CBSE Class–12 Mathematics

NCERT solution

Chapter - 5

Continuity & Differentiability - Exercise 5.3

Find $\frac{dy}{dx}$ in the following Exercise 1 to 15.

1. $2x+3y=\sin x$

Ans. Given: $2x+3y=\sin x$

$\Rightarrow \frac{d}{dx}(2x)+\frac{d}{dx}(3y)=\frac{d}{dx}\sin x$

$\Rightarrow 2+3\frac{dy}{dx}=\cos x$

$\Rightarrow 3\frac{dy}{dx}=\cos x-2$

$\Rightarrow \frac{dy}{dx}=\frac{\cos x-2}{3}$

2. $2x+3y=\sin y$

Ans. Given: $2x+3y=\sin y$

$\Rightarrow \frac{d}{dx}(2x)+\frac{d}{dx}(3y)=\frac{d}{dx}\sin y$

$\Rightarrow 2+3\frac{dy}{dx}=\cos y\frac{dy}{dx}$

$\Rightarrow -\cos y\frac{dy}{dx}+3\frac{dy}{dx}=-2$

$$\Rightarrow -\frac{dy}{dx}(\cos y - 3) = -2$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{\cos y - 3}$$

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3. $ax + by^2 = \cos y$

Ans. Given: $ax + by^2 = \cos y$

$$\Rightarrow \frac{d}{dx}(ax) + \frac{d}{dx}(by^2) = \frac{d}{dx}\cos y$$

$$\Rightarrow a + b.2y\frac{dy}{dx} = -\sin y\frac{dy}{dx}$$

$$\Rightarrow 2by\frac{dy}{dx} + \sin y\frac{dy}{dx} = -a$$

$$\Rightarrow \frac{dy}{dx}(2by + \sin y) = -a$$

$$\Rightarrow \frac{dy}{dx} = \frac{-a}{2by + \sin y}$$

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4. $xy + y^2 = \tan x + y$

Ans. Given: $xy + y^2 = \tan x + y$

$$\Rightarrow \frac{d}{dx}(xy) + \frac{d}{dx}(y^2) = \frac{d}{dx}\tan x + \frac{d}{dx}y$$

$$\Rightarrow x\frac{d}{dx}y + y\frac{d}{dx}x + 2y\frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$$ [By Product Rule]

$$\Rightarrow x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$$

$\Rightarrow x\frac{dy}{dx}+2y\frac{dy}{dx}-\frac{dy}{dx}=\sec^2 x-y$

$\Rightarrow (x+2y-1)\frac{dy}{dx}=\sec^2 x-y$

$\Rightarrow \frac{dy}{dx}=\frac{\sec^2 x-y}{x+2y-1}$

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5. $x^2+xy+y^2=100$

Ans. Given: $x^2+xy+y^2=100$

$\Rightarrow \frac{d}{dx}x^2+\frac{d}{dx}xy+\frac{d}{dx}y^2=\frac{d}{dx}100$

$\Rightarrow 2x+\left(x\frac{d}{dx}y+y\frac{d}{dx}x\right)+2y\frac{dy}{dx}=0$

$\Rightarrow 2x+x\frac{dy}{dx}+y+2y\frac{dy}{dx}=0$

$\Rightarrow (x+2y)\frac{dy}{dx}=-2x-y$

$\Rightarrow \frac{dy}{dx}=\frac{-(2x+y)}{x+2y}$

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6. $x^3+x^2y+xy^2+y^3=81$

Ans. Given: $x^3+x^2y+xy^2+y^3=81$

$\Rightarrow \frac{d}{dx}x^3+\frac{d}{dx}x^2y+\frac{d}{dx}xy^2+\frac{d}{dx}y^3=\frac{d}{dx}81$

$\Rightarrow 3x^2+\left(x^2\frac{dy}{dx}+y.\frac{d}{dx}x^2\right)+x\frac{d}{dx}y^2+y^2\frac{d}{dx}x+3y^2\frac{dy}{dx}=0$

$\Rightarrow 3x^2 + x^2\frac{dy}{dx} + y.2x + x.2y\frac{dy}{dx} + y^2.1 + 3y^2\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx}\left(x^2 + 2xy + 3y^2\right) = -3x^2 - 2xy - y^2$

$\Rightarrow \frac{dy}{dx} = \frac{-\left(3x^2+2xy+y^2\right)}{x^2+2xy+3y^2}$

7. $\sin^2 y + \cos xy = \pi$

Ans. Given: $\sin^2 y + \cos xy = \pi$

$\Rightarrow \frac{d}{dx}(\sin y)^2 + \frac{d}{dx}\cos xy = \frac{d}{dx}(\pi)$

$\Rightarrow 2\sin y\frac{d}{dx}\sin y - \sin xy\frac{d}{dx}(xy) = 0$

$\Rightarrow 2\sin y\cos y\frac{dy}{dx} - \sin xy\left(x\frac{dy}{dx} + y.1\right) = 0$

$\Rightarrow \sin 2y\frac{dy}{dx} - x\sin xy\frac{dy}{dx} - y\sin xy = 0$

$\Rightarrow (\sin 2y - x\sin xy)\frac{dy}{dx} = y\sin xy$

$\Rightarrow \frac{dy}{dx} = \frac{y\sin xy}{\sin 2y - x\sin xy}$

8. $\sin^2 x + \cos^2 y = 1$

Ans. Given: $\sin^2 x + \cos^2 y = 1$

$\implies \frac{d}{dx}(sin^2x) + \frac{d}{dx}(cos^2y) = \frac{d}{dx}(1)$

$$\Rightarrow 2\sin x\frac{d}{dx}\sin x+2\cos y\frac{d}{dx}\cos y=0$$

$$\Rightarrow 2\sin x\cos x+2\cos y\left(-\sin y\frac{dy}{dx}\right)=0$$

$$\Rightarrow \sin 2x-\sin 2y\frac{dy}{dx}=0$$

$$\Rightarrow -\sin 2y\frac{dy}{dx}=-\sin 2x$$

$$\Rightarrow \frac{dy}{dx}=\frac{\sin 2x}{\sin 2y}$$

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9. $y=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Ans. Given: $y=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

To simplify the given Inverse Trigonometric function, putting $x=\tan\theta$

$$\Rightarrow y=\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)=\sin^{-1}(\sin 2\theta)=2\theta$$

$$\Rightarrow y=2\tan^{-1}x$$

$$\Rightarrow \frac{dy}{dx}=2.\frac{1}{1+x^2}=\frac{2}{1+x^2}$$

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10. $y=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), \frac{-1}{\sqrt{3}}<x<\frac{1}{\sqrt{3}}$

Ans. Given: $y=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), \frac{-1}{\sqrt{3}}<x<\frac{1}{\sqrt{3}}$

To simplify the given Inverse Trigonometric function, putting $x = \tan\theta$

$$\Rightarrow\ y = \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right)$$

$$\Rightarrow\ y = \tan^{-1}(\tan 3\theta) = 3\theta$$

$$\Rightarrow\ y = 3\tan^{-1}x$$

$$\Rightarrow\ \frac{dy}{dx} = 3.\frac{1}{1+x^2} = \frac{3}{1+x^2}$$

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11. $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$

Ans. Given: $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$

Putting $x = \tan\theta$

$$y = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$= \cos^{-1}(\cos 2\theta) = 2\theta = 2\tan^{-1}x$$

$$\therefore\ \frac{dy}{dx} = 2.\frac{1}{1+x^2} = \frac{2}{1+x^2}$$

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12. $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$

Ans. Given: $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$

Putting $x = \tan\theta$

$y=\sin^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$=\sin^{-1}(\cos 2\theta)$

$=\sin^{-1}\sin\left(\frac{\pi}{2}-2\theta\right)=\frac{\pi}{2}-2\theta$

$=\frac{\pi}{2}-2\tan^{-1}x$

$\therefore\ \frac{dy}{dx}=0-2.\frac{1}{1+x^2}=\frac{-2}{1+x^2}$

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13. $y=\cos^{-1}\left(\frac{2x}{1+x^2}\right), -1<x<1$

Ans. Given: $y=\cos^{-1}\left(\frac{2x}{1+x^2}\right), -1<x<1$

Putting $x=\tan\theta$

$y=\cos^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$=\cos^{-1}(\sin 2\theta)$

$=\cos^{-1}\cos\left(\frac{\pi}{2}-2\theta\right)=\frac{\pi}{2}-2\theta$

$=\frac{\pi}{2}-2\tan^{-1}x$

$\therefore\ \frac{dy}{dx}=0-2.\frac{1}{1+x^2}=\frac{-2}{1+x^2}$

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14. $y=\sin^{-1}\left(2x\sqrt{1-x^2}\right), \frac{-1}{\sqrt{2}}<x<\frac{1}{\sqrt{2}}$

Ans. Given: $y=\sin^{-1}\left(2x\sqrt{1-x^2}\right), -\frac{1}{\sqrt{2}}<x<\frac{1}{\sqrt{2}}$

Putting $x=\sin\theta$

$y=\sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$

= $\sin^{-1}\left(2\sin\theta\sqrt{\cos^2\theta}\right)$

= $\sin^{-1}\left(2\sin\theta\cos\theta\right)$

= $\sin^{-1}(\sin 2\theta)=2\theta=2\sin^{-1}x$

$\therefore\ \frac{dy}{dx}=2.\frac{1}{\sqrt{1-x^2}}=\frac{2}{\sqrt{1-x^2}}$

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15. $y=\sec^{-1}\left(\frac{1}{2x^2-1}\right), 0<x<\frac{1}{\sqrt{2}}$

Ans. Given: $y=\sec^{-1}\left(\frac{1}{2x^2-1}\right), 0<x<\frac{1}{\sqrt{2}}$

Putting $x=\cos\theta$

$y=\sec^{-1}\left(\frac{1}{2\cos^2\theta-1}\right)$

= $y=\sec^{-1}\left(\frac{1}{\cos 2\theta}\right)$

= $\sec^{-1}(\sec 2\theta)$

= $2\theta=2\cos^{-1}x$

$\therefore\ \frac{dy}{dx}=2.\frac{-1}{\sqrt{1-x^2}}=\frac{-2}{\sqrt{1-x^2}}$