CBSE Class–10 Mathematics

NCERT solution

Chapter - 8

Introduction to Trigonometry - Exercise 8.4

1. Express the trigonometric ratios $\sin A, \sec A$ and $\tan A$ in terms of $\cot A$.

Ans. For $\sin A$,

By using identity $\operatorname{cosec}^2 A - \cot^2 A = 1$

$$\Rightarrow \operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\Rightarrow \frac{1}{\sin^2 A} = 1 + \cot^2 A$$

$$\Rightarrow \sin^2 A = \frac{1}{1 + \cot^2 A}$$

$$\Rightarrow \sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$$

For $\sec A$,

By using identity $\sec^2 A - \tan^2 A = 1$

$$\Rightarrow \sec^2 A = 1 + \tan^2 A$$

$$\Rightarrow \sec^2 A = 1 + \frac{1}{\cot^2 A} = \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\Rightarrow \sec^2 A = \frac{1 + \cot^2 A}{\cot^2 A}$$

$$\Rightarrow \sec A = \frac{\sqrt{1 + \cot^2 A}}{\cot A}$$

For $\tan A$,

$$\tan A = \frac{1}{\cot A}$$

2. Write the other trigonometric ratios of A in terms of $\sec A$.

Ans. For $\sin A$,

By using identity, $\sin^2 A + \cos^2 A = 1$

$$\Rightarrow \sin^2 A = 1 - \cos^2 A$$

$$\Rightarrow \sin^2 A = 1 - \frac{1}{\sec^2 A} = \frac{\sec^2 A - 1}{\sec^2 A}$$

$$\Rightarrow \sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

For $\cos A$,

$$\cos A = \frac{1}{\sec A}$$

For $\tan A$,

By using identity $\sec^2 A - \tan^2 A = 1$

$$\Rightarrow \tan^2 A = \sec^2 A - 1$$

$$\Rightarrow \tan A = \sqrt{\sec^2 A - 1}$$

For $\cos ecA$,

$$\cos ecA = \frac{1}{\sin A} = \frac{1}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}$$

$$\Rightarrow \cos ecA = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

For $\cot A$,

$$\cot A = \frac{1}{\tan A}$$

$$\Rightarrow \cot A = \frac{1}{\sqrt{\sec^2 A - 1}}$$

3. Evaluate:

(i) $\dfrac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

(ii) $\sin 25° \cos 65° + \cos 25° \sin 65°$

Ans. (i) $\dfrac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

$$= \frac{\sin^2 63° + \sin^2 (90° - 63°)}{\cos^2 (90° - 73°) + \cos^2 73°}$$

$$= \frac{\sin^2 63° + \cos^2 63°}{\sin^2 73° + \cos^2 73°}$$

$$\left[\because \sin(90° - \theta) = \cos\theta, \cos(90° - \theta) = \sin\theta\right]$$

$$= \frac{1}{1} = 1 \quad \left[\because \sin^2\theta + \cos^2\theta = 1\right]$$

(ii) $\sin 25° \cos 65° + \cos 25° \sin 65°$

$$= \sin 25°.\cos(90° - 25°) + \cos 25°.\sin(90° - 25°)$$

$$= \sin 25°.\sin 25° + \cos 25°.\cos 25°$$

$\left[\because \sin\left(90^\circ-\theta\right)=\cos\theta, \cos\left(90^\circ-\theta\right)=\sin\theta\right]$

= $\sin^2 25^\circ+\cos^2 25^\circ$ = 1

$\left[\because \sin^2\theta+\cos^2\theta=1\right]$

4. Choose the correct option. Justify your choice:

(i) $9\sec^2 A-9\tan^2 A$ =

(A) 1

(B) 9

(C) 8

(D) 0

(ii) $\left(1+\tan\theta+\sec\theta\right)\left(1+\cot\theta-\operatorname{cosec}\theta\right)$ =

(A) 0

(B) 1

(C) 2

(D) none of these

(iii) $\left(\sec A+\tan A\right)\left(1-\sin A\right)$ =

(A) $\sec A$

(B) $\sin A$

(C) $\operatorname{cosec} A$

(D) $\cos A$

(iv) $\frac{1+\tan^2 A}{1+\cot^2 A}$ =

(A) $\sec^2 A$

(B) -1

(C) $\cot^2 A$

(D) none of these

Ans. (i) (B) $9\sec^2 A - 9\tan^2 A$

= $9\left(\sec^2 A - \tan^2 A\right)$

= $9\times 1 = 9$ [Since $\sec^2\theta - \tan^2\theta = 1$]

(ii) (C) $\left(1+\tan\theta+\sec\theta\right)\left(1+\cot\theta-\operatorname{cosec}\theta\right)$

= $\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$

= $\left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$

= $\frac{\left(\cos\theta+\sin\theta\right)^2-(1)^2}{\cos\theta.\sin\theta}$

= $\frac{\cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta-1}{\cos\theta.\sin\theta}$

= $\frac{1+2\cos\theta\sin\theta-1}{\cos\theta.\sin\theta}$

$\left[\because \sin^2\theta+\cos^2\theta=1\right]$

= $\frac{2\cos\theta\sin\theta}{\cos\theta.\sin\theta}$ = 2

(iii)(D) $(\sec A+\tan A)(1-\sin A)$

$= \left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A)$

$= \left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)$

$= \frac{1-\sin^2 A}{\cos A}$ [Since $(a+b)(a-b)=a^2-b^2$]

$= \frac{\cos^2 A}{\cos A}$

$= \cos A \left[\because 1-\sin^2 A=\cos^2 A\right]$

(iv)(D) $\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sec^2 A-\tan^2 A+\tan^2 A}{\operatorname{cosec}^2 A-\cot^2 A+\cot^2 A}$

$= \frac{\sec^2 A}{\operatorname{cosec}^2 A} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$

$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$

5. Prove the following identities, where the angles involved are acute angles for which the expressions are defined:

(i) $(\operatorname{cosec}\theta-\cot\theta)^2=\frac{1-\cos\theta}{1+\cos\theta}$

(ii) $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$

(iii) $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\,\mathrm{cosec}\,\theta$

(iv) $\frac{1+\sec A}{\sec A}=\frac{\sin^2 A}{1-\cos A}$

(v) $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\mathrm{cosec}\,A+\cot A$, **using the identity** $\mathrm{cosec}^2 A=1+\cot^2 A$

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}}=\sec A+\tan A$

(vii) $\frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}=\tan\theta$

(viii) $(\sin A+\mathrm{cosec}\,A)^2+(\cos A+\sec A)^2$

$=7+\tan^2 A+\cot^2 A$

(ix) $(\mathrm{cosec}\,A-\sin A)(\sec A-\cos A)$

$=\frac{1}{\tan A+\cot A}$

(x) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\frac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A$

Ans. (i) L.H.S. $(\mathrm{cosec}\,\theta-\cot\theta)^2$

= $\mathrm{cosec}^2\theta+\cot^2\theta-2\,\mathrm{cosec}\,\theta\cot\theta$ [Since $(a-b)^2=a^2+b^2-2ab$]

= $\frac{1}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}-2\times\frac{1}{\sin\theta}.\frac{\cos\theta}{\sin\theta}$

$$= \frac{1+\cos^2\theta}{\sin^2\theta} - \frac{2\cos\theta}{\sin^2\theta}$$

$$= \frac{1+\cos^2\theta - 2\cos\theta}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta} \quad \left[\because a^2+b^2-2ab=(a-b)^2\right]$$

$$= \frac{(1-\cos\theta)(1-\cos\theta)}{1-\cos^2\theta}$$

$$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{1-\cos\theta}{1+\cos\theta} = \text{R.H.S.}$$

(ii) L.H.S. $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$

$$= \frac{\cos^2\theta + 1 + \sin^2\theta + 2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{\cos^2\theta + \sin^2\theta + 1 + 2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{1+1+2\sin A}{(1+\sin A)\cos A} \quad \left[\because \sin^2\theta + \cos^2\theta = 1\right]$$

$$= \frac{2+2\sin A}{(1+\sin A)\cos A} = \frac{2(1+\sin A)}{(1+\sin A)\cos A}$$

$$= \frac{2}{\cos A} = 2\sec A = \text{R.H.S}$$

(iii) L.H.S. $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}$

$$=\frac{\dfrac{\sin\theta}{\cos\theta}}{1-\dfrac{\cos\theta}{\sin\theta}}+\frac{\dfrac{\cos\theta}{\sin\theta}}{1-\dfrac{\sin\theta}{\cos\theta}}$$

$$=\frac{\dfrac{\sin\theta}{\cos\theta}}{\dfrac{\sin\theta-\cos\theta}{\sin\theta}}+\frac{\dfrac{\cos\theta}{\sin\theta}}{\dfrac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$=\frac{\sin\theta}{\cos\theta}\times\frac{\sin\theta}{\sin\theta-\cos\theta}+\frac{\cos\theta}{\sin\theta}\times\frac{\cos\theta}{\cos\theta-\sin\theta}$$

$$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$=\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$$

$$=\frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$$

$$\left[\because a^3-b^3=(a-b)(a^2+b^2+ab)\right]$$

$$=\frac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}\left[\because \sin^2\theta+\cos^2\theta=1\right]$$

$$=\frac{1}{\sin\theta\cos\theta}+\frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$=\frac{1}{\sin\theta\cos\theta}+1=1+\frac{1}{\sin\theta\cos\theta}$$

$$=1+\sec\theta\cos ec\theta$$

(iv) L.H.S. $\frac{1+\sec A}{\sec A} = \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}} = \frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}}$

$= \frac{\cos A+1}{\cos A} \times \frac{\cos A}{1} = 1+\cos A$

$= 1+\cos A \times \frac{1-\cos A}{1-\cos A}$

$= \frac{1-\cos^2 A}{1-\cos A}$ [Since $(a+b)(a-b) = a^2 - b^2$]

$= \frac{\sin^2 A}{1-\cos A}$ = R.H.S.

(v) L.H.S. $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}$

Dividing all terms by $\sin A$,

$= \frac{\cot A-1+\cos ecA}{\cot A+1-\cos ecA} = \frac{\cot A+\cos ecA-1}{\cot A-\cos ecA+1}$

$= \frac{(\cot A+\cos ecA)-(\cos ec^2 A-\cot^2 A)}{(1+\cot A-\cos ecA)}$ [Since $\cos ec^2\theta - \cot^2\theta = 1$]

$= \frac{(\cot A+\cos ecA)+(\cot^2 A-\cos ec^2 A)}{(1+\cot A-\cos ecA)}$

$= \frac{(\cot A+\cos ecA)+(\cot A+\cos ecA)(\cot A-\cos ecA)}{(1+\cot A-\cos ecA)}$

$= \frac{(\cot A+\cos ecA)(1+\cot A-\cos ecA)}{(1+\cot A-\cos ecA)}$

$= \cot A+\cos ecA$ = R.H.S.

(vi) L.H.S. $\sqrt{\frac{1+\sin A}{1-\sin A}}$

$= \sqrt{\frac{1+\sin A}{1-\sin A}} \times \sqrt{\frac{1+\sin A}{1+\sin A}}$

$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}} \left[\because (a+b)(a-b) = a^2 - b^2\right]$

$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} \left[\because 1-\sin^2\theta = \cos^2\theta\right]$

$= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$

$= \sec A + \tan A$ = R.H.S.

(vii) L.H.S. $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$= \frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(2\cos^2\theta - 1\right)}$

$= \frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left[2\left(1-\sin^2\theta\right)-1\right]}$

$\left[\because 1-\sin^2\theta = \cos^2\theta\right]$

$= \frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(2-2\sin^2\theta-1\right)}$

$= \frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(1-2\sin^2\theta\right)} = \frac{\sin\theta}{\cos\theta}$

= $\tan\theta$ = R.H.S

(viii) L.H.S. $(\sin A+\operatorname{cosec}A)^2+(\cos A+\sec A)^2$

$$= \left(\sin A+\frac{1}{\sin A}\right)^2+\left(\cos A+\frac{1}{\cos A}\right)^2$$

$$= \sin^2 A+\frac{1}{\sin^2 A}+2.\sin A.\frac{1}{\sin A}+\cos^2 A+\frac{1}{\cos^2 A}+2.\cos A.\frac{1}{\cos A}$$

$$= 2+2+\sin^2 A+\cos^2 A+\frac{1}{\sin^2 A}+\frac{1}{\cos^2 A}$$

$$= 4+1+\frac{1}{\sin^2 A}+\frac{1}{\cos^2 A}$$

$$= 5+\operatorname{cosec}^2 A+\sec^2 A$$

$$= 5+1+\cot^2 A+1+\tan^2 A$$

$$\left[\because \operatorname{cosec}^2\theta=1+\cot^2\theta, \sec^2\theta=1+\tan^2\theta\right]$$

$$= 7+\tan^2 A+\cot^2 A$$

= R.H.S.

(ix) L.H.S. $(\operatorname{cosec}A-\sin A)(\sec A-\cos A)$

$$= \left(\frac{1}{\sin A}-\sin A\right)\left(\frac{1}{\cos A}-\cos A\right)$$

$$= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)$$

$$= \frac{\cos^2 A}{\sin A}\times\frac{\sin^2 A}{\cos A} = \sin A.\cos A$$

$= \dfrac{\sin A.\cos A}{\sin^2 A + \cos^2 A}$ $\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

Dividing all the terms by $\sin A.\cos A$,

$$= \frac{\dfrac{\sin A.\cos A}{\sin A.\cos A}}{\dfrac{\sin^2 A}{\sin A.\cos A} + \dfrac{\cos^2 A}{\sin A.\cos A}}$$

$$= \frac{1}{\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}}$$

$= \dfrac{1}{\tan A + \cot A}$ = R.H.S.

(x) L.H.S. $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right) = \dfrac{\sec^2 A}{\operatorname{cosec}^2 A}$

$\left[\because 1+\tan^2\theta = \sec^2\theta, 1+\cot^2\theta = \operatorname{cosec}^2\theta\right]$

$= \dfrac{1}{\cos^2 A} \times \dfrac{\sin^2 A}{1} = \tan^2 A$ = R.H.S.

Now, Middle side = $\left(\dfrac{1-\tan A}{1-\cot A}\right)^2 = \left(\dfrac{1-\tan A}{1-\dfrac{1}{\tan A}}\right)^2$

$$= \left(\frac{1-\tan A}{\dfrac{\tan A - 1}{\tan A}}\right)^2$$

$= \left(\dfrac{1-\tan A}{\dfrac{-(1-\tan A)}{\tan A}}\right)^2 = (-\tan A)^2$

$= \tan^2 A$ = R.H.S.