = -(2+10) = -12$$

$$A_{13} = + \begin{vmatrix} 2 & 3 \\ -2 & 0 \end{vmatrix} = 6$$

$$A_{21} = - \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} - (-1) = 1$$

$$A_{22} = + \begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} = 1+4 = 5$$

$$A_{23} = - \begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = -(-2) = 2$$

$$A_{31} = + \begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} = -5-6 = -11$$

$$A_{32} = - \begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} = -(5-4) = -1$$

$$A_{33} = + \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 3+2=5$$

$$\therefore \text{adj. A} = \begin{vmatrix} 3 & -12 & 6 \\ 1 & 5 & 2 \\ -11 & -1 & 5 \end{vmatrix}'$$

$$= \begin{vmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{vmatrix}$$

Verify $A (\text{adj. A}) = (\text{adj. A}) A = |A| I$ in Exercise 3 and 4.

3. $\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

Ans. Let $A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

$$\Rightarrow \text{adj. A} = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$$

$$\Rightarrow A(\text{adj. A}) = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix} \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -12+12 & -6+6 \\ 24-24 & 12-12 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \dots(i)$$

Again $(\text{adj. A}). A = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

$$= \begin{bmatrix} -12+12 & -18+18 \\ 8-8 & 12-12 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \dots(ii)$$

$$\text{And } |A| = \begin{vmatrix} 2 & 3 \\ -4 & -6 \end{vmatrix} = 2(-6) - 3(-4) = -12 + 12 = 0$$

$$\text{Again } |A|I = |A|I_2 = (0) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \dots(iii)$$

\therefore From eq. (i), (ii) and (iii) $A \cdot (\text{adj. } A) = (\text{adj. } A) \cdot A = |A|I$

$$4. \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$\text{Ans. Let } A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$\Rightarrow |A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix}$$

$$\therefore A_{11} = + \begin{vmatrix} 0 & -2 \\ 0 & 3 \end{vmatrix} = +0+0=0 \quad A_{12} = - \begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} = -(9+2) = -11$$

$$A_{13} = + \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} = +(0-0)=0 \quad A_{21} = - \begin{vmatrix} -1 & 2 \\ 0 & 3 \end{vmatrix} = -(-3-0)=3$$

$$A_{22} = + \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 3-2=1 \quad A_{23} = - \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = -(0+1) = -1$$

$$A_{31} = + \begin{vmatrix} -1 & 2 \\ 0 & -2 \end{vmatrix} = 2 - 0 = 2 \quad A_{32} = - \begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} = -(-2 - 6) = 8$$

$$A_{33} = + \begin{vmatrix} 1 & -1 \\ 3 & 0 \end{vmatrix} = 3 + 0 = 3$$

$$\therefore \text{adj. A} = \begin{vmatrix} 0 & -11 & 0 \\ 3 & 1 & -1 \\ 2 & 8 & 3 \end{vmatrix}'$$

$$= \begin{vmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{vmatrix}$$

$$\therefore A. (\text{adj. A}) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+11+0 & 3-1-2 & 2-8+6 \\ 0-0-0 & 9+0+2 & 6+0-6 \\ 0+0+0 & 3+0-3 & 2+0+9 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} \dots\dots\dots(i)$$

$$\text{Again } (\text{adj. A}). A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+9+2 & 0+0+0 & 0-6+6 \\ -11+3+8 & 11+0+0 & -22-2+24 \\ 0-3+3 & 0-0+0 & 0+2+9 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} \dots\dots\dots(ii)$$

$$\text{And } |A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix}$$

$$= 1(0-0) - (-1)(9+2) + 2(0-0) = 0+11+0 = 11$$

$$\text{Also } |A|I = |A|I_3 = 11 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} \dots\dots\dots(iii)$$

\therefore From eq. (i), (ii) and (iii) $A \cdot (\text{adj. } A) = (\text{adj. } A) \cdot A = |A|I$

Find the inverse of the matrix (if it exists) given in Exercise 5 to 11.

$$5. \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$$

$$\text{Ans. Let } A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} 2 & -2 \\ 4 & 3 \end{vmatrix} = 6 - (-8) = 6 + 8 = 14 \neq 0$$

\therefore Matrix A is non-singular and hence A^{-1} exist.

$$\text{Now adj. } A = \begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix} \text{ And } A^{-1} = \frac{1}{|A|} \text{adj. } A = \frac{1}{14} \begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$$

6. $\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$

Ans. Let $A = \begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} -1 & 5 \\ -3 & 2 \end{vmatrix} = -2 - (-15) = -2 + 15 = 13 \neq 0$$

\therefore Matrix A is non-singular and hence A^{-1} exist.

Now $\text{adj. } A = \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$ And $A^{-1} = \frac{1}{|A|} \text{adj. } A = \frac{1}{13} \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$

7. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

Ans. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{vmatrix} = 1(10 - 0) - 2(0 - 0) + 3(-0) = 10 \neq 0$$

$\therefore A^{-1}$ exists.

$$A_{11} = + \begin{vmatrix} 2 & 4 \\ 0 & 5 \end{vmatrix} = +(10 - 0) = 10, \quad A_{12} = - \begin{vmatrix} 0 & 4 \\ 0 & 5 \end{vmatrix} = -(0 - 0) = 0,$$

$$A_{13} = + \begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix} = +(0 - 0) = 0, \quad A_{21} = - \begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix} = -(10 - 0) = -10,$$

$$A_{22} = + \begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix} = +(5-0) = 5, \quad A_{23} = - \begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} = -(0-0) = 0,$$

$$A_{31} = + \begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} = +(8-6) = 2, \quad A_{32} = - \begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix} = -(4-0) = -4,$$

$$A_{33} = + \begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = +(2-0) = 2$$

$$\therefore \text{adj. } A = \begin{vmatrix} 10 & 0 & 0 \\ -10 & 5 & 0 \\ 2 & -4 & 2 \end{vmatrix}'$$

$$= \begin{vmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{vmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj. } A = \frac{1}{10} \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

8. $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$

Ans. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{vmatrix} = 1(-3-0) - 0 + 0 = -3 \neq 0$$

$\therefore A^{-1}$ exists.

$$A_{11} = + \begin{vmatrix} 3 & 0 \\ 2 & -1 \end{vmatrix} = +(-3-0) = -3, A_{12} = - \begin{vmatrix} 3 & 0 \\ 5 & -1 \end{vmatrix} = -(-3-0) = 3,$$

$$A_{13} = + \begin{vmatrix} 3 & 3 \\ 5 & 2 \end{vmatrix} = +(6-15) = -9, A_{21} = - \begin{vmatrix} 0 & 0 \\ 2 & -1 \end{vmatrix} = -(0-0) = 0,$$

$$A_{22} = + \begin{vmatrix} 1 & 0 \\ 5 & -1 \end{vmatrix} = +(-1-0) = -1, A_{23} = - \begin{vmatrix} 1 & 0 \\ 5 & 2 \end{vmatrix} = -(2-0) = -2,$$

$$A_{31} = + \begin{vmatrix} 0 & 0 \\ 3 & 0 \end{vmatrix} = +(0-0) = 0, A_{32} = - \begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix} = -(0-0) = 0,$$

$$A_{33} = + \begin{vmatrix} 1 & 0 \\ 3 & 3 \end{vmatrix} = +(3-0) = 3$$

$$\therefore \text{adj. } A = \begin{bmatrix} -3 & 3 & -9 \\ 0 & -1 & -2 \\ 0 & 0 & 3 \end{bmatrix}^t = \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj. } A = \frac{-1}{3} = \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

9. $\begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$

Ans. Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{vmatrix} = 2\{(-1)-(4)\} + 3(8-7) = -3 \neq 0$$

$\therefore A^{-1}$ exists.

$$A_{11} = + \begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix} = +(-1-0) = -1, A_{12} = - \begin{vmatrix} 4 & 0 \\ -7 & 1 \end{vmatrix} = -(4-0) = -4,$$

$$A_{13} = + \begin{vmatrix} 4 & -1 \\ -7 & 2 \end{vmatrix} = +(8-7) = 1, A_{21} = - \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = -(1-6) = 5,$$

$$A_{22} = + \begin{vmatrix} 2 & 3 \\ -7 & 1 \end{vmatrix} = +(2+21) = 23, A_{23} = - \begin{vmatrix} 2 & 1 \\ -7 & 2 \end{vmatrix} = -(4+7) = -11,$$

$$A_{31} = + \begin{vmatrix} 1 & 3 \\ -1 & 0 \end{vmatrix} = +(0+3) = 3, A_{32} = - \begin{vmatrix} 2 & 3 \\ 4 & 0 \end{vmatrix} = -(0-12) = 12,$$

$$A_{33} = + \begin{vmatrix} 2 & 1 \\ 4 & -1 \end{vmatrix} = +(-2-4) = -6$$

$$\therefore \text{adj. } A = \begin{bmatrix} -1 & 4 & 1 \\ 5 & 23 & -11 \\ 3 & 12 & -6 \end{bmatrix}' = \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj. } A = \frac{-1}{3} = \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$$

10. $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$

Ans. Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{vmatrix} = 1(8-6) - (-1)(0+9) + 2(0-6) = -1 \neq 0$$

$\therefore A^{-1}$ exists.

$$A_{11} = + \begin{vmatrix} 2 & -3 \\ -2 & 4 \end{vmatrix} = +(8-6) = 2, A_{12} = - \begin{vmatrix} 0 & -3 \\ 3 & 4 \end{vmatrix} = -(0+9) = -9,$$

$$A_{13} = + \begin{vmatrix} 0 & 2 \\ 3 & -2 \end{vmatrix} = +(0-6) = -6, A_{21} = - \begin{vmatrix} -1 & 2 \\ -2 & 4 \end{vmatrix} = -(-4+4) = 0,$$

$$A_{22} = + \begin{vmatrix} 2 & 2 \\ 3 & 4 \end{vmatrix} = +(4-6) = -2, A_{23} = - \begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} = -(-2+3) = -1,$$

$$A_{31} = + \begin{vmatrix} -1 & 2 \\ 2 & -3 \end{vmatrix} = +(3-4) = -1, A_{32} = - \begin{vmatrix} 1 & 2 \\ 0 & -3 \end{vmatrix} = -(-3-0) = 3,$$

$$A_{33} = + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} = +(2-0) = 2$$

$$\therefore \text{adj. } A = \begin{bmatrix} 2 & -9 & -6 \\ 0 & -2 & -1 \\ -1 & 3 & 2 \end{bmatrix}' = \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj. } A = \frac{-1}{1} \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

11.
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & \sin \alpha & -\cos \alpha \end{bmatrix}$$

Ans. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & \sin \alpha & -\cos \alpha \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & \sin \alpha & -\cos \alpha \end{vmatrix}$$

$$= 1(-\cos^2 \alpha - \sin^2 \alpha) - 0 + 0 = -(\cos^2 \alpha + \sin^2 \alpha) = -1 \neq 0$$

$\therefore A^{-1}$ exists.

$$A_{11} = + \begin{vmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & -\cos \alpha \end{vmatrix} = +(-\cos^2 \alpha - \sin^2 \alpha),$$

$$= -(\cos^2 \alpha + \sin^2 \alpha) = -1$$

$$A_{12} = - \begin{vmatrix} 0 & \sin \alpha \\ 0 & -\cos \alpha \end{vmatrix} = -(0 - 0) = 0, A_{13} = + \begin{vmatrix} 0 & \cos \alpha \\ 0 & \sin \alpha \end{vmatrix} = +(0 - 0) = 0,$$

$$A_{21} = - \begin{vmatrix} 0 & 0 \\ \sin \alpha & -\cos \alpha \end{vmatrix} = -(0 - 0) = 0, A_{22} = + \begin{vmatrix} 1 & 0 \\ 0 & -\cos \alpha \end{vmatrix} = +(-\cos \alpha - 0) = -\cos \alpha$$

,

$$A_{23} = - \begin{vmatrix} 1 & 0 \\ 0 & \sin \alpha \end{vmatrix} = -(\sin \alpha - 0) = -\sin \alpha, A_{31} = + \begin{vmatrix} 0 & 0 \\ \cos \alpha & \sin \alpha \end{vmatrix} = (0 - 0) = 0,$$

$$A_{32} = - \begin{vmatrix} 1 & 0 \\ 0 & \sin \alpha \end{vmatrix} = -(\sin \alpha - 0) = -\sin \alpha, A_{33} = + \begin{vmatrix} 1 & 0 \\ 0 & \cos \alpha \end{vmatrix} = +(\cos \alpha - 0) = \cos \alpha$$

$$\therefore \text{adj. } A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos \alpha & -\sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos \alpha & -\sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj. } A = - \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos \alpha & -\sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & \sin \alpha & -\cos \alpha \end{bmatrix}$$

12. Let $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

Ans. Given: Matrix $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 3 & 7 \\ 2 & 5 \end{vmatrix} = 15 - 14 = 1 \neq 0$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj. } A = \frac{1}{1} \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

Matrix $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$

$$\therefore |B| = \begin{vmatrix} 6 & 8 \\ 7 & 9 \end{vmatrix} = 54 - 56 = -2 \neq 0$$

$$\therefore B^{-1} = \frac{1}{|B|} \text{adj. } B = \frac{1}{-2} \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$$

$$\text{Now } AB = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 18+49 & 24+63 \\ 12+35 & 16+45 \end{bmatrix} = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$$

$$\therefore |AB| = \begin{vmatrix} 67 & 87 \\ 47 & 61 \end{vmatrix} = 67(61) - 87(47) = 4087 - 4089 = -2 \neq 0$$

$$\text{Now L.H.S.} = (AB)^{-1} = \frac{1}{|AB|} \text{adj. } (AB) = \frac{1}{-2} \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix} \dots\dots\dots(i)$$

$$\text{R.H.S.} = B^{-1}A^{-1} = \frac{1}{-2} \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix} \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} 45+16 & -63-24 \\ -35-12 & 49+18 \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix} \dots\dots\dots(ii)$$

\therefore From eq. (i) and (ii), we get

$$\text{L.H.S.} = \text{R.H.S.}$$

$$\Rightarrow (AB)^{-1} = B^{-1}A^{-1}$$

13. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$. Hence find A^{-1} .

$$\text{Ans. Given: } A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\therefore A^2 = A.A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$\text{L.H.S.} = A^2 - 5A + 7I = A^2 - 5A + 7I_2$$

$$\begin{aligned}
 &= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\
 &= \begin{bmatrix} 8-15 & 5-5 \\ -5+5 & 3-10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\
 &= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\
 &= \begin{bmatrix} -7+7 & 0+0 \\ 0+0 & -7+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0
 \end{aligned}$$

= R.H.S.

$$\Rightarrow A^2 - 5A + 7I_2 = 0 \dots\dots(i)$$

To find: A^{-1} , multiplying eq. (i) by A^{-1} .

$$\Rightarrow A^2 A^{-1} - 5A A^{-1} + 7I_2 A^{-1} = 0 A^{-1}$$

$$\Rightarrow A - 5I_2 + 7A^{-1} = 0$$

$$\Rightarrow 7A^{-1} = -A + 5I_2$$

$$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{7} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

14. For the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find numbers a and b such that $A^2 + aA + bI = 0$.

Ans. Given: $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

$$\therefore A^2 = A.A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9+2 & 6+2 \\ 3+1 & 2+1 \end{bmatrix} = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$$

$$\therefore A^2 + aA + bI_2 = 0$$

$$\Rightarrow \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + a \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + b \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 3a & 2a \\ a & a \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 11+3a+b & 8+2a+0 \\ 4+a+0 & 3+a+b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\therefore \text{We have } 11+3a+b=0 \dots\dots\dots(i)$$

$$8+2a+0=0 \dots\dots\dots(ii)$$

$$\Rightarrow 2a = -8$$

$$\Rightarrow a = -4$$

Here $a = -4$ satisfies $4+a+0=0$ also, therefore $a = -4$

Putting $a = -4$ in eq. (i), $11-12+b=0 \Rightarrow b-1=0 \Rightarrow b=1$

Here also $b=1$ satisfies $3+a+b=0$, therefore $b=1$

Therefore, $a = -4$ and $b = 1$

15. For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$, show that $A^3 - 6A^2 + 5A + 11I = 0$. Hence find A^{-1} .

Ans. Given: $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

$$\therefore A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1+1+2 & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

Now $A^3 = A^2A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 4+2+2 & 4+4-1 & 4-6+3 \\ -3+8-28 & -3+16+14 & -3-24-42 \\ 7-3+28 & 7-6-14 & 7+9+42 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$

$$\text{L.H.S.} = A^3 - 6A^2 + 5A + 11I$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - 6 \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 5 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 11 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 8-24+5 & 7-12+5 & 1-6+5 \\ -23+18+5 & 27-48+10 & -69+84-15 \\ 32-42+10 & -13+18-5 & 58-84+15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & 0 & 0 \\ 0 & -11 & 0 \\ 0 & 0 & -11 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 = \text{R.H.S.}$$

Now, to find A^{-1} , multiplying $A^3 - 6A^2 + 5A + 11I = 0$ by A^{-1}

$$\Rightarrow A^3 A^{-1} - 6A^2 A^{-1} + 5A A^{-1} + 11I A^{-1} = 0 A^{-1}$$

$$\Rightarrow A^2 - 6A + 5I + 11A^{-1} = 0$$

$$\Rightarrow 11A^{-1} = 6A - 5I - A^2$$

$$\Rightarrow 11A^{-1} = 6 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} - 5 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$\Rightarrow 11A^{-1} = \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & -6 & 18 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} - \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$\Rightarrow 11A^{-1} = \begin{bmatrix} 6-5-4 & 6-2 & 6-1 \\ 6+3 & 12-5-8 & -18+14 \\ 12-7 & -6+3 & 18-5-14 \end{bmatrix}$$

$$\Rightarrow 11A^{-1} = \begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{11} \begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

16. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that $A^3 - 6A^2 + 9A + 4I = 0$ and hence find A^{-1} .

Ans. Given: $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$$\therefore A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 4+1+1 & -2-2-1 & 1+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$\text{Now } A^3 = A^2 A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5-10-6 & 5+5+12 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

$$\text{L.H.S.} = A^3 - 6A^2 + 9A - 4I$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6 \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9 \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix} + \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 22-36 & -21+30 & 21-30 \\ -21+30 & 22-36 & -21+30 \\ 21-30 & -21+30 & 22-36 \end{bmatrix} + \begin{bmatrix} 18-4 & -9-0 & 9-0 \\ -9-0 & 18-4 & -9-0 \\ 9-0 & -9-0 & 18-4 \end{bmatrix}$$

$$= \begin{bmatrix} -14 & 9 & -9 \\ 9 & -14 & 9 \\ -9 & 9 & -14 \end{bmatrix} + \begin{bmatrix} -14 & 9 & -9 \\ 9 & -14 & 9 \\ -9 & 9 & -14 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 = \text{R.H.S.}$$

Now, to find A^{-1} , multiplying $A^3 - 6A^2 + 9A - 4I$ by A^{-1}

$$\Rightarrow A^3 A^{-1} - 6A^2 A^{-1} + 9A A^{-1} - 4I A^{-1} = 0 A^{-1}$$

$$\Rightarrow A^2 - 6A + 9I - 4A^{-1} = 0$$

$$\Rightarrow 4A^{-1} = A^2 - 6A + 9I$$

$$\Rightarrow 4A^{-1} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6 \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow 4A^{-1} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -6 & 6 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\Rightarrow 4A^{-1} = \begin{bmatrix} 6-12+9 & -5+6+0 & 5-6+0 \\ -5+6+0 & 6-12+9 & -5+6+0 \\ 5-6+0 & -5+6+0 & 6-12+9 \end{bmatrix} = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{4} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

17. Let A be a non-singular matrix of order 3 x 3. Then $|\text{adj. } A|$ is equal to:

(A) $|A|$

(B) $|A|^2$

(C) $|A|^3$

(D) $3|A|$

Ans. If A is a non-singular matrix of order $n \times n$, then $|\text{adj. } A| = |A|^{n-1}$

\therefore Putting $n = 3$, $|\text{adj. } A| = |A|^2$

Therefore, option (B) is correct.

18. If A is an invertible matrix of order 2, then $\det (A^{-1})$ is equal to:

(A) $\det A$

(B) $\frac{1}{\det A}$

(C) 1

(D) 0

Ans. Since $AA^{-1} = I$

$$\therefore |AA^{-1}| = |I|$$

$$\Rightarrow |A||A^{-1}| = 1$$

$$\Rightarrow |A^{-1}| = \frac{1}{|A|}$$

Therefore, option (B) is correct.