CBSE Class–12 Mathematics

NCERT solution

Chapter - 1

Relations & Functions -Exercise 1.3

1. Let f : {1, 3, 4} $\rightarrow$ {1, 2, 5} and g : {1, 2, 5} $\rightarrow$ {1, 3} be given by f = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}. Write down gof.

Ans. f = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}

Now, $f(1)=2, f(3)=5, f(4)=1$ and $g(1)=3, g(2)=3, g(5)=1$

$(gof)(n)=g[f(x)]=g[f(1)]=g(2)=3$

$g[f(3)]=g(5)=1$ and $g[f(4)]=g(1)=3$

Hence, $gof =$ {(1, 3), (3, 1), (4, 3)}

2. Let f, g and h be functions from R $\rightarrow$ R. Show that:

$(f+g)oh = foh + goh$

$(f.g)\,oh = (foh).(goh)$

Ans. (a) To prove: $(f+g)oh = foh + goh$

L. H. S. = $(f+g)oh = (f+g)[h(x)]$ = $f[h(x)]+g[h(x)]$ = $foh + goh$ = R. H. S.

(b) To prove: $(f.g)\,oh = (foh).(goh)$

L. H. S. = $(f.g)oh = (f.g)[h(x)]$ = $f[h(x)].g[h(x)]$ = $foh.goh$ = R. H. S.

3. Find gof and fog, if:

(i) $f(x) = |x|$ **and** $g(x) = |5x - 2|$

(ii) $f(x) = 8x^3$ **and** $g(x) = x^{\frac{1}{3}}$

Ans. To find: gof and fog

(i) $f(x) = |x|$ and $g(x) = |5x - 2|$

$gof = g[f(x)] = g[|x|]$ and $fog = f[g(x)] = f[(5x-2)]$ = $|5x-2|$ = $|5|x|-2|$

(ii) $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$

$gof = g[f(x)] = g[8x^3]$ $(8x^3)^{\frac{1}{3}} = 2x$

and $fog = f[g(x)] = f\left[\left(x^{\frac{1}{3}}\right)\right]$ = $8\left(x^{\frac{1}{3}}\right)^3$ $= 8x$

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4. If $f(x) = \dfrac{(4x+3)}{(6x-4)}, x \neq \dfrac{2}{3}$, **show that** $fof(x) = x$, **for all** $x \neq \dfrac{2}{3}$. **What is the inverse of** f**?**

Ans. Given: $f(x) = \dfrac{(4x+3)}{(6x-4)}, x \neq \dfrac{2}{3}$

L.H.S. = $fof(x) = f[f(x)]$ = $f\left[\dfrac{4x+3}{6x-4}\right]$ = $\dfrac{4\left(\dfrac{4x+3}{6x-4}\right)+3}{6\left(\dfrac{4x+3}{6x-4}\right)-4}$ = $\dfrac{16x+12+18x-12}{24x+18-24x+16}$

= $\dfrac{34x}{34} = x$ = R.H.S.

Now, $y=\dfrac{4x+3}{6x-4}$

$\Rightarrow 6xy-4y=4x+3$

$\Rightarrow 6xy-4x=4y+3$

$\Rightarrow x(6y-4)=4y+3$

$\Rightarrow x=\dfrac{4y+3}{6y-3}$

$\Rightarrow y=\dfrac{4x+3}{6x-3}$

Hence inverse of $f=f$.

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5. State with reason whether following functions have inverse:

(i) f : {1, 2, 3, 4} $\rightarrow$ {10} with f = {(1, 10), (2, 10), (3, 10), (4, 10)}

(ii) g : {5, 6, 7, 8} $\rightarrow$ {1, 2, 3, 4} with g = {(5, 4), (6, 3), (7, 4), (8, 2)}

(iii) h : {2, 3, 4, 5} $\rightarrow$ {7, 9, 11, 13} with h = {(2, 7), (3, 9), (4, 11), (5, 13)}

Ans. (i) f = {(1, 10), (2, 10), (3, 10), (4, 10)}

It is many-one function, therefore f has no inverse.

(ii) g = {(5, 4), (6, 3), (7, 4), (8, 2)}

It is many-one function, therefore g has no inverse.

(iii) h = {(2, 7), (3, 9), (4, 11), (5, 13)}

h is one-one onto function, therefore, h has an inverse.

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6. Show that $f:[-1,1]\rightarrow$ R given by $f(x)=\frac{x}{(x+2)}$ is one-one. Find the inverse of the function $f:[-1,1]\rightarrow$ Range f.

Ans. Part I: $f:[-1,1]\rightarrow$ R given by $f(x)=\frac{x}{(x+2)}, x\neq -2$

Let $x_1, x_2 \in [-1,1]$, then $f(x_1)=\frac{x_1}{x_1+2}$ and $f(x_2)=\frac{x_2}{x_2+2}$

When $f(x_1)=f(x_2)$ then $\frac{x_1}{x_1+2}=\frac{x_2}{x_2+2}$

$\Rightarrow x_1x_2+2x_1=x_1x_2+2x_2$

$\Rightarrow x_1=x_2 \therefore f$ is one-one.

Part II: Let $y\in$ Range of f

$\Rightarrow y=f(x)=\frac{x}{x+2}$ for some x in $[-1,1]$

As $y=\frac{x}{x+2}$

$\Rightarrow yx+2y=x$

$\Rightarrow x(1-y)=2y$

$\Rightarrow x=\frac{2y}{1-y}$

$\Rightarrow f^{-1}(y)=\frac{2y}{1-y} \quad \therefore f$ is onto.

Therefore, $f^{-1}x=\frac{2x}{1-x}$

7. Consider f : R $\rightarrow$ R given by $f(x)=4x+3$. Show that f is invertible. Find the inverse of f. [Hint: $f^{-1}(y) = \frac{y-3}{4}$]

Ans. Consider f : R $\rightarrow$ R given by $f(x)=4x+3$

Let $x_1, x_2 \in$ R, then $f(x_1)=4x_1+3$ and $f(x_2)=4x_2+3$

Now, for $f(x_1)=f(x_2)$, then $4x_1+3=4x_2+3 \Rightarrow x_1=x_2 \therefore f$ is one-one.

Let $y \in$ Range of f

$\Rightarrow y=4x+3$

$\Rightarrow x=\dfrac{y-3}{4}$

$\therefore f\left(\dfrac{y-3}{4}\right)=4\left(\dfrac{y-3}{4}\right)+3=y$

$\Rightarrow f(x)=y \therefore f$ is onto.

Therefore, f is invertible and hence, $x=f^{-1}(y)=\dfrac{y-3}{4}$.

8. Consider f: $R_+ \rightarrow [4,\infty]$ given by $f(x)=x^2+4$. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(y)=\sqrt{y-4}$, where R_+ is the set of all non-negative real numbers.

Ans. Consider f: $R_+ \rightarrow [4,\infty]$ and $f(x)=x^2+4$.

Let $x_1, x_2 \in$ R $\rightarrow [4,\infty]$, then $f(x_1)=x_1^2+4$ and $f(x_2)=x_2^2+4$

$\Rightarrow x_1^2+4=x_2^2+4$

$\Rightarrow x_1=x_2 \therefore f$ is one-one.

Now $y = x^2 + 4$

$\Rightarrow x = \sqrt{y-4}$ as $x > 0$

$\therefore f\left(\sqrt{y-4}\right) = \left(\sqrt{y-4}\right)^2 + 4 = y$

$\Rightarrow f(x) = y \quad \therefore f$ is onto.

Therefore, $f(x)$ is invertible and $f^{-1}(y) = \sqrt{y-4}$.

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9. Consider $f : R_+ \to [-5, \infty]$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \left(\dfrac{\left(\sqrt{y+6}\right) - 1}{3}\right)$.

Ans. Consider $f : R_+ \to [-5, \infty]$ and $f(x) = 9x^2 + 6x - 5$.

Let $x_1, x_2 \in R \to [-5, \infty]$, then $f(x_1) = 9x_1^2 + 6x_1 - 5$ and $f(x_2) = 9x_2^2 + 6x_2 - 5$

Now, $f(x_1) = f(x_2)$ then $9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$

$\Rightarrow 9x_1^2 + 6x_1 = 9x_2^2 + 6x_2$

$\Rightarrow 9\left(x_1^2 - x_2^2\right) + 6\left(x_1 - x_2\right) = 0$

$\Rightarrow (x_1 - x_2)\left[9(x_1 + x_2) + 6\right] = 0$

$\Rightarrow x_1 - x_2 = 0$

$\Rightarrow x_1 = x_2 \quad \therefore f$ is one-one.

Now, again $y = 9x^2 + 6x - 5$

$\Rightarrow 9x^2 + 6x - (5 + y) = 0$

$\Rightarrow x = \frac{-6 \pm \sqrt{(6)^2 + 4 \times 9(5+y)}}{18} = \frac{-6 \pm 6\sqrt{1+5+y}}{18} = \frac{-6 \pm 6\sqrt{y+6}}{18} = \frac{\sqrt{y+6}-1}{3}$

$\therefore f(x) = f\left(\frac{\sqrt{y+6}-1}{3}\right) = 9\left(\frac{\sqrt{y+6}-1}{3}\right)^2 + 6\left(\frac{\sqrt{y+6}-1}{3}\right) - 5$

$= 9\left(\frac{y+6+1-2\sqrt{y+6}}{9}\right) + 2\left(\sqrt{y+6}-1\right) - 5$

$= y+7-2\sqrt{y+6}+2\sqrt{y+6}-2-5 = y \quad \therefore f$ is onto.

Therefore, $f(x)$ is invertible and $f^{-1}(x) = \frac{\sqrt{y+6}-1}{3}$.

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10. Let $f : X \rightarrow Y$ be an invertible function. Show that f has unique inverse.

(Hint: Suppose g_1 and g_2 are two inverses of f. Then for all $f \in Y, fog_1(y) = 1_Y(y) = fog_2(y)$. Use one-one ness of f).

Ans. Given: $f : X \rightarrow Y$ be an invertible function.

Thus f is 1 – 1 and onto and therefore f^{-1} exists.

Let g_1 and g_2 be two inverses of f. Then for all $y \in$ Y,

$fog_1(y) = I_y(y) = fog_2(y) \quad \therefore fog_1(y) = fog_2(y)$

$\Rightarrow f[g_1(y)] = f[g_2(y)]$

$\Rightarrow g_1(y) = g_2(y)$

$\therefore$ The inverse is unique and hence f has a unique inverse.

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11. Consider f : {1, 2, 3} $\rightarrow \{a, b, c\}$ given by $f(1) = a, f(2) = b$ and $f(3) = c$. Find

f^{-1} **and show that** $\left(f^{-1}\right)^{-1}=f$.

Ans. $f=\{(1,a),(2,b),(3,c)\}$, then it is clear that f is 1 – 1 and onto and therefore f^{-1} exists.

Also, $f^{-1}=\{(1,a),(b,2),(c,3)\}$ and $\left(f^{-1}\right)^{-1}=\{(1,a),(2,b),(3,c)\}=f$

Hence, $\left(f^{-1}\right)^{-1}=f$

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12. Let $f: \mathrm{X} \rightarrow \mathrm{Y}$ be an invertible function. Show that the inverse of f^{-1} is f, i.e., $\left(f^{-1}\right)^{-1}=f$.

Ans. Let $f: \mathrm{X} \rightarrow \mathrm{Y}$ be an invertible function.

Then f is one-one and onto

$\Rightarrow g: y \rightarrow$ X where g is also one-one and onto such that

$gof(x)=\mathrm{I}_x$ and $fog(y)=\mathrm{I}_y$

$\Rightarrow g=f^{-1}$

Now, $f^{-1}o\left(f^{-1}\right)^{-1}=\mathrm{I}$ and $fo\left[f^{-1}o\left(f^{-1}\right)^{-1}\right]=fo\mathrm{I}$

$\Rightarrow \left[fof^{-1}\right]o\left(f^{-1}\right)^{-1}=f$

$\Rightarrow \mathrm{I}o\left(f^{-1}\right)^{-1}=f$

$\Rightarrow \left(f^{-1}\right)^{-1}=f$

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13. If f : R $\rightarrow$ R given by $f(x)=\left(3-x^3\right)^{\frac{1}{3}}$, then $fof(x)$ is:

(A) $x^{\frac{1}{3}}$

(B) x^3

(C) x

(D) $\left(3-x^3\right)$

Ans. f : R $\rightarrow$ R and $f(x)=\left(3-x^3\right)^{\frac{1}{3}}$

$\Rightarrow f[f(x)]=\left[3-[f(x)]^3\right]^{\frac{1}{3}} = \left[3-\left\{\left(3-x^3\right)^{\frac{1}{3}}\right\}^3\right]^{\frac{1}{3}}$

$= \left[3-\left(3-x^3\right)\right]^{\frac{1}{3}} = \left(3-3+x^3\right)^{\frac{1}{3}} = x$

Therefore, option (C) is correct.

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14. Let f : R – $\left\{\frac{-4}{3}\right\} \rightarrow$ R be a function defined as $f(x)=\frac{4x}{3x+4}$. The inverse of f is the map g : Range of $f \rightarrow \mathrm{R}-\left\{\frac{-4}{3}\right\}$ given by:

(A) $g(y)=\frac{3y}{3-4y}$

(B) $g(y)=\frac{4y}{4-3y}$

(C) $g(y)=\frac{4y}{3-4y}$

(D) $g(y)=\frac{3y}{4-3y}$

Ans. Given: $f : R - \left\{\frac{-4}{3}\right\} \rightarrow$ R and $f(x) = \frac{4x}{3x+4}$

Now, Range of $f \rightarrow R - \left\{-\frac{4}{3}\right\}$

Let $y = f(x)$

$\therefore \quad y = \frac{4x}{3x+4}$

$\Rightarrow 3xy + 4y = 4x$

$\Rightarrow x(4-3y) = 4y$

$\Rightarrow x = \frac{4y}{4-3y}$

$\therefore \quad f^{-1}(y) = g(y) = \frac{4y}{3-4y}$

Therefore, option (B) is correct.